IPPP Internal Seminar

McMule

QED Corrections for Low-Energy Experiments

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Monte Carlo for MUons and other LEptons

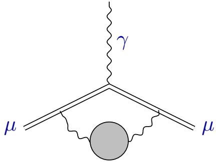
- framework for fully-differential (N)NLO in QED with $m_f > 0$
- given matrix elements, McMule takes care of everything else
- subtraction scheme: FKS^ℓ
- quite a few processes already

$e\mu \to e\mu$	NNLO (dominant)	$L \to \nu\bar{\nu}\ell$	NNLO
$\ell p \to \ell p$	NNLO (dominant)	$L \to \nu\bar{\nu}\ell\gamma$	NLO
$ee \to ee$	NNLO (ongoing)	$L \to \nu\bar{\nu}\ell l^+ l^-$	NLO
$ee \to \gamma\gamma$	NNLO (ongoing)	$L \to eX$	NLO
$ee \to \mu\mu$	NNLO (ongoing)		

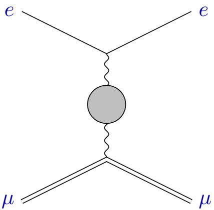
why bother?

precision physics at low-energy: $e\mu \to e\mu$

To get this to $\mathcal{O}(1\%)$ ...



... measure this at 10^{-5}

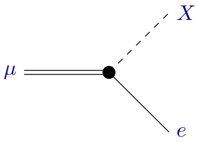


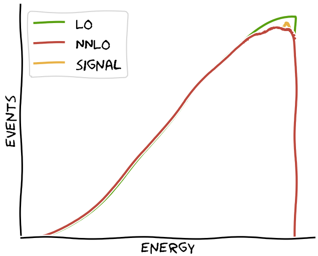
$\Rightarrow$ NNLO QED with $m_e > 0$ + PS + ...
(review: [MUonE theory initiative 20])

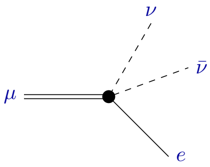
why bother?

precision physics at low-energy: $\mu \to \nu\bar{\nu}e$ and $\mu \to eX$

- new light cLFV ALP with $m_X \ll m_\mu$ [Calibbi, Redigolo, Ziegler, Zupan 20]
- potentially visible at MEG [Banerjee, Engel, Gurgone, Papa, Schwendimann, Signer, YU 2?]
- need $\mu \to \nu\bar{\nu}e$ really precisely







two-loop (if unavailable)

- $\mathcal{M}_n^{(2)}(m=0)$ known [Mastrolia, Prima, et al.] (full $e\mu \to e\mu$)
- massify $\mathcal{M}_n^{(2)}(m) = \mathcal{M}_n^{(2)}(z) + \mathcal{O}(z)$

one-loop and tree-level

- $\mathcal{M}_{n+1}^{(1)}(m)$ and $\mathcal{M}_{n+2}^{(0)}(m)$ using OpenLoops [Pozzorini, Zoller, Lindert et al. 19]
- or by hand using Package-X [Patel 15] / FDF [Fazio, Mastrolia, Marabella, Torres 14]

phase space

- subtraction scheme FKS^ℓ [Engel, Signer YU 19]
- numerical integration vegas [Lepage 80]

massification /ˈmasɪˈfɪkeɪʃ(ə)n/

(noun)

Methods to add leading mass effects to amplitudes in perturbation theory

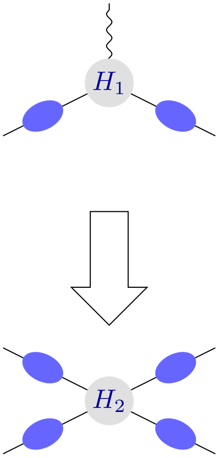
Derivatives:

- **massify** verb
- **massified** adjective

- SCET inspired $\sim$ fragmentation fct.
- drop polynomially surpressed terms

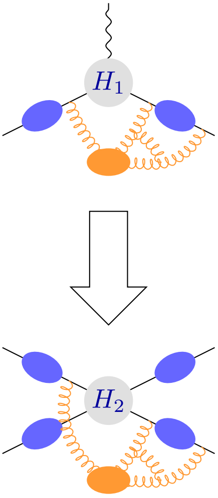
one external mass $m \ll Q^2 = s$

- Bhabha scattering (photonic) [Penin 2005]
- QCD with $n_f = n_m = n_h = 0$ [Mitov, Moch 2006]
- $F_{\gamma^*}(m) \overset{!}{\simeq} \sqrt{Z_J} \times \sqrt{Z_J} \times F(0)$
- $Z_J \supset \ln(m^2/\mu^2)$ but universal
- $\mathcal{A}_{ee\to ee}(m)|_{C_F^2} \simeq Z_J^{4/2} \times \mathcal{A}(0)$



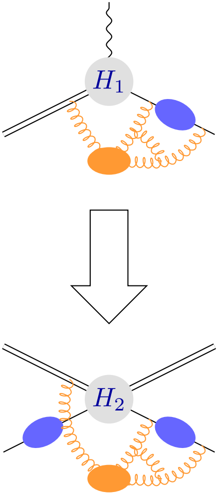
light fermion loops

- n_m terms [Becher, Melnikov 07]
- $F_{\gamma^*}(m) \overset{!}{\simeq} \mathcal{S} \times \sqrt{Z_J} \times \sqrt{Z_J} \times F(0)$
- $\mathcal{S}(s,m)$: from vacuum polarisation with massive fermions, $\supset \ln(m^2/s)$
- $\mathcal{A}_{ee\to ee}(m) \simeq \mathcal{S}' \times Z_J^{4/2} \times \mathcal{A}(0)$



including heavy fermions

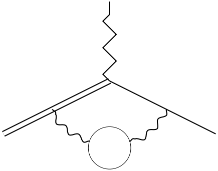
- two masses $s \sim M \gg m \gg 0$
  [Engel, Gnendiger, Signer, YU 18]
- $F_\mu(m) \overset{!}{\simeq} \mathcal{S} \times \sqrt{Z_q} \times F_\mu(0)$
- $\mathcal{A}_{\mu e}(m) \simeq \mathcal{S}' \times Z_q^{2/2} \times \mathcal{A}(0)$



- soft region using light-cone coordinates for q

$$\propto \int \mathrm{d}^d k \frac{\Pi^{(n_m)}(k)}{(k^2)^2(2p\cdot k)(2q_-\cdot k)}$$

$$\propto \int_0^\infty \mathrm{d}x \frac{m^{2-4\epsilon}\Gamma(\epsilon)\Gamma(1-\epsilon)}{x(s+M^2x)} = \infty$$



- integral is not regularised in d dimensions

$\Rightarrow$ analytic regularisation: $\frac{1}{p\cdot k} \to \frac{1}{(p\cdot k)^{1+\eta}}$ [Smirnov 97, Becher, Broggio, Ferroglia 14]

$$\propto \int \mathrm{d}x \frac{m^{2-4\epsilon-\eta}\Gamma(\epsilon+\frac{\eta}{2})}{x^{1-\eta/2}(s+M^2x)^{1+\eta/2}}$$

$$\propto (mM)^{-\eta}\Gamma(\epsilon+\tfrac{\eta}{2})\Gamma(\tfrac{\eta}{2}) \propto \frac{(mM)^{-\eta}}{\eta}$$

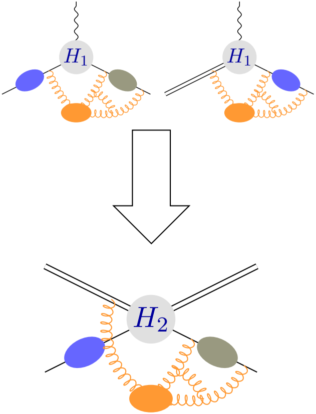
- $Z_q \supset -\frac{s^\eta}{\eta} \to \ Z_q \times \mathcal{S} \supset \log \frac{s}{mM}$

$\Rightarrow$ breakdown of naive factorisation $\sim$ factorisation anomaly [Beneke 05, Becher, Bell, Neubert 11]

- new feature for $0 \ll m \ll M$

$\Rightarrow$ two matching calculations

- now $-\frac{s^\eta}{\eta} \subset Z_q \neq \bar{Z}_q \supset \frac{m^{-2\eta}}{\eta}$



0. (anti-)collinear contributions $\sqrt{Z_q}$ and $\sqrt{\bar{Z}_q}$ known
1. calculate $\mathcal{M}_n^{(2)}(0)$ aka. the difficult bit
2. calculate $\mathcal{O}(\epsilon^2)$ of $\mathcal{M}_n^{(1)}(0)$ (you probably already have that)
3. process dependent soft function $\mathcal{S}$ in eikonal theory with analytic regulator
4. $\mathcal{M}_n^{(2)}(m) = \prod_i \sqrt{Z_i} \times \prod_i \sqrt{\bar{Z}_i} \times \mathcal{S} \times \mathcal{M}_n^{(2)}(0) + \mathcal{O}(z)$

n. resum if needed

FKS^ℓ

or tricking YFS into a full-flegded subtraction scheme where we stand

- process with extra photon, let $\xi \propto E_\gamma$
- no collinear singularity
- $\mathcal{M}^{(0)}_{n+1} \propto 1/\xi^2$ is divergent
- phase space $\mathrm{d}\phi_1 = \mathrm{d}\Upsilon\ \mathrm{d}\xi\ \xi^{1-2\epsilon}$

credo: keep splitting until vegas can integrate it

$$\mathrm{d}\sigma = \mathrm{d}\Phi_n \mathrm{d}\phi_1\ \mathcal{M}^{(0)}_{n+1} = \mathrm{d}\Phi_n \mathrm{d}\Upsilon\ \mathrm{d}\xi\ \xi^{-1-2\epsilon} \underbrace{\Big(\xi^2 \mathcal{M}^{(0)}_{n+1}\Big)}_{\to\text{finite}}$$

$$= \mathrm{d}\Phi_n \mathrm{d}\Upsilon\ \mathrm{d}\xi \Bigg(\underbrace{-\frac{\xi_c^{-2\epsilon}}{2\epsilon}\delta(\xi)}_{\supset \mathrm{d}\sigma^{(s)}} + \underbrace{\Big(\frac{1}{\xi^{1+2\epsilon}}\Big)_c}_{\supset \mathrm{d}\sigma^{(h)}} \Bigg) \Big(\xi^2 \mathcal{M}^{(0)}_{n+1}\Big)$$

- defined c-distribution $\int \mathrm{d}\xi\, \big(\frac{1}{\xi}\big)_c f(\xi) = \int \mathrm{d}\xi\, \frac{f(\xi) - f(0)\theta(\xi_c - \xi)}{\xi}$

- $\mathrm{d}\sigma^{(s)}$ easy because $\delta(\xi) \to$ eikonal $\hat{\mathcal{E}}$
- $\mathrm{d}\sigma^{(h)}$ is finite $\epsilon \to 0$

$$\mathrm{d}\sigma^{(s)} \subset \mathrm{d}\sigma_n(\xi_c) = \mathrm{d}\Phi_n\Big(\mathcal{M}_n^{(1)} + \hat{\mathcal{E}}(\xi_c)\mathcal{M}_n^{(0)}\Big)$$

$$\mathrm{d}\sigma^{(h)} \equiv \mathrm{d}\sigma_{n+1}(\xi_c) = \mathrm{d}\Phi_{n+1}\Big(\frac{1}{\xi}\Big)_c\Big(\xi^2\mathcal{M}_{n+1}^{(0)}\Big)$$

ξ_c is unphysical

- both bits depend on ξ_c but σ doesn't

$\Rightarrow$ dependence drops out exactly

- use to test implementation and stability credo: keep splitting until vegas can integrate it

- no one-loop eikonals $\to$ like normal FKS
- $\mathrm{d}\sigma^{(h)}$ is not yet finite ($\mathcal{M}^{(1)}_{n+1}$ has $1/\epsilon$ poles)
- use KLN theorem / YFS for $(n+1)$-particle process

$$\underbrace{\mathcal{M}^{(1)f}_{n+1}}_{\supset \mathrm{d}\sigma^{(fin)}} = \mathcal{M}^{(1)}_{n+1} + \underbrace{\hat{\mathcal{E}}(\xi_c)\mathcal{M}^{(0)}_{n+1}}_{\supset -\mathcal{I}} = \text{finite}$$

$\Rightarrow$ eikonal subtraction

- $\mathrm{d}\sigma^{(h)}(\xi_c) = \mathrm{d}\sigma^{(fin)}(\xi_c) - \mathcal{I}(\xi_c)$
- $\mathcal{I}$ is process- and observable dependent, not finite
- deal with it later

credo: keep splitting until vegas can integrate it

- two soft photons $\to$ just do FKS twice

$$\mathrm{d}\sigma = \mathrm{d}\sigma^{(hh)} + 2\times\mathrm{d}\sigma^{(hs/sh)} + \mathrm{d}\sigma^{(ss)}$$

- $\mathrm{d}\sigma^{(ss)} \propto \frac{1}{2!}\hat{\mathcal{E}}^2(\xi_c)\mathcal{M}_n^{(0)}$
- $\mathrm{d}\sigma^{(hh)} = \frac{1}{2!}\mathrm{d}\Phi_{n+2}\Big(\frac{1}{\xi_1}\Big)_c\Big(\frac{1}{\xi_2}\Big)_c\Big(\xi_1^2\xi_2^2\mathcal{M}_{n+2}^{(0)}\Big)$
- $\mathrm{d}\sigma^{(hs/sh)} = 2\times\frac{1}{2!}\mathrm{d}\Upsilon\mathrm{d}\xi\Big(\frac{1}{\xi^{1+2\epsilon}}\Big)_c\Big(\xi^2\hat{\mathcal{E}}(\xi_c)\mathcal{M}_{n+1}^{(0)}\Big) = \mathcal{I}(\xi_c)$

everything complicated drops out!

- adding two-loop $\mathcal{M}_n^{(2)}$

$$\mathrm{d}\sigma_n(\xi_c) = \mathrm{d}\Phi_n \underbrace{\left(\mathcal{M}_n^{(2)} + \hat{\mathcal{E}}\mathcal{M}_n^{(1)} + \frac{1}{2!}\mathcal{M}_n^{(0)}\hat{\mathcal{E}}^2\right)}_{\mathcal{M}_n^{(2)f}}$$

$$\mathrm{d}\sigma_{n+1}(\xi_c) = \mathrm{d}\Phi_{n+1}\left(\frac{1}{\xi_1}\right)_c\left(\xi_1^2\mathcal{M}_{n+1}^{(1)f}\right)$$

$$\mathrm{d}\sigma_{n+2}(\xi_c) = \mathrm{d}\Phi_{n+2}\left(\frac{1}{\xi_1}\right)_c\left(\frac{1}{\xi_2}\right)_c\left(\xi_1^2\xi_2^2\mathcal{M}_{n+2}^{(0)}\right)$$

- never need $\mathcal{M}_{n+i}^{(j)}$ higher than $\mathcal{O}(\epsilon^0)$
- YFS build-up
- can be extended to N$^\ell$LO $\Rightarrow$ FKS^ℓ

McMule

putting everything together

- public repository

  `https://gitlab.com/mule-tools/mcmule/`

- core written in Fortran 95, helper scripts in python
- 7 kLoC (core) + 630 kLoC (matrix elements) + 5 kLoC (helper)
- use COLLIER for some loop integrals [Denner, Dittmaier, Hofer 16]
- adding new processes is simple
- pre-compiled Docker container available
- all published data avaible online
- ease of use (both for theory and experimental)

$\rightarrow$ user needs to only touch one file specifying observable

$\rightarrow$ arbitrary cuts and 1D histograms

$e\mu \to e\mu$ for the MUonE experiment @ CERN's M2

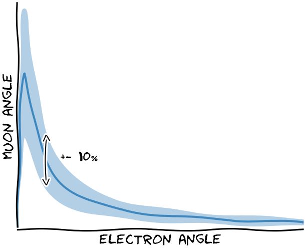
- 150 GeV $\mu^\pm$, atomic e^-

$\Rightarrow$ everything in the labframe

- can't measure energies well

$\Rightarrow$ only got $\theta_{e,\mu}$ and $\varphi_{e,\mu}$

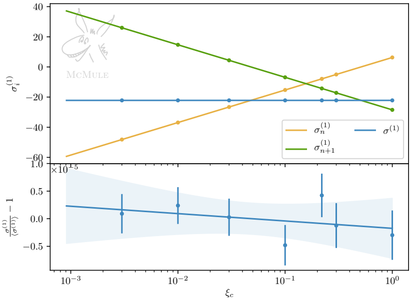
- LO kinematics $\theta_\mu = \theta_\mu(\theta_e)$
- only dominant NNLO



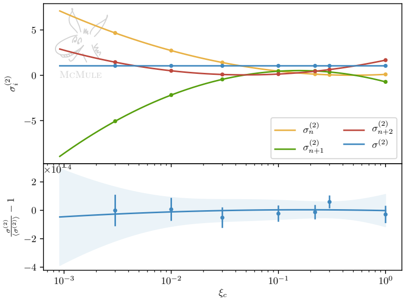
cuts

- $E_e > 1\,\text{GeV}$, $\theta_\mu > 0.3\,\text{mrad}$
- $b = |\frac{\theta_\mu}{\theta_\mu^{\text{el}}} - 1| < 10\%$ (optional)

- slightly different observable

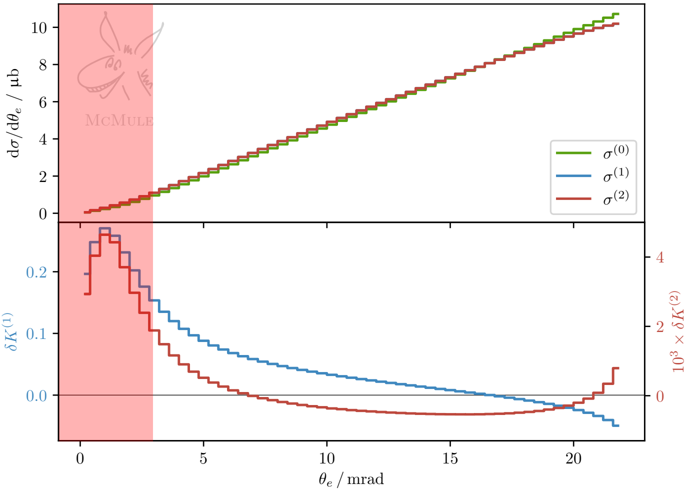


verified with [Alacevich, Carloni Calame, Chiesa, Montagna, Nicrosini, Piccinini 19]

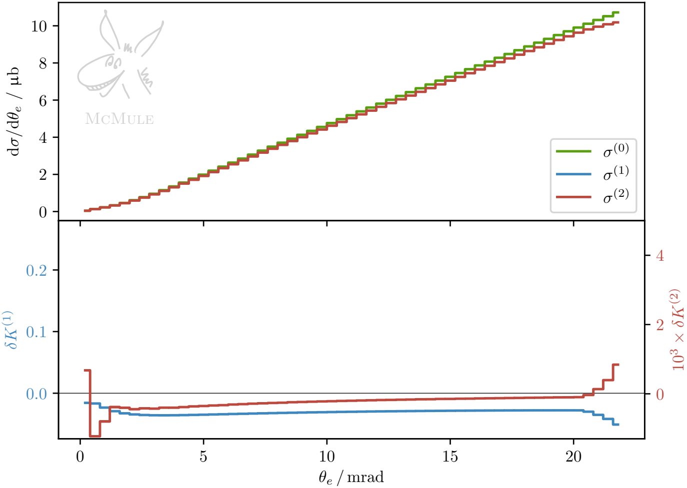


verified with [Carloni Calame, Chiesa, Hasan, Montagna, Nicrosini, Piccinini 20]

$\Rightarrow$ need to resum

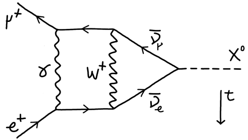


- corrections are smaller and flatter



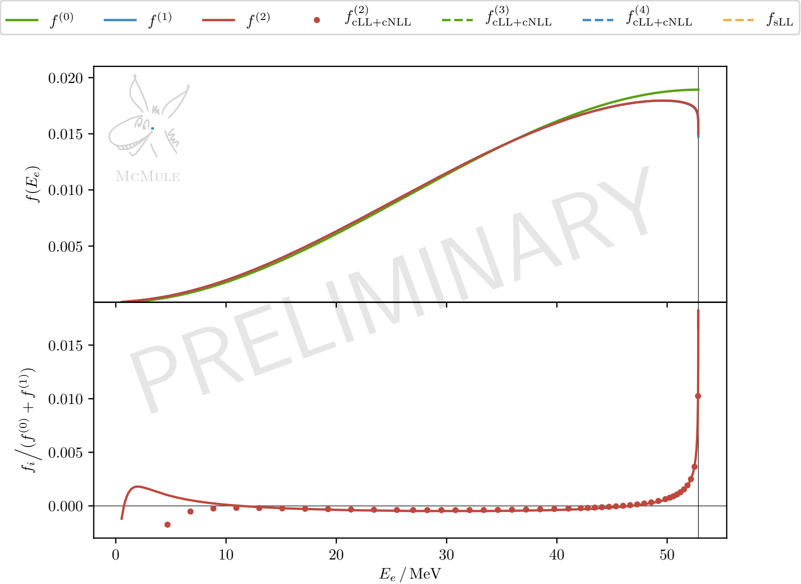
joint project with the MEG collaboration [Banerjee, Engel, Gurgone, Papa, Schwendimann, Signer, YU 2?]

- search for cLFV ALP X
- looking for small bumps in muon decay spectrum
- full detector simulation interfaced to McMule
- $$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}E_e\mathrm{d}\cos\theta} = \Gamma_0\Big(F(E_e) + G(E_e)P\cos\theta\Big)$$
- need best possible F and G (NNLO + sLL + ...)
- signal at NLO

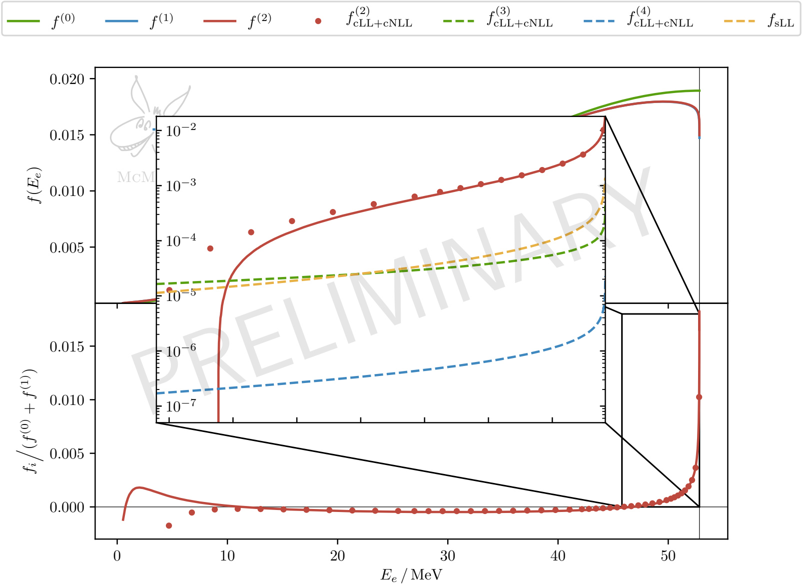


[Gurgone 20]

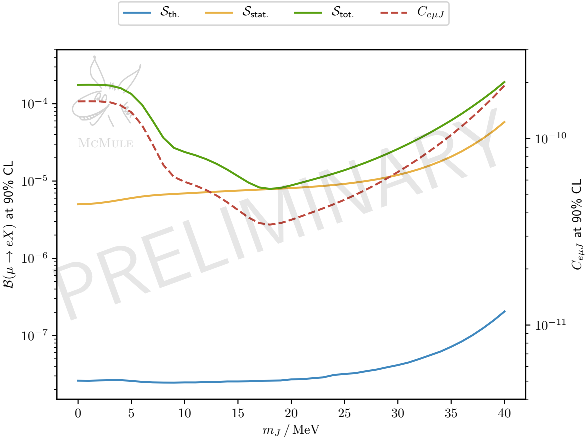
$\mu \to e\nu\bar{\nu}$: the F function



$\mu \to e\nu\bar{\nu}$: the F function



- combine theory with simplified (but vetted) detector model
- realisitic $N = 10^8 \mu$ (due to trigger)
- analysis for dedicated search ongoing



conclusion

- low-energy QED is very important for many experiments
- McMule is a framework to implement these
- used in experimental analysis (MEG, MUonE)

future, in somewhat order

- upcoming processes $ee \to ee$ ($\to$ PRad II), $ee \to \gamma\gamma$ ($\to$ PADME), $ee \to \mu\mu$ ($\to$ Belle), full $e\mu \to e\mu$ at NNLO
- adding parton shower
- improve stability as $\xi \to 0$ in $\mathcal{M}^{(1)}_{n+1}$ $\to$ next-to-eikonal
- 2γ exchange in $\ell p \to \ell p$ ($\to$ MUSE, PRad, ..)
- use FKS^3 for dominant N^{3}LO in $e\mu \to e\mu$

McMule

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[d]TU Dresden, Germany
[e]ETH Zurich, CH
[f]Durham, UK

[*] master students

abbreviated user file without cut on b

```
integer, parameter :: nr_q = 2
integer, parameter :: nr_bins = 300
real, parameter :: min_val(nr_q) = (/  0.    , 0.    /)
real, parameter :: max_val(nr_q) = (/ 30.E-3, 6.E-3 /)

FUNCTION QUANT(p1, p2, p3, p4)
real(kind=prec) :: p1(4), p2(4), p3(4), p4(4), quant

q3lab = boost_rf(q1,q3) ; theta_e = acos(cos_th(ez,q3lab))
q4lab = boost_rf(q1,q4) ; theta_m = acos(cos_th(ez,q4lab))

pass_cut = .true.
if(q4lab(4) < 1000.) pass_cut = .false.
if(theta_m<0.3E-3) pass_cut = .false.

names(1) = "thetae" ; quant(1) = theta_e
names(2) = "thetam" ; quant(2) = theta_m

END FUNCTION QUANT
```

prepare code for execution

```
$ pymule create -i
What generic process? em2em
Which flavour combination? muone
How many / which seeds? 4
Which xi cuts? 0.1,0.3,0.5,1.0
Where to store data? test-run
Which pieces? 0,F,R
How much statistics for 0 (pc, pi, c, i)? 5M,20,10M,70
How much statistics for F (pc, pi, c, i)? 5M,20,10M,70
How much statistics for R (pc, pi, c, i)? 20M,20,80M,70
$ ./test-run/submit.sh
```

Data can be found on `gitlab.com/mule-tools/user-library`