



UCLA Mani L. Bhaumik Institute
for Theoretical Physics

Leading Colour Two-Loop QCD Corrections for Three-Photon Production at Hadron Colliders

in collaboration with Ben Page, Evgenij Pascual, Vasily Sotnikov

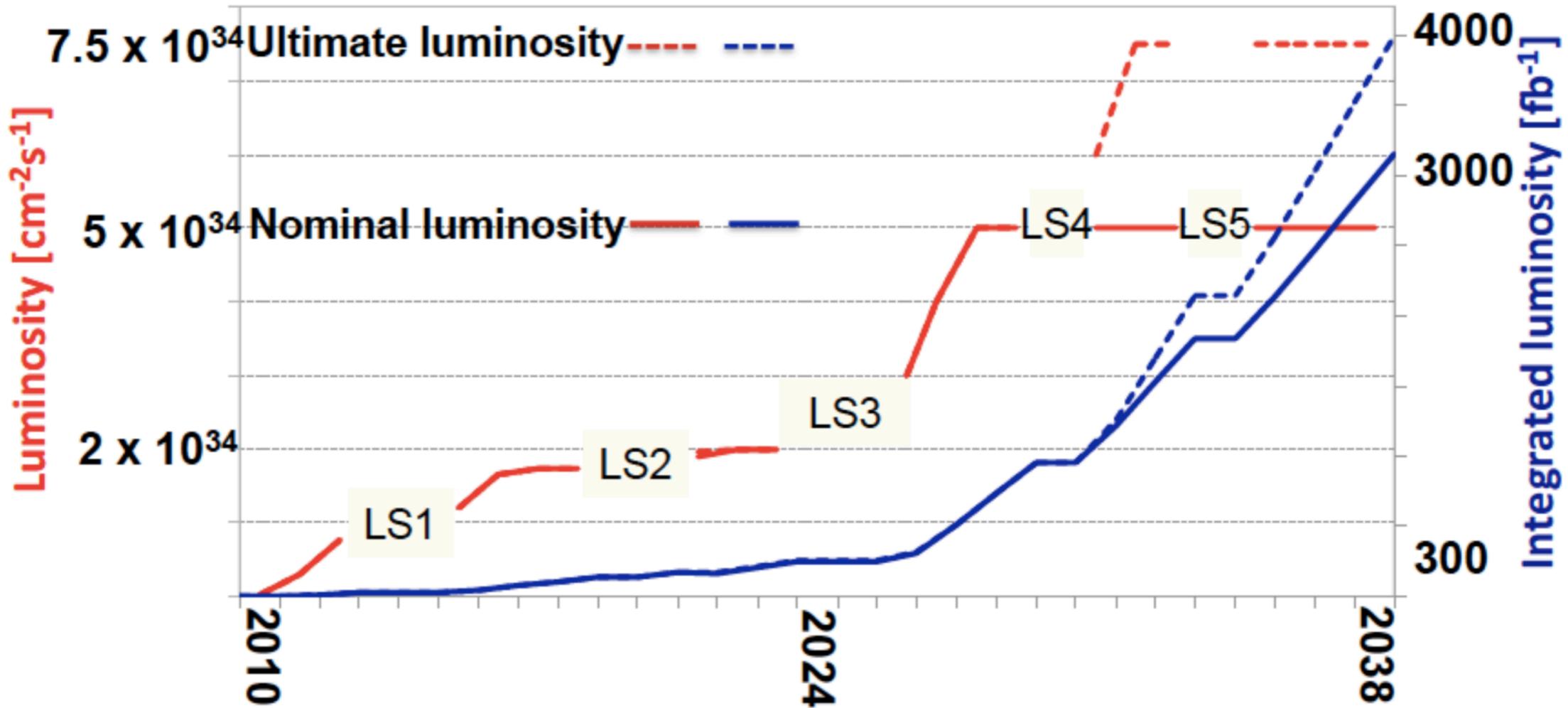
Samuel Abreu
CERN/Bhaumik fellow

IPPP - Durham — 22nd of October 2020

Outline

- ◆ Motivation: Two-Loop Multi-Leg Amplitudes
- ◆ Two-Loop numerical unitarity
- ◆ Two-Loop $q\bar{q} \rightarrow 3\gamma$ Amplitudes

Motivation



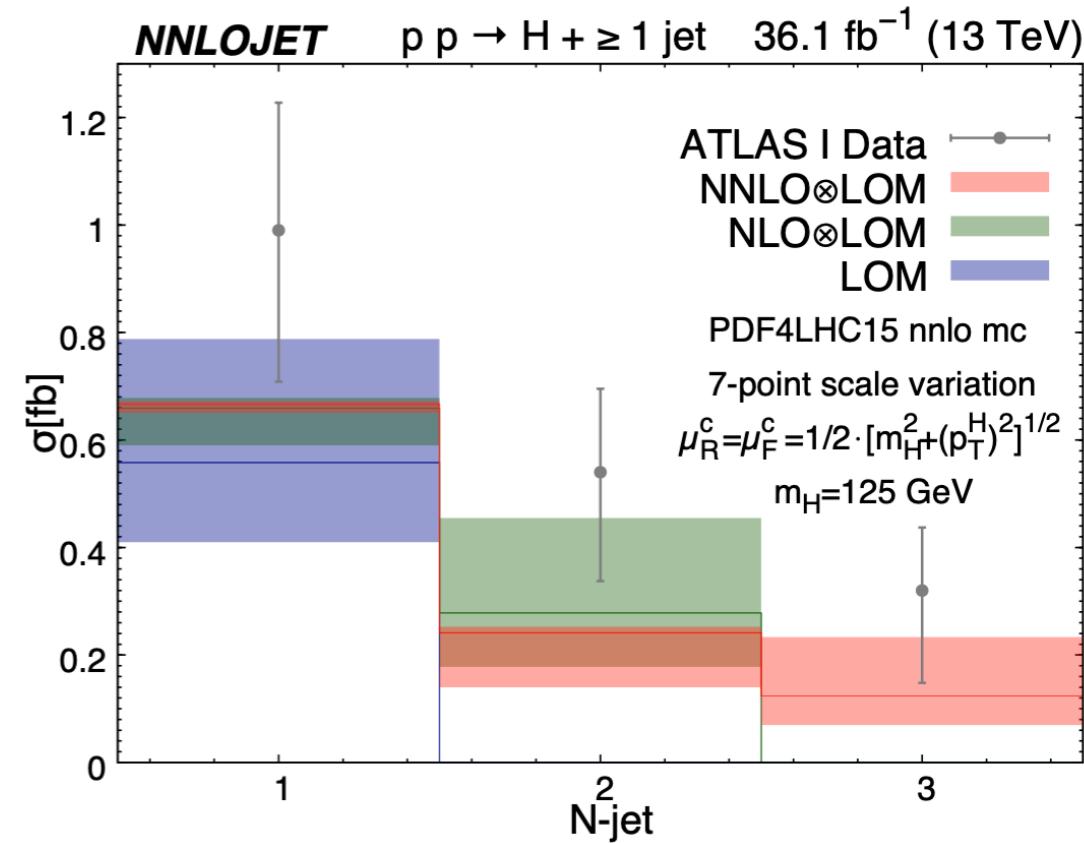
The LHC will still be running for many years, probing the Standard Model at the percent level for many observables: **theory must keep up with experiments**

Motivation

Exciting times for precision QCD!

Precise theoretical predictions required to match precise measurements

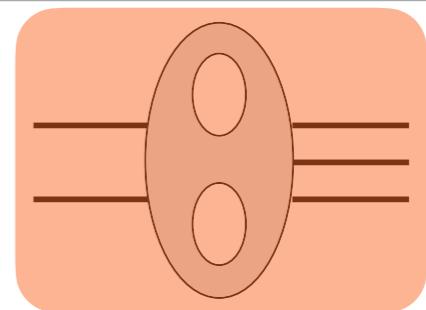
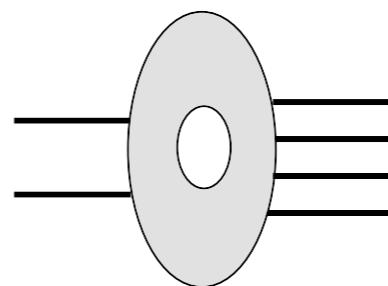
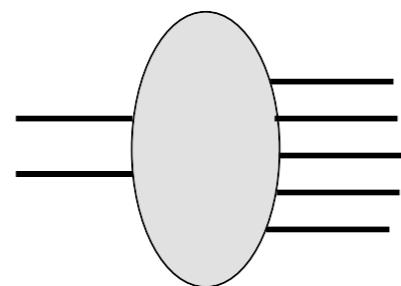
- ◆ 20 times more data to be delivered
- ◆ Multi-leg and multi-loop amplitudes are needed
- ◆ State of the art: 2-to-2 processes at 2 loops
- ◆ Need for 2-loop accuracy for 2-to-3 processes:
 - ✓ 3- γ
 - ✓ 3-jets
 - ✓ Vector/Higgs boson in association with two jets
 - ✓ ...



JHEP 1907 (2019) 052, X. Chen et al

Two-loop (five-point) amplitudes

Cross-section contributions:

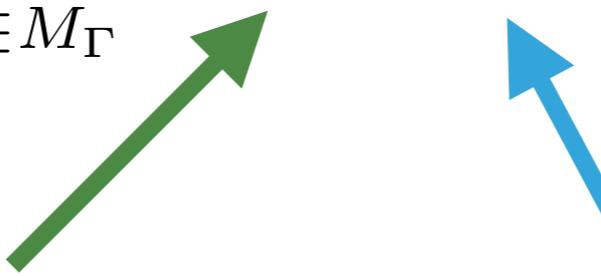


$$\mathcal{A} = \sum_{\Gamma} \sum_{i \in M_{\Gamma}} c_{\Gamma,i}(D) I_{\Gamma,i}(D)$$

Master coefficients:

process specific

Rational (algebraic)
functions



Master integrals:
depend only on the
kinematics

Special functions
(polylogarithms, elliptic, ...)

- ◆ Important contribution to NNLO corrections
- ◆ Laboratory for complex multi-scale calculation
- ◆ Analytic results: fast, stable and flexible
- ◆ Study interesting mathematical properties

Standard Modern Approach

Step 1: From Feynman rules to form-factors

- ▶ QGRAF/FeynArts
- ▶ Projectors onto form-factors

Step 2: reduction to master integrands

- ▶ Integration-by-parts (IBP) relations: main bottleneck!
- ▶ //Simplify: much more compact expression than previous steps

Step 3: insert expressions for master integrals

- ▶ Differential equations [Kotikov 91; Gehrmann, Remiddi 01; Henn 13]
- ▶ Direct integration [HyperInt, ...]
- ▶ Numerical integration [pySecDec, Fiesta, diffexp, ...]

Two-loop five-point massless amplitudes: this approach struggles

- ▶ Integrand is large and hard to manipulate
- ▶ IBPs are hard to get [H. A. Chawdhry, M. A. Lim, A. Mitov 18] and nearly impossible to use (22GB of compressed text file)

- ◆ When analytics are too hard, there is a lot (all??) to be learnt from numerical evaluations
- ◆ Ansatzt our way until the final result
 - ✓ Coefficients of amplitudes
 - ✓ Differential equations for master integrals

Two-Loop Numerical Unitarity

Two-loop (numerical) unitarity

[Abreu, Febres Cordero, Ita, Jaquier, Page, Zeng, 17,18,19]

- ◆ Computing the amplitude, $D = 4 - 2\epsilon$

$$\mathcal{A} = \sum_{\Gamma} \sum_{i \in M_{\Gamma}} c_{\Gamma,i}(D) I_{\Gamma,i}(D)$$

- ◆ Decomposition of integrand

$$\mathcal{A}(\ell_l) = \sum_{\Gamma} \sum_{i \in \textcolor{red}{M}_{\Gamma} \cup \textcolor{blue}{S}_{\Gamma}} c_{\Gamma,i}(D) \frac{m_{\Gamma,i}(\ell_l, D)}{\prod_{j \in P_{\Gamma}} \rho_j}$$

- ✓ Dimension $|\textcolor{red}{M}_{\Gamma} \cup \textcolor{blue}{S}_{\Gamma}|$ simple to determine
- ✓ Construct surface $\textcolor{blue}{S}_{\Gamma}$ terms that integrate to zero
- ✓ Compute coefficients $c_{\Gamma,i}$ from generalised unitarity

Two-loop (numerical) unitarity — surface terms I

10

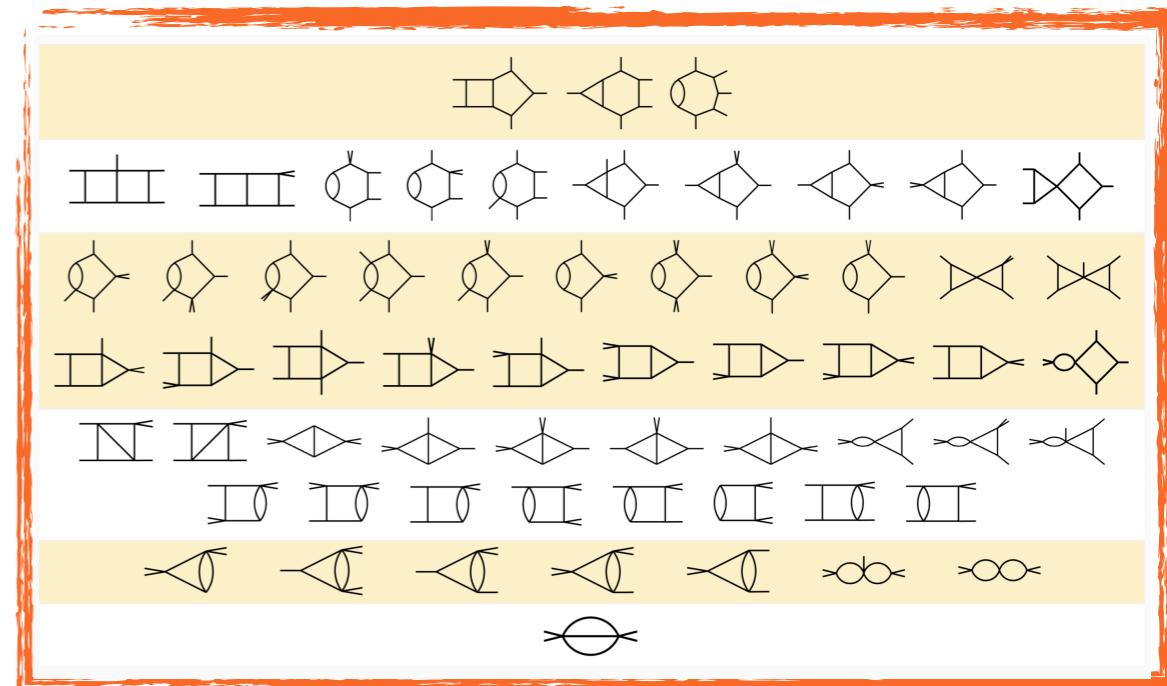
[Ita 15; Abreu, Febres Cordero, Ita, Jaquier, Page, Zeng 17]

- ◆ For each Γ , construct numerators such that

$$\int [d^D \ell_l] \frac{m_{\Gamma,i}(\ell_l, D)}{\prod_{j \in P_\Gamma} \rho_j} = 0$$

- ◆ “Integration-by-parts” IBP relations ...

$$\int [d^D \ell_l] \frac{\partial}{\partial \ell_i^\nu} \left[\frac{u_i^\nu}{\prod_{k \in P_\Gamma} \rho_k} \right] = 0$$



- ◆ ... that don't increase propagator power

$$u_i^\nu \frac{\partial}{\partial \ell_i^\nu} \rho_j = f_j \rho_j$$

[Gluza, Kajda, Kosower 10;
Schabinger 11]

- ✓ Compute the u_i^ν by solving syzygy equations

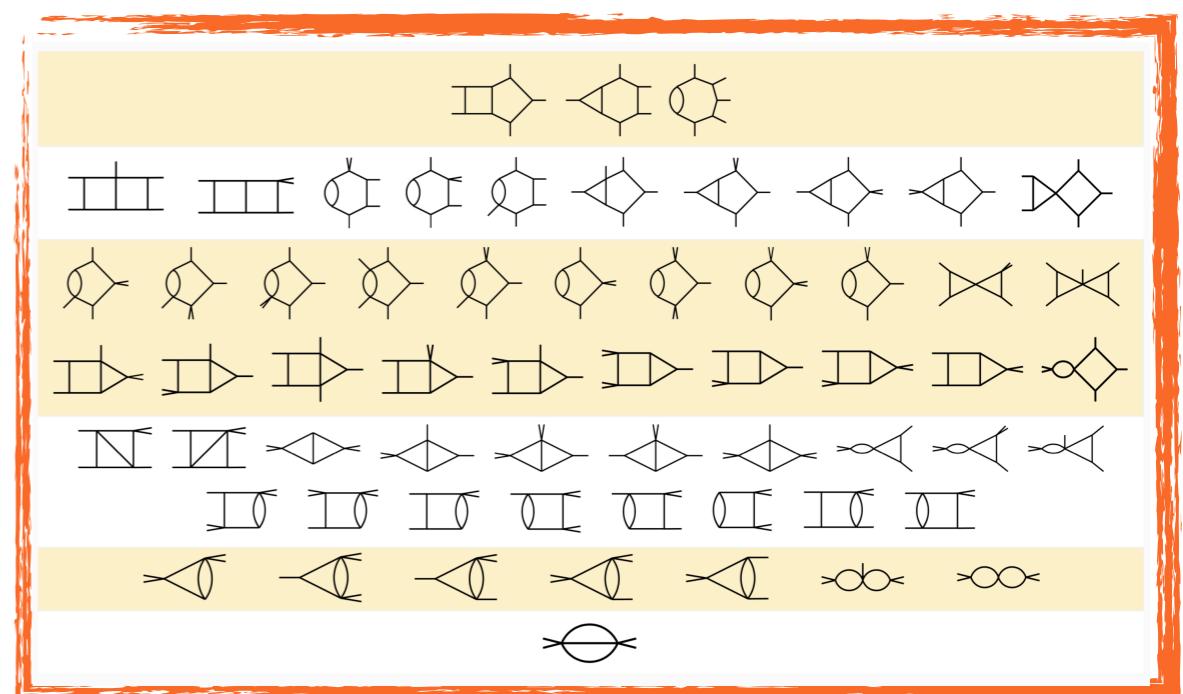
see also [Boehm, Georgoudis, Larsen, Schulze, Zhang, ... 16 - 19]
and [Agarwal, von Manteuffel 19]

Two-loop (numerical) unitarity — surface terms II

[Ita 15; Abreu, Febres Cordero, Ita, Jaquier, Page, Zeng 17,18]

- ◆ Surface terms with u_i^ν and $t(\ell_l)$: $m_{\Gamma,k} = u_i^\nu \frac{\partial t(\ell_l)}{\partial \ell_i^\nu} + t(\ell_l) \left(\frac{\partial u_i^\nu}{\partial \ell_i^\nu} - \sum_{k \in P_\Gamma} f_k \right)$
 - ◆ Linear algebra: independent surface terms, determine $|S_\Gamma|$, infer $|M_\Gamma|$
 - ◆ u_i^ν are power-counting/theory independent

Analytic decomposition of integrand into master integrands and surface terms



Two-loop numerical unitarity — coefficients

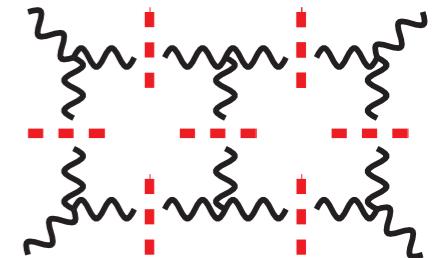
12

$$\mathcal{A}(\ell_l) = \sum_{\Gamma} \sum_{i \in M_{\Gamma} \cup S_{\Gamma}} c_{\Gamma,i}(D) \frac{m_{\Gamma,i}(\ell_l, D)}{\prod_{j \in P_{\Gamma}} \rho_j}$$

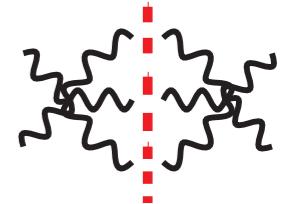
[Bern, Dixon, Dunbar, Kosower 94,95]

- ♦ In on-shell configuration of ℓ_l , integrand factorises

$$\sum_{\text{states}} \prod_{k \in T_{\Gamma}} \mathcal{A}_k^{\text{tree}}(\ell_l^{\Gamma}) = \sum_{\Gamma' \geq \Gamma} \frac{c_{\Gamma'} m_{\Gamma'}(\ell_l^{\Gamma})}{\prod_{j \in (P_{\Gamma'} \setminus P_{\Gamma})} \rho_j(\ell_l^{\Gamma})}$$



- ♦ Need efficient computation of tree amplitudes (numerical)



✓ Berends-Giele recursion [Berends, Giele, 88]

✓ Dimensional dependence: decomposition by particle content

[Anger, Sotnikov 18; Abreu et al. 19; Sotnikov 19]

- ♦ Solve cut equations numerically: never construct analytic integrand

Compute coefficients at a
numerical phase-space point

Master Integrals and Pentagon Functions

13

$$\mathcal{A} = \sum_{\Gamma} \sum_{i \in M_{\Gamma}} c_{\Gamma,i}(\epsilon, D_s, \vec{x}) I_{\Gamma,i}$$

- ◆ $I_{\Gamma,i}$ are linear combinations of **multiple polylogarithms (MPLs)**
 - ✓ Complicated multivalued functions
 - ✓ Rich algebraic structure: find **basis in this space of functions**

$$I_j(\epsilon, \vec{x}) = \sum_{i \in B} \sum_{k=-4} \epsilon^k d_{j,i,k} h_i(\vec{x})$$

- ◆ Classified for **five-point massless amplitudes: pentagon functions**
 - ✓ Analytic expressions known [Gehrmann, Henn, Lo Presti 18]
[D. Chicherin, V. Sotnikov 20]
 - ✓ Libraries for efficient numerical evaluation

Compute amplitude at a
numerical phase-space point

Two-loop numerical unitarity — analytics

14

$$\mathcal{A} = \sum_{\Gamma} \sum_{i \in M_{\Gamma}} c_{\Gamma,i}(\epsilon, D_s, \vec{x}) I_{\Gamma,i}$$

[Peraro, 16]

[Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov 18, 19]

- ◆ Rational dependence on all variables: compute in **finite field**

- ✓ No loss of precision in numerical manipulations

[von Manteuffel, Schabinger, 15], [Peraro, 16]

- ◆ Analytic ϵ and D_s dependence

- ✓ Compute at enough values to determine rational function

- ◆ Reconstruct analytic form of **finite remainder** ($D_s = 4 - 2\epsilon$)

- ✓ Remove infrared $\mathcal{R}^{(2)} = \mathcal{A}^{(2)} - \mathbf{I}_1 \mathcal{A}^{(1)} - \mathbf{I}_2 \mathcal{A}^{(0)}$

- ✓ Decomposition in terms of **pentagon functions**

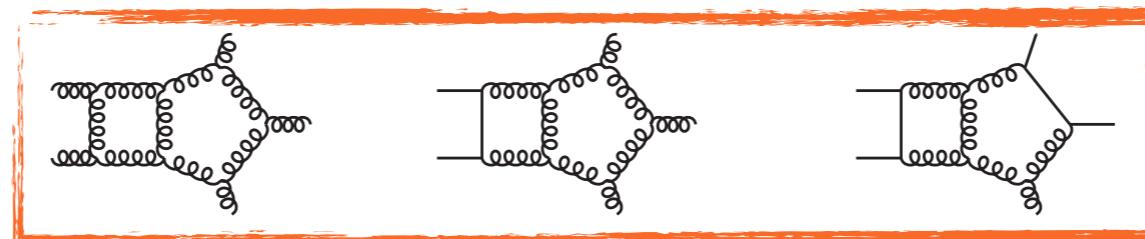
$$\mathcal{R}^{(2)} = \sum_i d_i(\vec{x}) h_i(\vec{x})$$

Compute **analytic amplitude** from numerics

- ◆ Fully automated in C++ framework [Abreu et al., 20]



- ◆ Growing list of amplitudes computed
 - ✓ 4-parton amplitudes: testing ground [Abreu et al., 17]
 - ✓ 5-parton amplitudes: major target in the community [Abreu et al.; 18, 19]
 - ✓ 4-graviton amplitudes: non-planar, very complicated integrand [Abreu et al., 20]
 - ✓ $q\bar{q} \rightarrow 3\gamma$: towards phenomenology [Abreu, Page, Pascual, Sotnikov, to appear]



- ◆ Using similar ideas (analytics from numerics) [Abreu, Dixon, Herrmann, Page, Zeng; 18, 19]
 - ✓ $\mathcal{N} = 4$ SYM: 6 numerical evaluations
 - ✓ $\mathcal{N} = 8$ SUGRA: 45 numerical evaluations

With different approaches [Chicherin, Gehrmann, Henn, Wasser, Zhang, Zoia 18,19]
[Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zoia, 19]

Two-Loop $q\bar{q} \rightarrow 3\gamma$ Amplitudes

3γ production at hadron colliders

17

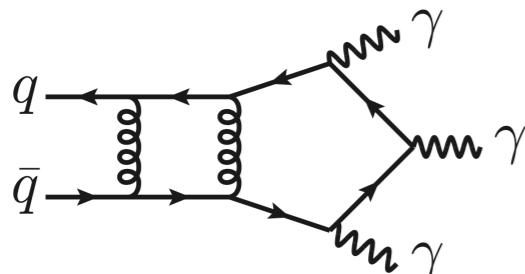
[Abreu, Page, Pascual, Sotnikov, to appear]

Also [Chawdhry, M. L. Czakon, A. Mitov, R. Poncelet, 19]

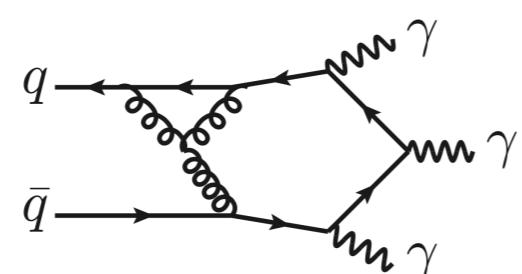
- ◆ Only one channel:

$$q(p_1, h_1) + \bar{q}(p_2, h_2) \rightarrow \gamma(p_3, h_3) + \gamma(p_4, h_4) + \gamma(p_5, h_5)$$

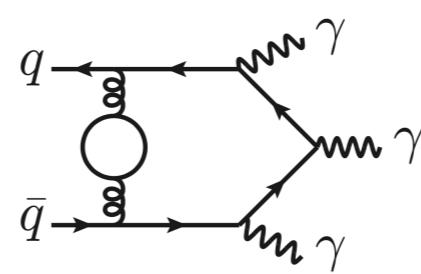
- ◆ Four different contributions:



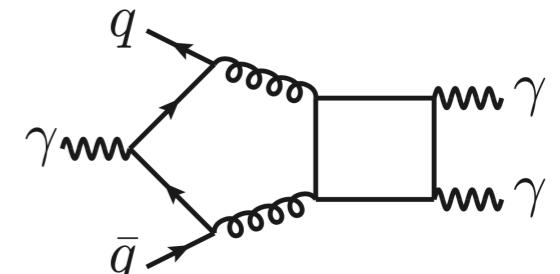
$$\propto C_F^2$$



$$\propto C_F C_A$$



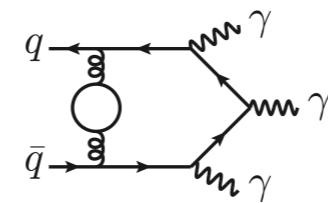
$$\propto C_F N_f$$



$$\propto C_F \sum_q Q_q^2$$

- ◆ Leading-colour planar contributions

$$\underbrace{\quad}_{+2} \quad \underbrace{\quad}_{\propto \frac{N_c^2}{4}}$$



$$\propto C_F N_f$$

[Abreu, Page, Pascual, Sotnikov, to appear]

$$\mathcal{M}(1_q^{h_1}, 2_{\bar{q}}^{h_2}, 3_\gamma^{h_3}, 4_\gamma^{h_4}, 5_\gamma^{h_5}) := e_q^3 \delta_{i_1 i_2} \Phi(1_q^{h_1}, 2_{\bar{q}}^{h_2}, 3_\gamma^{h_3}, 4_\gamma^{h_4}, 5_\gamma^{h_5}) \mathcal{A}(1_q^{h_1}, 2_{\bar{q}}^{h_2}, 3_\gamma^{h_3}, 4_\gamma^{h_4}, 5_\gamma^{h_5})$$

♦ Independent helicity amplitudes: two independent

$$\mathcal{A}_{+++} = \mathcal{A}(1_q^+, 2_{\bar{q}}^-, 3_\gamma^+, 4_\gamma^+, 5_\gamma^+) \quad \mathcal{A}_{-++} = \mathcal{A}(1_q^+, 2_{\bar{q}}^-, 3_\gamma^-, 4_\gamma^+, 5_\gamma^+)$$

→ Kinematics

$$s_{12} = (p_1 + p_2)^2, s_{23} = (p_2 + p_3)^2, s_{34} = (p_3 + p_4)^2, s_{45} = (p_4 + p_5)^2, s_{15} = (p_1 + p_5)^2$$

$$\text{tr}_5 = 4 i \epsilon(p_1, p_2, p_3, p_4)$$

→ Physical region

$$s_{12}, s_{34}, s_{45} > 0, \quad s_{23}, s_{15} < 0, \quad \text{tr}_5^2 < 0$$

→ No photon ordering, all permutations appear

[Abreu, Page, Pascual, Sotnikov, to appear]

◆ Master integral decomposition

$$A_h = \sum_i c_i(\epsilon, \vec{s}, \text{tr}_5) m_i(\epsilon, \vec{s}, \text{tr}_5)$$

◆ Coefficient ansatz

$$c_i(\epsilon, \vec{s}, \text{tr}_5) = \frac{1}{P_i(\epsilon)} \sum_{k=0}^{\kappa_i} \epsilon^k c_{i,k}(\vec{s}, \text{tr}_5)$$

$$c_{i,k}(\vec{s}, \text{tr}_5) = c_{i,k}^+(\vec{s}) + \text{tr}_5 c_{i,k}^-(\vec{s})$$

◆ Denominator ansatz

[Abreu et al.; 18, 19]

$$c_{i,k}^\pm(\vec{s}) = \frac{n_{i,k}^\pm(\vec{s})}{\prod_j W_j^{q_{i,k}^j}(\vec{s})}$$

Letters of the alphabet
of five-point massless
nonplanar integrals

Only need to reconstruct the numerator

[Abreu, Page, Pascual, Sotnikov, to appear]

Helicity	Max degree	# independent $c_{i,k}^\pm$
$A_{+++}^{(2,N_f)}$	18	130
$A_{+++}^{(2,0)}$	27	1244
$A_{-++}^{(2,N_f)}$	20	203
$A_{-++}^{(2,0)}$	32	1320

$$c_{i,k}(\vec{s}, \mathbf{tr}_5) = c_{i,k}^+(\vec{s}) + \mathbf{tr}_5 c_{i,k}^-(\vec{s})$$

- ✓ All-order expression for the amplitude
- ✓ Compact expressions: O(80MB) for the four amplitudes in .m files
- * Too much information for NNLO corrections \Rightarrow 2-loop remainders

◆ **Pentagon function decomposition**

[Abreu, Page, Pascual, Sotnikov, to appear]

$$R_h^{(2,j)} = \sum_{i \in B} r_i(\vec{s}, \text{tr}_5) h_i(\vec{s}, \text{tr}_5)$$

◆ **Coefficient ansatz**

$$r_i(\vec{s}, \text{tr}_5) = r_i^+(\vec{s}) + \text{tr}_5 r_i^-(\vec{s})$$

◆ **Same denominator ansatz**

NB: tr_5^2 appears in the denominator of master integral coefficients, but not in remainder

Helicity	Max degree	# independent r_i^\pm	Max weight
$R_{+++}^{(2,N_f)}$	12	12	1
$R_{+++}^{(2,0)}$	16	62	2
$R_{-++}^{(2,N_f)}$	13	57	3
$R_{-++}^{(2,0)}$	30	171	4

- ✓ **Very compact** expressions: O(1.6MB) for the four amplitudes in .m files
- ✓ All that is required for NNLO phenomenology
- ✓ Available as a public code

✓ Recomputed 1-loop and 2-loop **4-point amplitudes**

[Anastasiou, Glover, Tejeda-Yeomans 02; Glover, Tejeda-Yeomans 03]

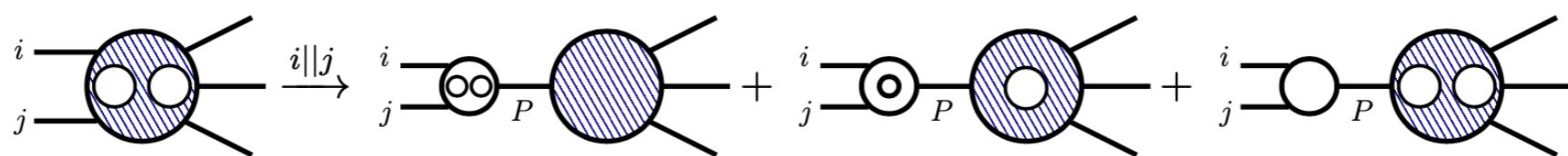
✓ Recomputed (to all orders) **1-loop 5-point amplitudes**

OpenLoops 2 [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller 19]

✓ Infrared pole structure

[Catani 98; Anastasiou, Glover, Tejeda-Yeomans 02; Glover, Tejeda-Yeomans 03]

✓ Collinear limits



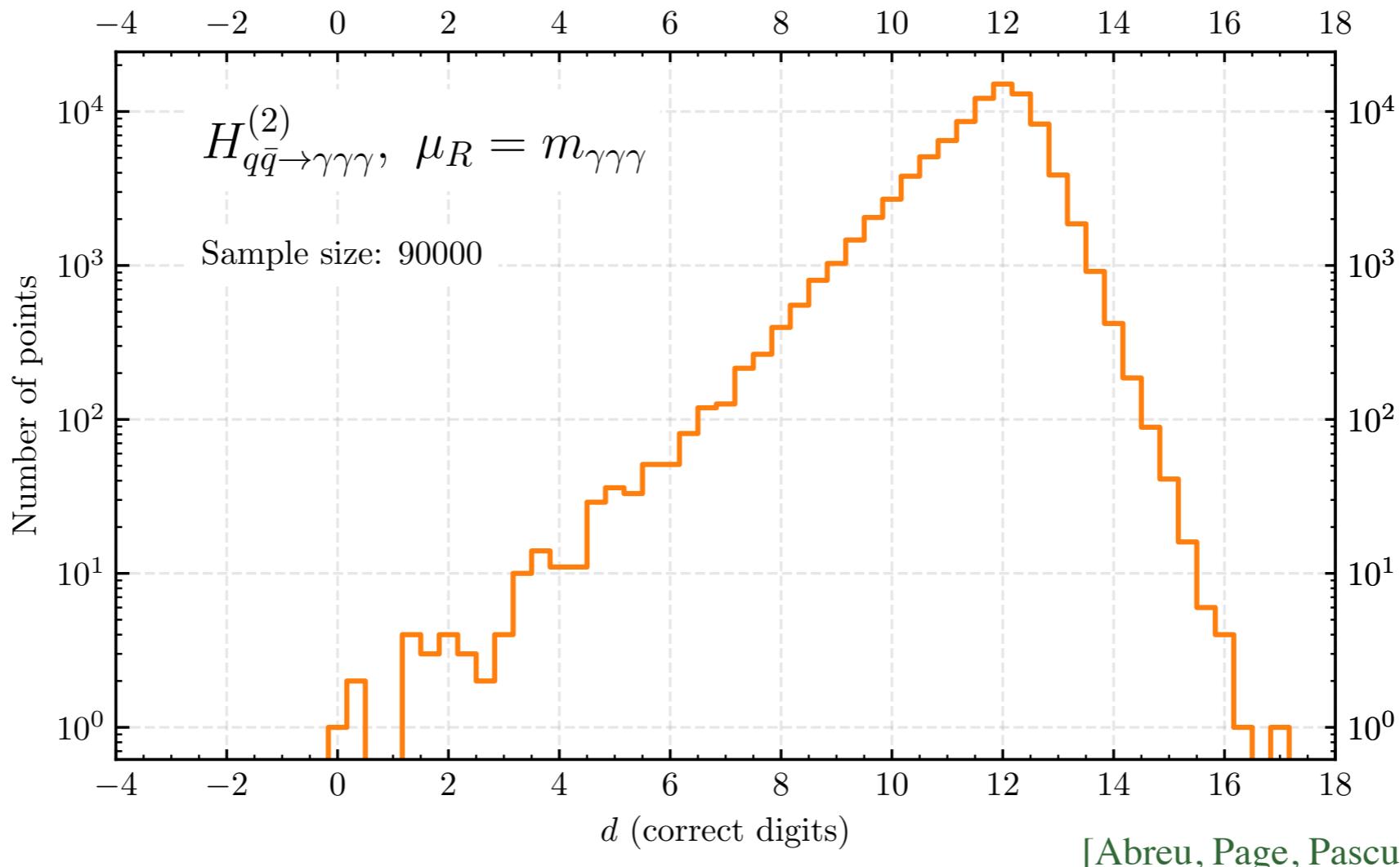
$q\bar{q}$ not accessible

$\gamma\gamma$ all helicities regular

$q\gamma$ all helicities checked

$\bar{q}\gamma$ all helicities checked

$$H^{(2)} \sim \langle R^{(1)} | R^{(1)} \rangle + 2\text{Re} \langle R^{(0)} | R^{(2)} \rangle$$



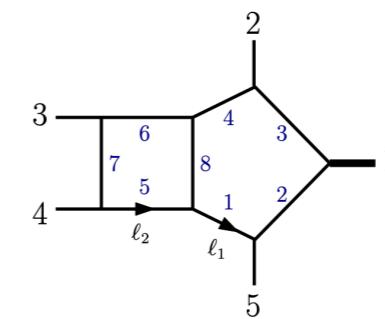
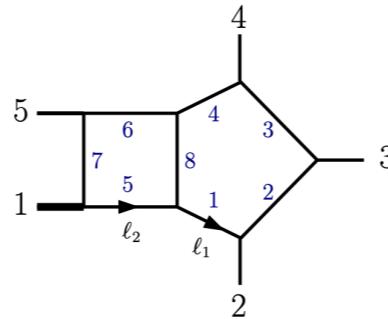
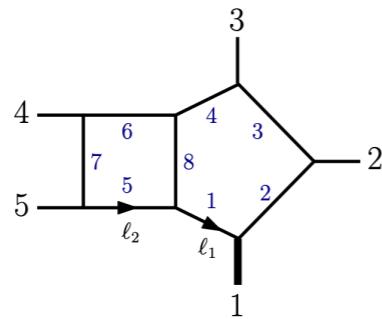
[Abreu, Page, Pascual, Sotnikov, to appear]

- ◆ Stable amplitude evaluations, ready phenomenology studies
- ◆ Less than 10 seconds per point

- ✓ Loss of precision fully understood ⇒ rescue system
- ✓ Already used for pheno!

[Kallweit, Sotnikov, Wiesemann 20]

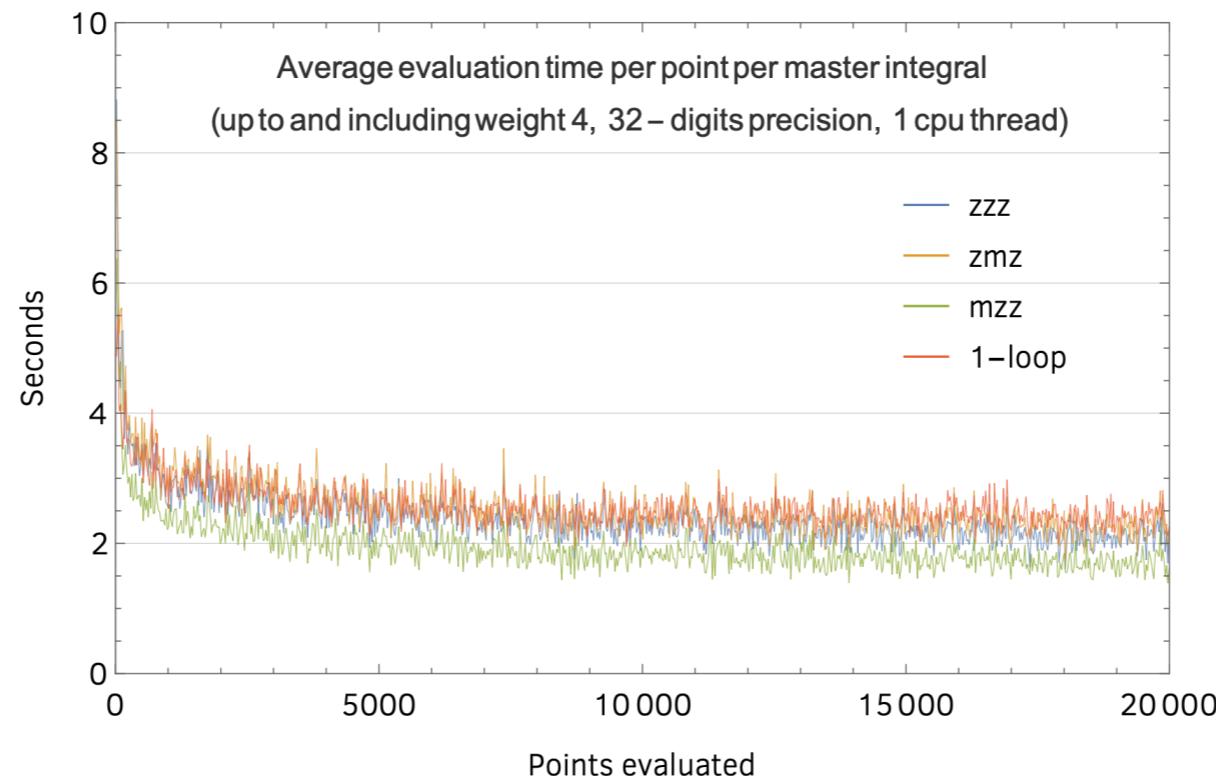
- ◆ The calculation of two-loop multi-leg amplitudes is developing fast
 - ✓ Massless amplitudes: first phenomenology results appearing
 - ✓ Massive amplitudes: next frontier, high-impact for LHC physics
- ◆ Simplification of results
 - ✓ Analytic structure of coefficient and master integrals
 - ✓ Crucial for phenomenology
 - ✓ New insights into the analytic structure of amplitudes
- ◆ Very complicated calculations
 - ✓ Bypass intermediate analytic complexity with numerics
 - ✓ Impossible without new insights from the amplitude community



- ◆ Analytic differential equations constructed from numerics

[Abreu, Page, Zeng, 20]

- ◆ Efficient and stable numerical solutions in physical region



- ◆ Integrals for $W + 2j$ production at the LHC

[Abreu, Ita, Moriello, Page, Tschernow, Zeng, 20]

[Canko, Papadopoulos, Syrrakos, 20]

- ◆ Target value for $W + 2j$ amplitude

[Badger, Brønnum-Hansen, Hartanto, Peraro 19]

THANK YOU!