DM of any spin

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Dark Matter of Any Spin -- an Effective Field Theory and Applications

Juan Carlos Criado, Niko Koivunen, Martti Raidal, Hardi Veermäe (2010.02224)

theoretical framework + application to DM

Higher-spin particles

massless fundamental particles: $j \leq 2$

massive higher-spin particles exist in nature!

DM is massive – could have any spin

Fields

	Spin	Lorentz	Phys. dof	Field dof
scalar	0	$(0, 0)$	1	1
spinor	$1/2$	$(1/2, 0)$	2	2
vector	1	$(1/2, 1/2)$	3	4
RS	$3/2$	$(1, 1/2)$	4	6

Unphysical dof can lead to problems such as faster-than-light propagation!

Is there a theory without extra dof?

Yes!

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Feynman Rules for Any Spin*

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The explicit Feynman rules are given for massive particles of any spin j, in both a $2j+1$-component and a $2(2j+1)$-component formalism. The propagators involve matrices which transform like symmetric traceless tensors of rank $2j$; they are the natural generalizations of the 2×2 four-vector σ^μ and 4×4 four-vector γ^μ for $j=\frac{1}{2}$. Our calculation uses field theory, but only as a convenient instrument for the construction of a Lorentz-invariant S matrix. This approach is also used to prove the spin-statistics theorem, crossing symmetry, and to discuss T, C, and P.

$(j, 0)$ field theory

$(j, 0)$ field dof $\quad = \quad 2j + 1 \quad = \quad$ spin-j physical dof

pert. unitarty violation $\rightarrow$ EFT

no free-theory Lagrangian

Why no simple Lagrangian?

$$\psi_{(a)}(x) = \int \frac{d^3p}{(2\pi)^3(2E_{\mathbf{p}})} \sum_\sigma \left[a_{p\sigma}\, u_{(a)}(p,\sigma)e^{ipx} + a^*_{p\sigma}\, v_{(a)}(p,\sigma)e^{-ipx} \right],$$

$$\Downarrow$$

$$(\Box + m^2)\psi = 0 \tag{1}$$

$$(\overbrace{\not\partial \otimes \ldots \otimes \not\partial}^{\longleftarrow\ 2j\ \longrightarrow})\psi = m^{2j}\psi^\dagger. \tag{2}$$

We don't need a free Lagrangian,

just the Feynman rules!

(Weinberg 1963)

TABLE I. The scalar matrix $\Pi(q) = (-)^{2j} t^{\mu_1 \mu_2 \cdots} q_{\mu_1} q_{\mu_2} \cdots$ for spins $j \leq 3$. In each case $\mathbf{J}$ is the usual $2j+1$-dimensional matrix representation of the angular momentum. The propagator for a particle of spin j is $S(q) = -i(-im)^{-2j}\Pi(q)/q^2+m^2-i\epsilon$.

$$\Pi^{(0)}(q) = 1$$

$$\Pi^{(1/2)}(q) = q^0 - 2(\mathbf{q}\cdot\mathbf{J})$$

$$\Pi^{(1)}(q) = -q^2 + 2(\mathbf{q}\cdot\mathbf{J})(\mathbf{q}\cdot\mathbf{J} - q^0)$$

$$\Pi^{(3/2)}(q) = -q^2(q^0 - 2\mathbf{q}\cdot\mathbf{J}) + \tfrac{1}{6}[(2\mathbf{q}\cdot\mathbf{J})^2 - \mathbf{q}^2][3q^0 - 2\mathbf{q}\cdot\mathbf{J}]$$

$$\Pi^{(2)}(q) = (-q^2)^2 - 2q^2(\mathbf{q}\cdot\mathbf{J})(\mathbf{q}\cdot\mathbf{J} - q^0) + \tfrac{2}{3}(\mathbf{q}\cdot\mathbf{J})[(\mathbf{q}\cdot\mathbf{J})^2 - \mathbf{q}^2][\mathbf{q}\cdot\mathbf{J} - 2q^0]$$

$$\Pi^{(5/2)}(q) = (-q^2)^2(q^0 - 2\mathbf{q}\cdot\mathbf{J}) - \tfrac{1}{6}q^2[(2\mathbf{q}\cdot\mathbf{J})^2 - \mathbf{q}^2][3q^0 - 2\mathbf{q}\cdot\mathbf{J}] + \frac{1}{120}[(2\mathbf{q}\cdot\mathbf{J})^2 - \mathbf{q}^2][(2\mathbf{q}\cdot\mathbf{J})^2 - 9\mathbf{q}^2][5q^0 - 2\mathbf{q}\cdot\mathbf{J}]$$

$$\Pi^{(3)}(q) = (-q^2)^3 + 2(-q^2)(\mathbf{q}\cdot\mathbf{J})(\mathbf{q}\cdot\mathbf{J} - q^0) - \tfrac{2}{3}q^2(\mathbf{q}\cdot\mathbf{J})[(\mathbf{q}\cdot\mathbf{J})^2 - \mathbf{q}^2][\mathbf{q}\cdot\mathbf{J} - 2q^0] + \frac{4}{45}(\mathbf{q}\cdot\mathbf{J})[(\mathbf{q}\cdot\mathbf{J})^2 - \mathbf{q}^2][(\mathbf{q}\cdot\mathbf{J})^2 - 4\mathbf{q}^2][\mathbf{q}\cdot\mathbf{J} - 3q^0]$$

Notation

$$\psi_{(a)} = \psi_{(a_1 a_2 \dots a_{2j})} \sim S^{2j}(1/2, 0) \sim (j, 0)$$

the a_i are two-component spinor indices

$$p_{a\dot{a}} \equiv \sigma^{\mu}_{a\dot{a}} p_{\mu}, \qquad t^a \equiv \epsilon^{ab} t_b$$

$$p_{(a)(\dot{a})} \equiv p_{a_1 \dot{a}_1} \dots p_{a_{2j} \dot{a}_{2j}}$$

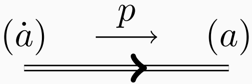
$$\delta^{(b)}_{(a)} \equiv \delta^{b_1}_{a_1} \dots \delta^{b_{2j}}_{a_{2j}}$$

$$\dots$$

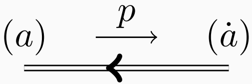
Some useful relations

$$p_{(a)(\dot{a})} q^{(a)(\dot{b})} + q_{(a)(\dot{a})} p^{(a)(\dot{b})} = 2(p \cdot q)^{2j} \delta^{(\dot{b})}_{(\dot{a})}$$

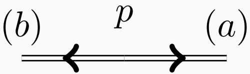
$$p_{(a)(\dot{a})} q^{(a)(\dot{a})} = (2p \cdot q)^{2j}$$



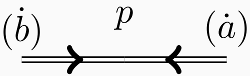
$$= \frac{i p_{(a)(\dot{a})}/m^{2j}}{p^2 - m^2},$$



$$= \frac{i p^{(\dot{a})(a)}/m^{2j}}{p^2 - m^2},$$

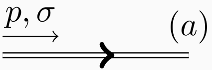


$$= \frac{i \delta_{(a)}{}^{(b)}}{p^2 - m^2},$$

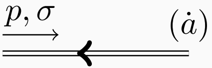


$$= \frac{i \delta^{(\dot{a})}{}_{(\dot{b})}}{p^2 - m^2}.$$

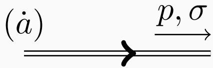
External lines

incoming left-handed = 

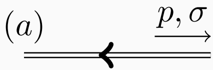
$= u_{(a)}(p, \sigma)$,

incoming right-handed = 

$= v^*_{(\dot{a})}(p, \sigma)$,

outgoing left-handed = 

$= u^*_{(\dot{a})}(p, \sigma)$,

outgoing right-handed = 

$= v_{(a)}(p, \sigma)$.

Mass dimension of the fields

from Weinberg's Feynman rules: 1 for any j

rescaling of the fields by m^j

restores the conventional normalization

$\implies$ effective dimension $\Delta = j + 1$

Parity and Lorentz irreps

$$(j, 0) \overset{\mathsf{P}}{\leftrightarrow} (0, j)$$

$$(\psi_L, \psi_R) \sim (j, 0) \oplus (0, j)$$

$$\text{Neutral: } \psi \equiv \psi_L = \psi_R^\dagger$$

Application to DM

Simplest field content:

singlet under SM symmetries, odd under $\mathbb{Z}_2$

Higgs portal

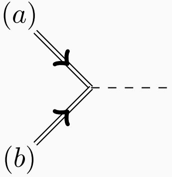
$$\mathcal{H}_{\text{portal}} = -\lambda\,\psi^{(a)}\psi_{(a)}(|\phi|^2 - v_h^2/2) + \text{h.c.}$$

(effective dimension: $2j+4$)

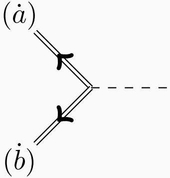
scale Λ_* of perturbative unitarity violation:

$$\left(\frac{m}{\Lambda_*}\right)^{2j} \equiv \frac{|\lambda|}{4\pi}$$

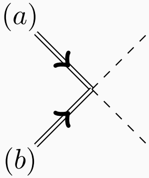
Vertices



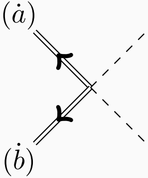
$$= 2iv_h\lambda\,\delta_{(a)}{}^{(b)},$$



$$= 2iv_h\lambda^*\,\delta^{(\dot{a})}{}_{(\dot{b})},$$



$$= 2i\lambda\,\delta_{(a)}{}^{(b)},$$



$$= 2i\lambda^*\,\delta^{(\dot{a})}{}_{(\dot{b})}.$$

Linear interactions

$$\mathcal{H}_{\text{linear}} = \frac{1}{(\Lambda_{\text{lin}})^n}\psi\mathcal{O}_{\text{SM}} \qquad (n \sim 3j \quad \text{for large } j)$$

$\Longrightarrow$ linear operators more suppressed than quadratic for $j = 5/2$ and $j > 3$

$\Longrightarrow$ accidental approximate $\mathbb{Z}_2$ rendering higher-spin particles metastable

DM observables

- Freeze-out and freeze-in abundance
- Invisible Higgs decays
- Direct detection
- Indirect detection

For any spin!

Freeze-in

$$\sigma \sim s^n \qquad \text{with } n \geq 0 \text{ for } j \geq 1/2$$

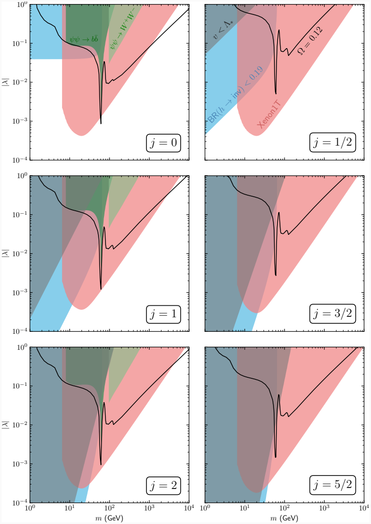
$$\implies \text{UV freeze-in}$$

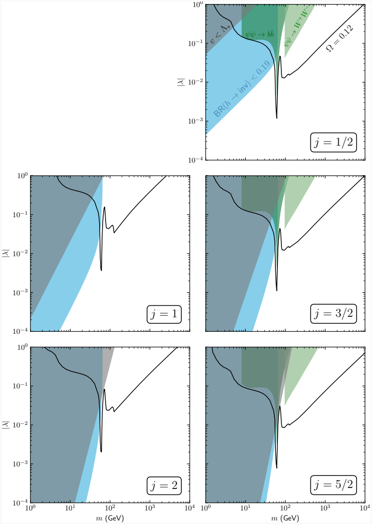
$$|\lambda| \simeq 7 \times 10^{-12} \frac{1}{\sqrt{A_j}} \left(\frac{m}{T_{\mathrm{RH}}} \right)^{2j-1/2}$$

Free parameters

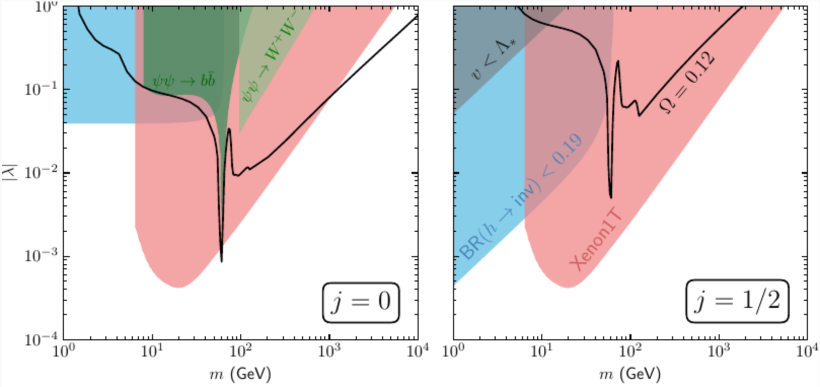
$$|\lambda| \qquad \theta = \arg(\lambda) \qquad m \qquad j$$

real λ (CP-even portal) | imaginary λ (CP-odd portal)

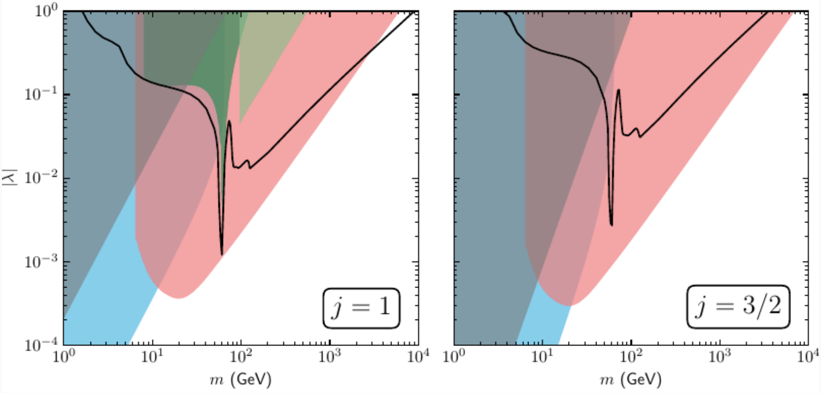




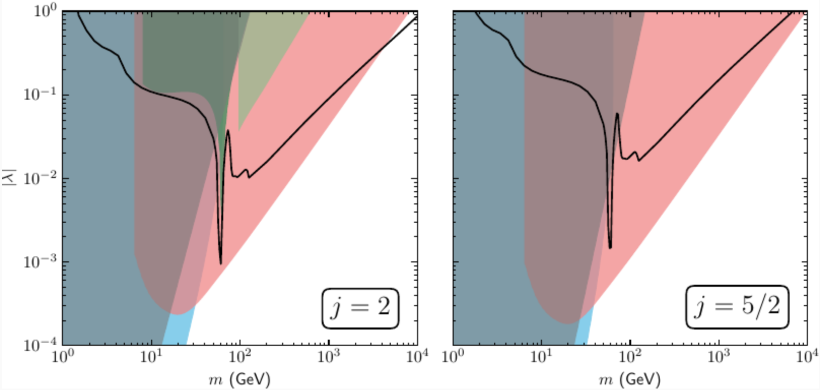
CP-even interaction



CP-even interaction



CP-even interaction



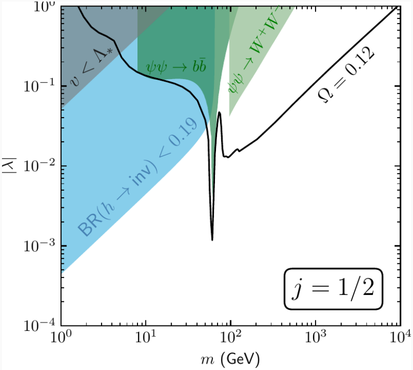
The direct detection cross section

$$\sigma_N = \frac{2^{2j+1} m_N^2 \mu_N^2 f_N^2 |\lambda|^2}{(2j+1)\pi m_h^4 m^2} \left[1 + \cos 2\theta + O(v^2)\right]$$

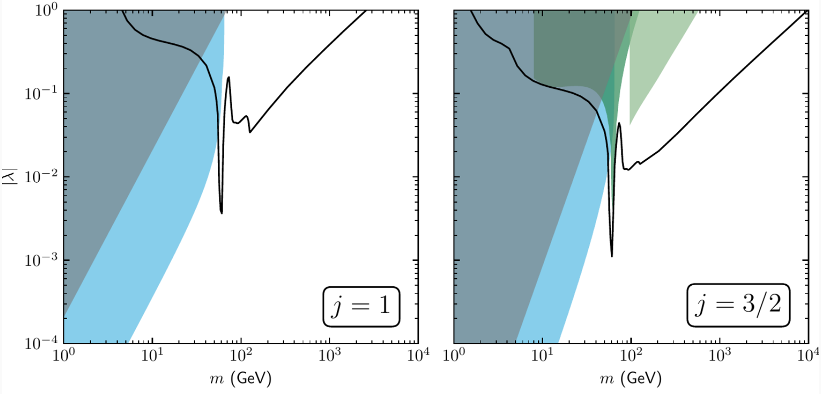
$$(\lambda = |\lambda| e^{i\theta})$$

very suppressed for $\cos 2\theta = -1$ (CP-odd portal)

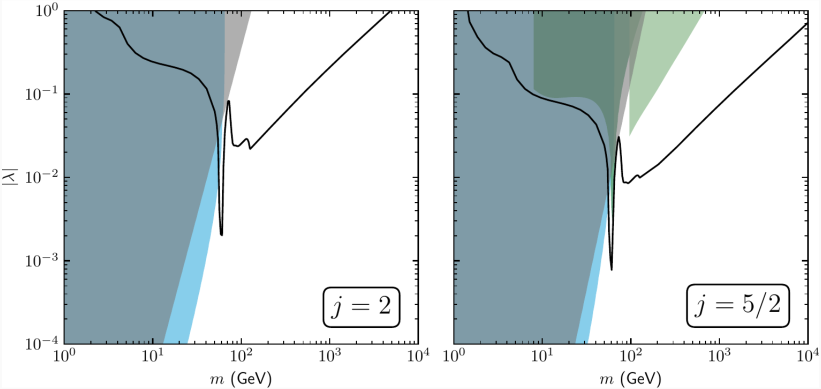
CP-odd interaction



CP-odd interaction



CP-odd interaction



Summary

General framework for any spin:

- $(j, 0)$ fields contain only physical dof
- usable reformulation of Weinberg's work
- no Lagrangian

DM pheno:

- Automatic $\mathbb{Z}_2$ for higher spin
- Any-j freeze-out DM for sufficiently high mass
- EW-scale masses allowed for CP-odd coupling

Future work

Collider physics

Nuclear physics

Non-gauge-singlet $(j, 0)$ fields