Improving the precision in modelling pion reactions in the NuWro event generator

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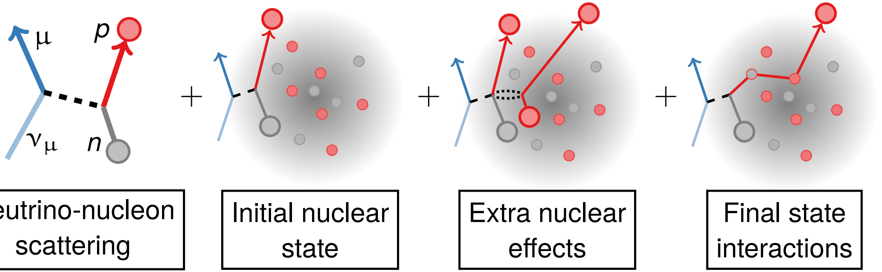
Cross section in the factorized scheme



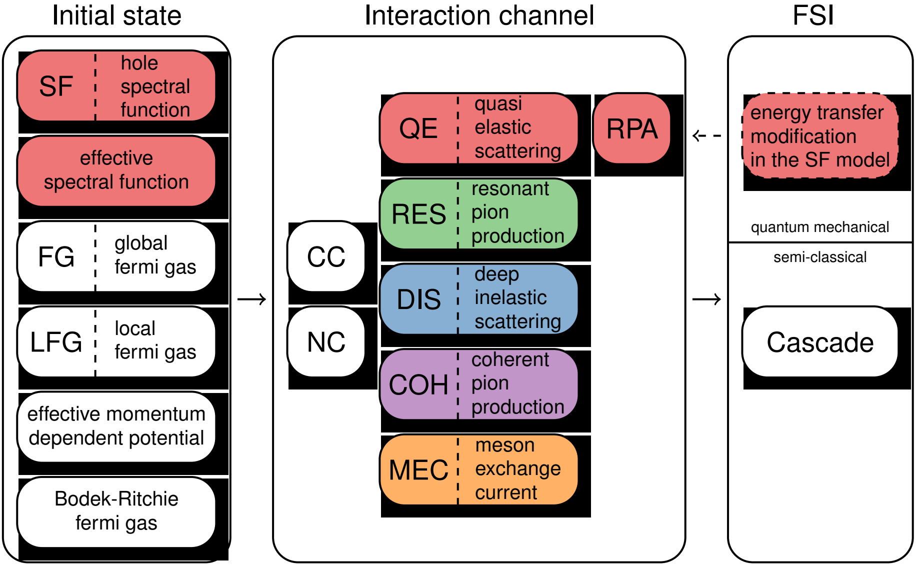
→ **Neutrino-nucleon scattering**: elementary interaction cross section

→ **Initial nuclear state**: modeling nucleons in the nuclear medium before the weak interaction

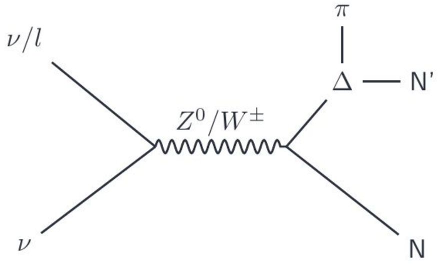
→ **Extra nuclear effects**: multiple-nucleon interactions or correlations

→ **Final state interactions**: in-medium outgoing particle propagation

NuWro blueprint



Single-pion production off the nucleon



The following **resonant channels** are considered in **NuWro**:

$\nu + p \to l^- + (\Delta^{++} \to p + \pi^+)$

$\nu + n \to l^- + (\Delta^{+} \to p + \pi^0 \text{ or } n + \pi^+)$

$\bar{\nu} + p \to l^+ + (\Delta^{0} \to p + \pi^- \text{ or } n + \pi^0)$

$\bar{\nu} + n \to l^+ + (\Delta^{-} \to n + \pi^-)$

$$\nu(\bar{\nu}) + p \to \nu(\bar{\nu}) + (\Delta^{+} \to p + \pi^0 \text{ or } n + \pi^+)$$

$$\nu(\bar{\nu}) + n \to \nu(\bar{\nu}) + (\Delta^{0} \to p + \pi^- \text{ or } n + \pi^0)$$

+ nonresonant background extrapolated from the DIS formalism into lower regions of W, Q^2

Single-pion production off the nucleon

To produce an event, one needs **information about** the **produced pion**

Delta decays in the **hadronic CMS**:

$$\frac{d^2\sigma_\Delta}{dQ^2 dW} \to \frac{d^4\sigma_\pi}{dQ^2 dW} \times \frac{df_\Delta(Q^2)}{d\Omega_\pi^*}$$

Pion angular distributions are essential to **generate** the **kinematics**

In **NuWro**, it is taken from **experimental results** (ANL or BNL):

S.J. Barish et al., Phys.Rev. D19 (1979) 2511

G.M. Radecky et al., Phys.Rev. D25 (1982) 1161

T. Kitagaki et al., Phys.Rev. D34 (1986) 2554

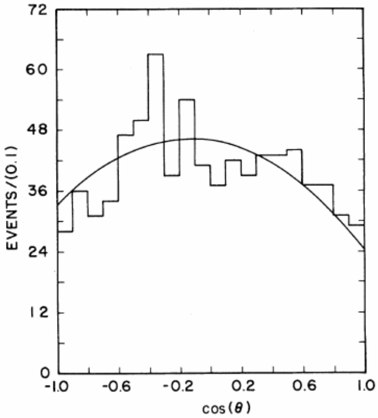


FIG. 15. Distribution of events in the pion polar angle cosθ for the final state $\mu^- p \pi^+$, with $M(p\pi^+) < 1.4$ GeV. The curve is the area-normalized prediction of the Adler model.

Radecky et al. [ANL Collaboration], PRD 25 (1982) 1161

Dimensionality of the problem

Δ-resonance excitation (free nucleon)	Pion production off the nucleon	Pion production on a nucleus
$\frac{d^2\sigma}{dQ^2 dW}$	$\frac{d^4\sigma}{dQ^2 dW d\Omega_\pi^*}$	$\frac{d^8\sigma}{dQ^2 dW d\Omega_\pi^* dE_m d\vec{p}_m}$

+2: *Include angular information about the Δ decay (Ω_π^*)*

+4: *Put the target nucleon inside a nucleus ($E_m, \vec{p}_m$)*

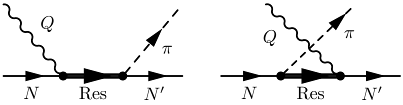
+1 invariant variable: the cross section is always symmetric w.r.t. 1 azimuthal angle, e.g., ϕ_μ

Ghent Low Energy Model

- The **model** of Ref. [R. González-Jiménez et al., **PRD 95 (2017) 113007**]
- The **low-energy** part based on the **Valencia** model

Resonances

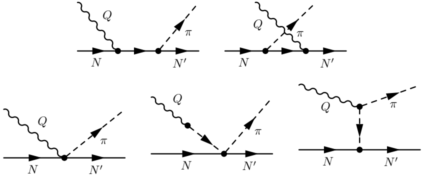
$P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$



based on [PRD 76 033005, PRD 87 113009, PRD 93 014016]

+

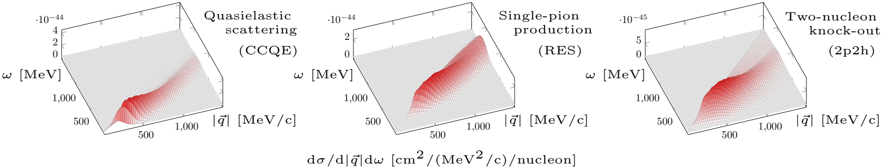
ChPT background



- **Bottleneck** for the implementation is the code **execution time**
- Adding a **nuclear model** will further **slow down** the implementation

Philosophy of the implementation

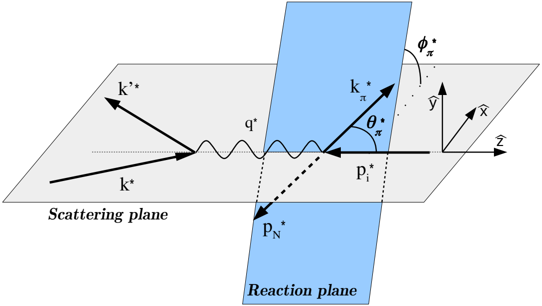
- It is common to use **tabularized** $(\mathrm{d}^2\sigma/\mathrm{d}|\vec{q}|\mathrm{d}\omega)$ **cross sections**
  - $\rightarrow$ enough to get the **lepton kinematics** right



- We add the information from more differential $\mathrm{d}^3\sigma$, $\mathrm{d}^4\sigma$...
  - $\rightarrow$ we sample the **next variable** with **previous** already **fixed**
  - $\rightarrow$ we **minimize** the number of **code evaluations**
- Mathematically, we do the **importance sampling**:

$$\int f_{4D}(x)[u(x)\mathrm{d}x] \rightarrow \int \frac{f_{4\mathrm{D}}(x)}{f_{3\mathrm{D}}(x)}[f_{3\mathrm{D}}(x)u(x)\mathrm{d}x] \rightarrow \int \frac{f_{4\mathrm{D}}(x)}{f_{3\mathrm{D}}(x)}\frac{f_{3\mathrm{D}}(x)}{f_{2\mathrm{D}}(x)}[f_{2\mathrm{D}}(x)u(x)\mathrm{d}x]$$

Single-pion production off the nucleon



Within such frame, one can factorize the ϕ_π^* dependence:

$$\frac{d^4\sigma}{dQ^2 dW d\Omega_\pi^*} = \frac{\mathcal{F}^2}{(2\pi)^4} \frac{k_\pi^*}{k_i^2} \left[A + B\cos(\phi_\pi^*) + C\cos(2\phi_\pi^*) + D\sin(\phi_\pi^*) + E\sin(2\phi_\pi^*)\right]$$

$\rightarrow$ where **A**,**B**,**C**,**D**,**E** are functions of $(Q^2, W, \cos\theta_\pi^*)$

J.E. Sobczyk et al., Phys.Rev. D98 (2018) 073001

4D algorithm

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}Q^2\mathrm{d}W\mathrm{d}\Omega^*_\pi} = \frac{\mathcal{F}^2}{(2\pi)^4}\frac{k^*_\pi}{k_i^2}\left[A + B\cos(\phi^*_\pi) + C\cos(2\phi^*_\pi) + D\sin(\phi^*_\pi) + E\sin(2\phi^*_\pi)\right]$$

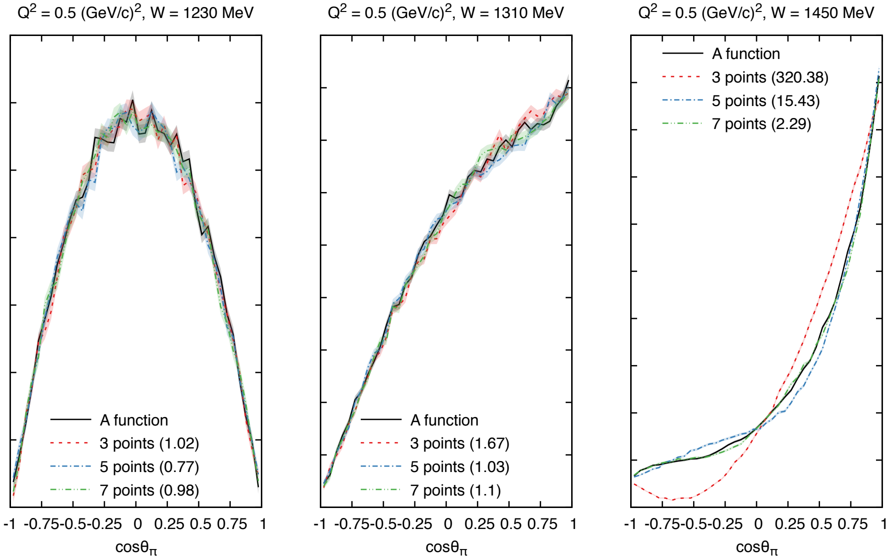
- Sample the 4D phase space (Q^2,W,$\cos\theta^*_\pi$,ϕ^*_π)
- For every point calculate the weight from an **explicit calculation**

3D algorithm

$$\int \frac{\mathrm{d}^4\sigma}{\mathrm{d}Q^2\mathrm{d}W\mathrm{d}\Omega^*_\pi}\mathrm{d}\phi^* = \frac{\mathrm{d}^3\sigma}{\mathrm{d}Q^2\mathrm{d}W\mathrm{d}\cos\theta^*_\pi} = \frac{\mathcal{F}^2}{(2\pi)^4}\frac{k^*_\pi}{k_i^2}\left[2\pi A(Q^2, W, \cos\theta^*_\pi)\right]$$

- Sample the 3D phase space (Q^2,W,$\cos\theta^*_\pi$)
- For every point calculate the weight from an **explicit calculation**
   - **Only for the accepted events** sample the forth variable ϕ^*_π from $\mathrm{d}^4\sigma/\mathrm{d}Q^2\mathrm{d}W\mathrm{d}\Omega^*_\pi$

The A function



- For most of the phase space, **the function is a parabola**

2D algorithm

$$\int \frac{\mathrm{d}^4\sigma}{\mathrm{d}Q^2\mathrm{d}W\mathrm{d}\Omega_\pi^*}\mathrm{d}\Omega_\pi^* = \frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2\mathrm{d}W} = \frac{\mathcal{F}^2}{(2\pi)^4}\frac{k_\pi^*}{k_i^2}\left[\int 2\pi A(Q^2, W, \cos\theta_\pi^*)\mathrm{d}\cos\theta_\pi^*\right]$$

- Sample the 2D phase space (Q^2,W)
- For every point calculate the weight using **precomputed arrays**

◦ **Only for the accepted events** sample the third variable $\cos\theta_\pi^*$:

→ ($k = 3$) Fitting a parabola and using an analytical inversion

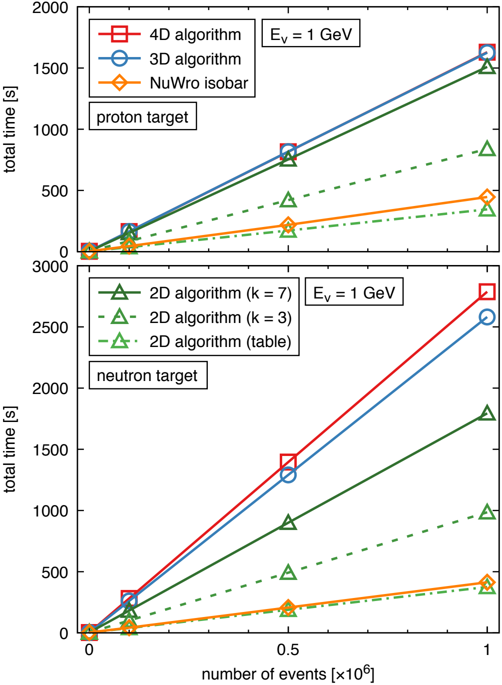
→ ($k = ?$) Fitting a polynomial of the order $k - 1$ and using a numerical inversion

→ (*table*) Using tabularized distributions

◦ **Only for the accepted events** sample the forth variable ϕ_π^* from $\mathrm{d}^4\sigma/\mathrm{d}Q^2\mathrm{d}W\mathrm{d}\Omega_\pi^*$

Performance

- How many times the cross section is calculated? (per accepted event)
  - 4D alg.: 1
  - 3D alg.: 2
  - 2D alg. ($k = 3$): 4
  - 2D alg. ($k = 7$): 8
  - 2D alg. (table): 1
- +1 for the neutron target because of 2 channels ($p + \pi^0$, $n + \pi^+$)



Performance

$$S_N = N \cdot \tau \cdot (1+\alpha) + (\frac{N}{\epsilon} - N) \cdot \tau = N \cdot \tau \cdot (\frac{1}{\epsilon} + \alpha).$$

N - accepted events τ *- trial event cost [arb.u.]* ϵ *- efficiency* α - additional cost for accepted events

	model	τ	ϵ	α	$S_{1\mathrm{M}}$
	4D alg.	8.01e-07	0.12	-	6.9
	3D alg.	8.02e-07	0.13	1.0	6.9
2D alg.	$(k=7)$	4.04e-08	0.16	143.9	6.1
	$(k=3)$	4.04e-08	0.16	72.0	3.2
	(table)	4.03e-08	0.16	18.6	1.0

(a) $E = 1.0$ GeV neutrinos off proton target.

	model	τ	ϵ	α	$S_{1\mathrm{M}}$
	4D alg.	1.83e-06	0.15	-	12.1
	3D alg.	1.83e-06	0.18	0.5	11.2
2D alg.	$(k=7)$	4.11e-08	0.21	169.4	7.2
	$(k=3)$	4.10e-08	0.21	85.1	3.7
	(table)	4.08e-08	0.21	22.0	1.1

(b) $E = 1.0$ GeV neutrinos off neutron target.

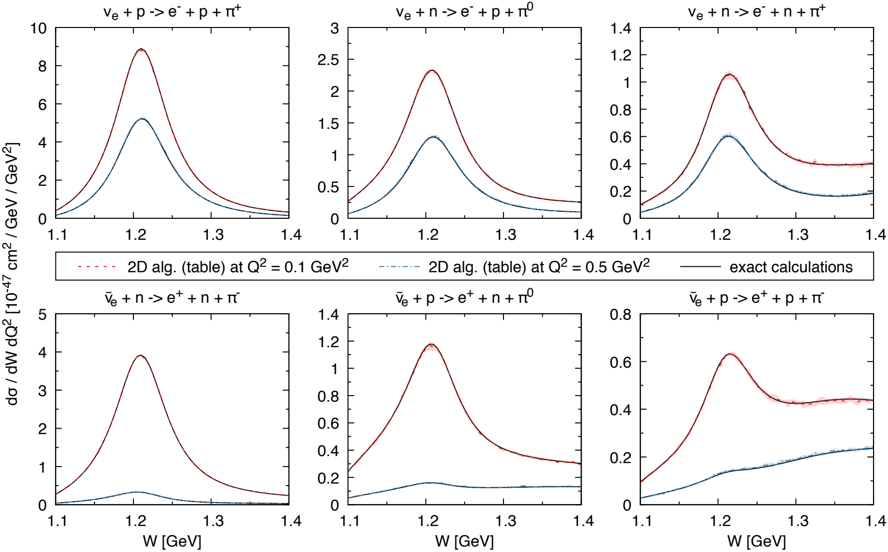
	model	τ	ϵ	α	$S_{1\mathrm{M}}$
	4D alg.	8.04e-07	0.08	-	9.9
	3D alg.	8.01e-07	0.10	1.0	8.8
2D alg.	$(k=7)$	3.98e-08	0.12	149.1	6.3
	$(k=3)$	4.08e-08	0.12	72.6	3.3
	(table)	4.04e-08	0.12	19.0	1.1

(c) $E = 2.5$ GeV neutrinos off proton target.

	model	τ	ϵ	α	$S_{1\mathrm{M}}$
	4D alg.	1.84e-06	0.14	-	13.5
	3D alg.	1.83e-06	0.18	0.5	11.4
2D alg.	$(k=7)$	4.19e-08	0.20	169.6	7.3
	$(k=3)$	4.13e-08	0.20	86.0	3.8
	(table)	4.12e-08	0.20	22.3	1.1

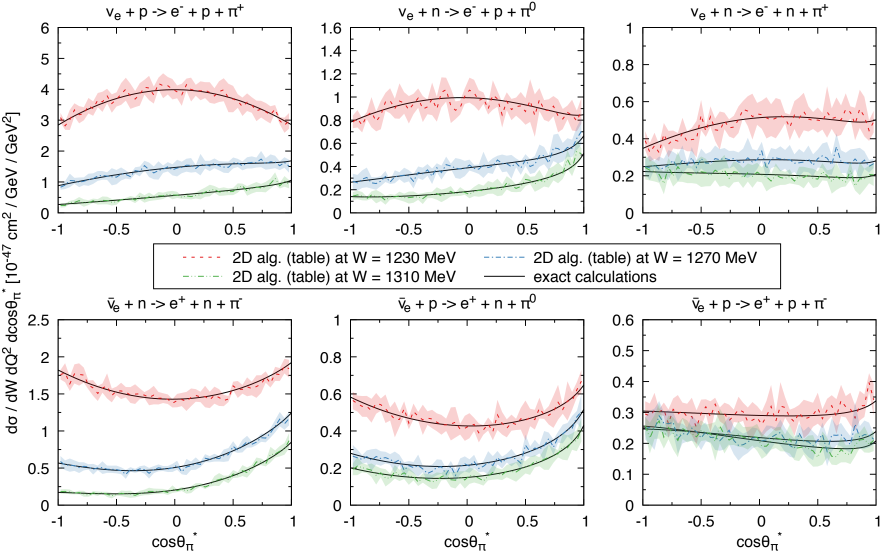
(d) $E = 2.5$ GeV neutrinos off neutron target.

Double-differential cross section



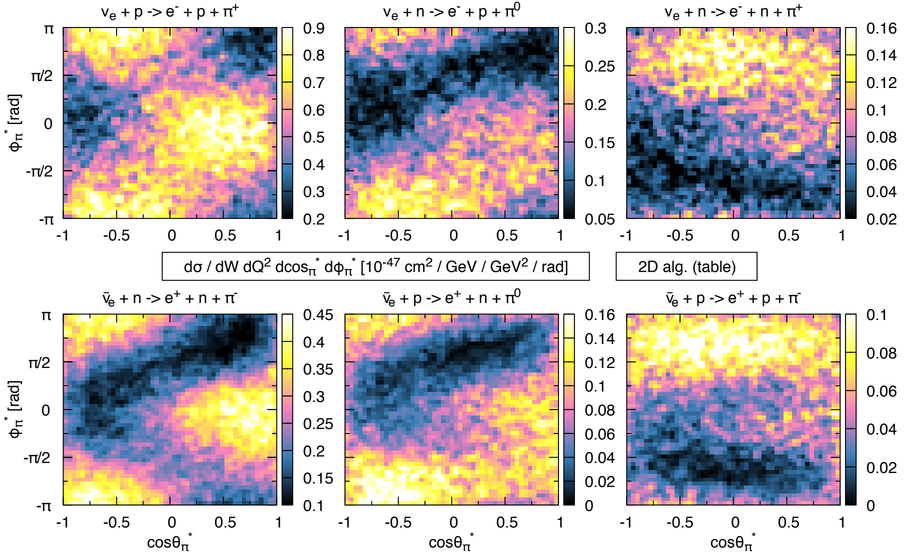
(total number of 10^7 ***events*** *across the* ***whole phase space****)*

Triple-differential cross section



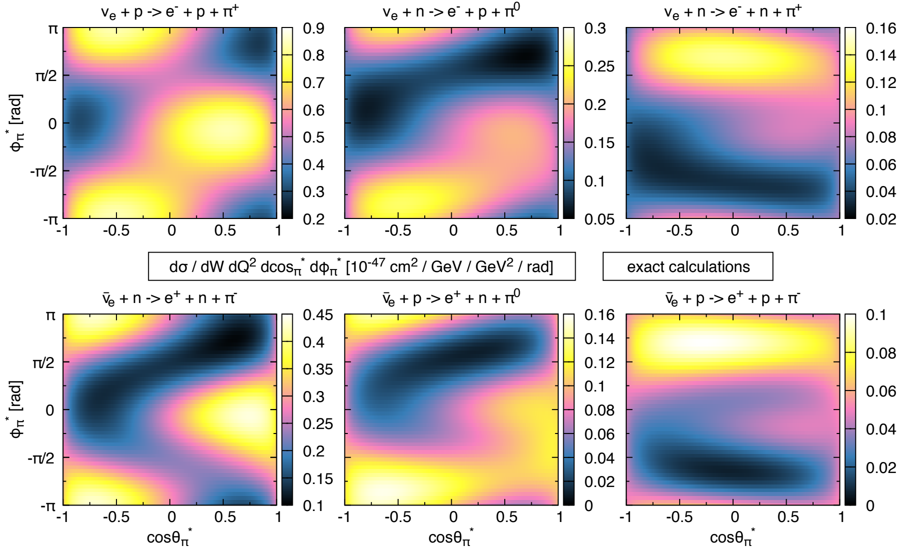
(total number of 10^7 ***events*** *across the* ***whole phase space****)*

Quadruple-differential cross section



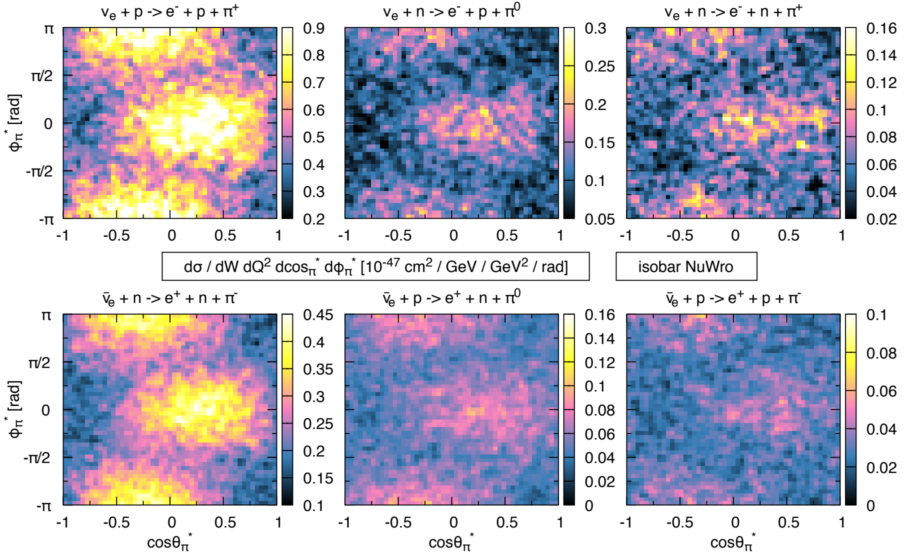
*(total number of 10^7 **events** across the **whole phase space**)*

Quadruple-differential cross section



*(numerical results on a **dense grid** with **selected kinematics**)*

Quadruple-differential cross section



(total number of 10^7 ***events*** *across the* ***whole phase space****)*

Summary

- We have implemented the **Ghent Low Energy Model** into **NuWro**
  - $\rightarrow$ **only off the nucleon**
  - $\rightarrow$ **theoretical predictions** for **angular pion distributions**
- We have investigated various **methods of optimization** in SPP
  - $\rightarrow$ different trade-offs between **efficiency**, **precision**, and **reliance on precomputed assets**
  - $\rightarrow$ the framework is **model-independent**

◦ **arXiv:2011.05269** (submitted to PRD)

Angular distributions in Monte Carlo event generation of weak single-pion production

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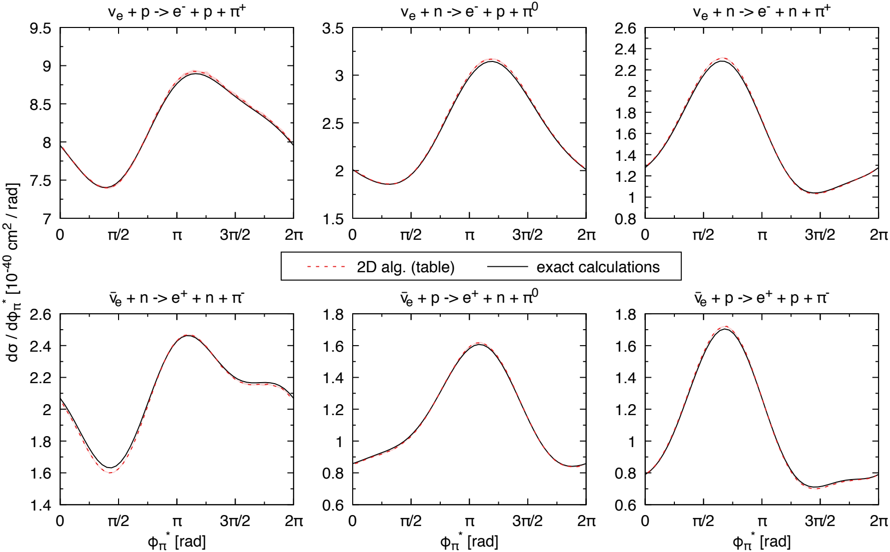
(Dated: November 16, 2020)

◦ we are investigating a consistent approach to the ν-nucleus case...

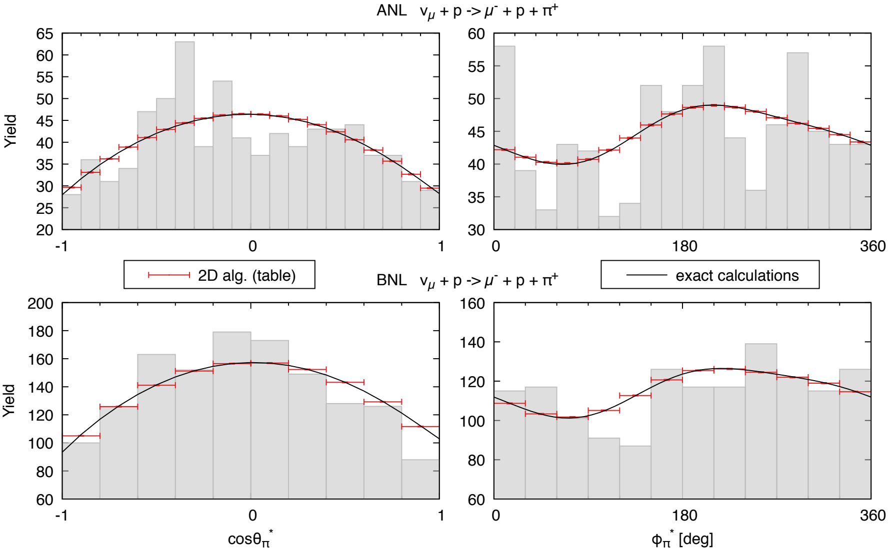
Backup slides

Single-differential cross section



(total number of 10^7 ***events*** *across the* ***whole phase space****)*

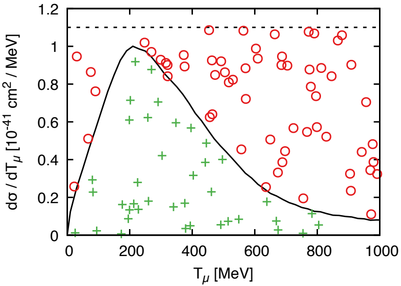
Single-differential cross section



(total number of 10^7 ***events*** *across the* ***whole phase space****)*

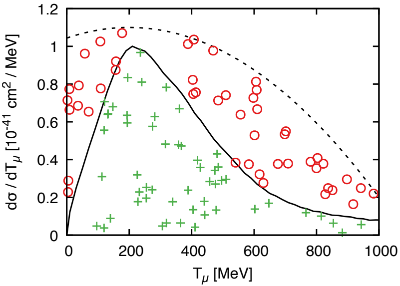
Importance sampling

efficiency $\approx$ 39%



$\rightarrow$

efficiency $\approx$ 53%



$$\int f(x)p(x)\mathrm{d}x \quad \rightarrow \quad \int \frac{f(x)p(x)}{g(x)} g(x)\mathrm{d}x$$

$p(x)$ - *nominal distribution* $g(x)$ - *importance distribution* $p(x)/g(x)$ - *likelihood ratio*

- useful, if we can sample $[g(x)\mathrm{d}x]$ analytically
- **also in many dimensions, if we sample more efficient ones first!**