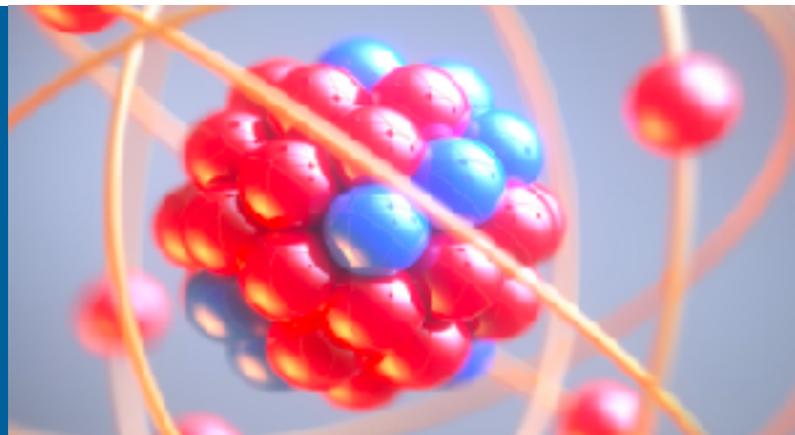


AB-INITIO NUCLEAR CALCULATIONS FOR NEUTRINO-NUCLEUS CROSS SECTIONS



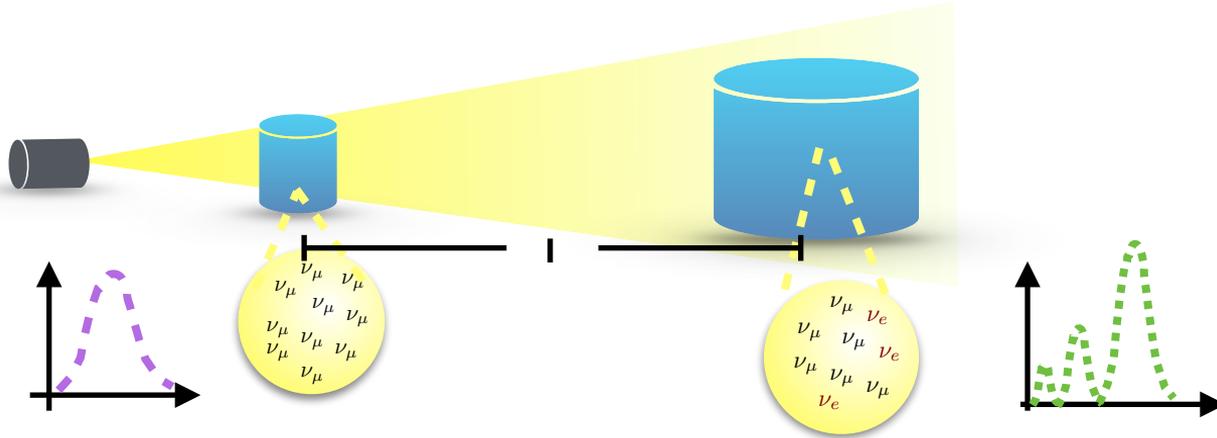
ALESSANDRO LOVATO
Argonne National Laboratory
&
Istituto Nazionale di Fisica Nucleare

10-11 February 2021

IPPP topical meeting on physics with high-brightness
stored muon beams

INTRODUCTION

Extracting oscillation parameters requires comparing the neutrino flux at near and far detectors



The flux is extracted from the measured neutrino-nucleus interactions in a detector

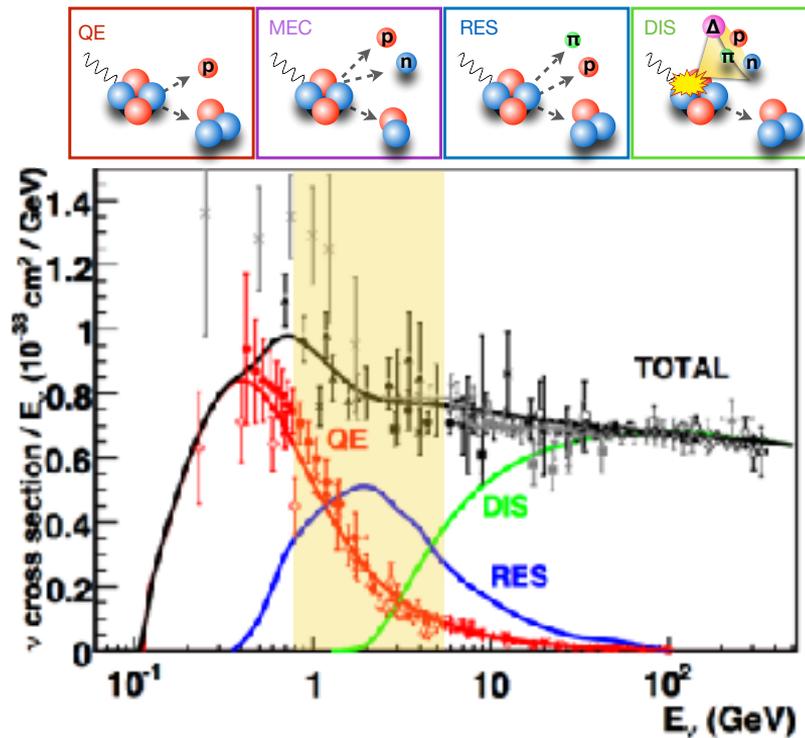
$$N_e(E_{\text{rec}}, L) \propto \sum_i \Phi_e(E, L) \sigma_i(E) f_{\sigma_i}(E, E_{\text{rec}}) dE$$

Knowledge of the neutrino-nucleus cross section \longrightarrow Precision on neutrino-oscillation parameters

INTRODUCTION

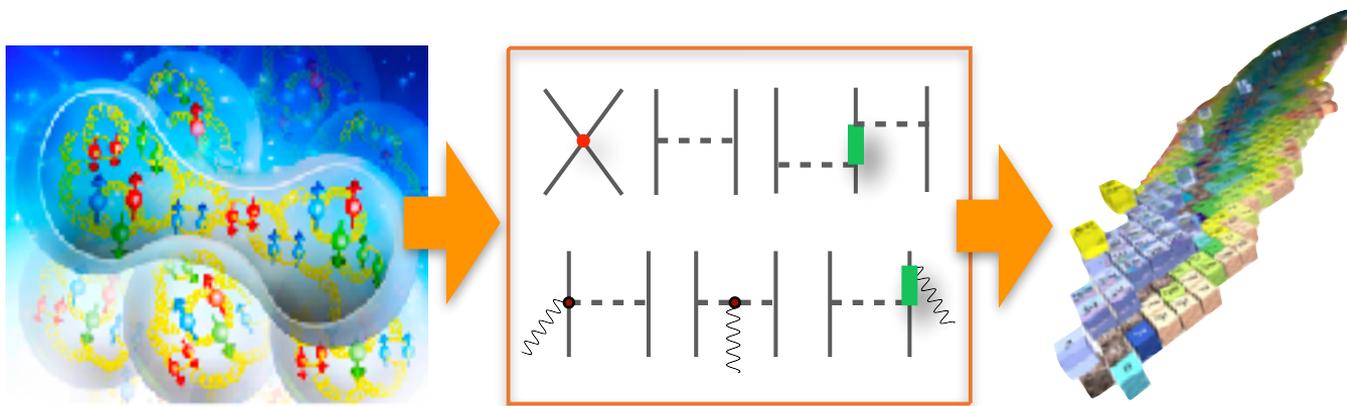
Achieving a robust description of the reaction mechanisms at play in the DUNE energy regime is a **formidable nuclear-theory challenge**

- Realistic description of nuclear correlations
- Relativistic effects in the current operators and kinematics
- Description of resonance-production and DIS region



MICROSCOPIC MODEL OF NUCLEAR THEORY

- In the low-energy regime, quark and gluons are confined within hadrons.
- The relevant degrees of freedom are protons, neutrons, and pions
- Effective field theories are the link between QCD and nuclear observables



Systematically improvable Hamiltonians and consistent electroweak currents

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

$$J = \sum_i j_i + \sum_{i<j} j_{ij} + \dots$$

NUCLEAR MANY-BODY METHODS



$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$



$$J_{mn} = \langle\Psi_m|J|\Psi_n\rangle$$

MANY-BODY SCHRÖDINGER EQUATION

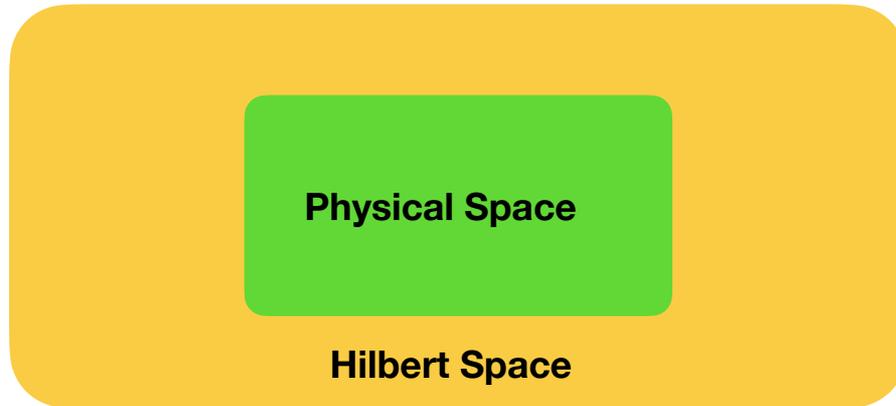
Non relativistic many body theory is aimed at solving the Schrödinger equation

$$H\Psi_n(x_1, \dots, x_A) = E_n\Psi_n(x_1, \dots, x_A) \iff x_i = \{\mathbf{r}_i, s_{i,z}, t_{i,z}\}$$

An exact solution of this equation is an exponentially hard problem

$$|\Psi\rangle = c_{\uparrow\uparrow\uparrow\dots}|\uparrow\uparrow\uparrow\dots\rangle + c_{\downarrow\uparrow\uparrow\dots}|\downarrow\uparrow\uparrow\dots\rangle + \dots + c_{\downarrow\downarrow\downarrow\dots}|\downarrow\downarrow\downarrow\dots\rangle$$

The majority of quantum states of interest for have distinctive features and intrinsic structure

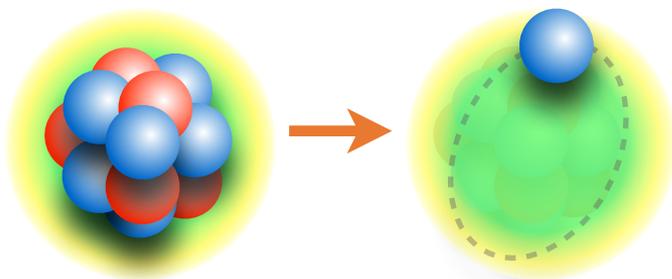


MEAN-FIELD METHODS

Mean-field theory: nucleons are independent particles subject to an average nuclear potential

$$\left[\sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} \right] \rightarrow \sum_i U_i$$

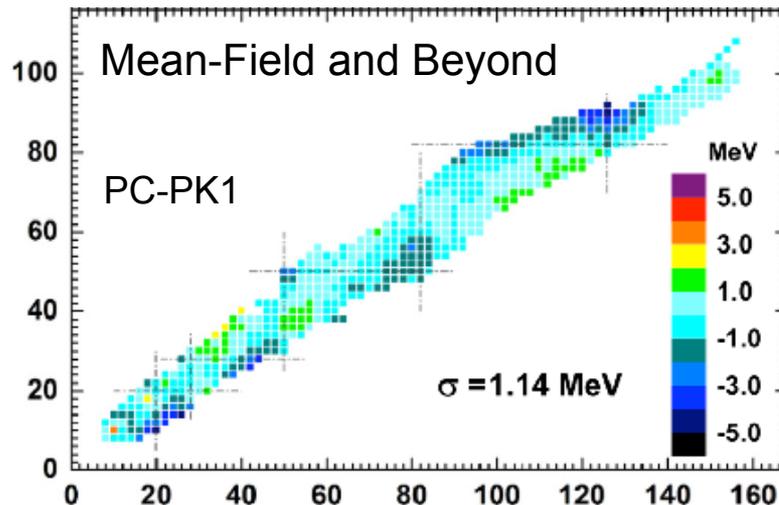
$$\Phi_0(x_1, \dots, x_A) = \mathcal{A}[\phi_{n_1}(x_1) \dots \phi_{n_A}(x_A)]$$



MFT is the tool of choice for describing large nuclei:

- Nucleon-nucleon scattering data and deuteron properties are ignored
- No clear way to derive effective currents

The interaction is fitted on nuclear binding energies and charge radii of stable nuclei



P. W. Zhao, et al. PRC 82, 054319 (2010)

BASIS-EXPANSION METHODS

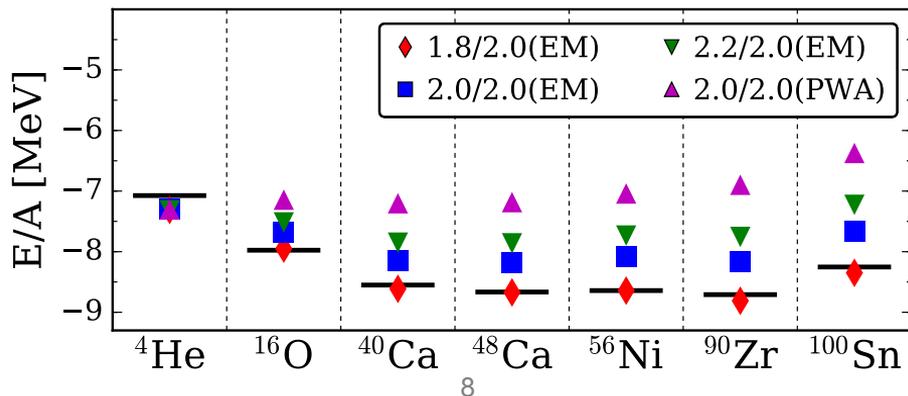
Any fermionic wave-function may be written as a linear combination of Slater determinants

$$|\Psi_0\rangle = \sum_n c_n |\Phi_n\rangle$$

Methods relying on single-particle basis expansions include the no-core shell model, the coupled-cluster theory, the in-medium similarity renormalization group method

The can describe nuclei with up to to $A=100$ protons and neutrons starting from the individual interactions among their constituents

T. D. Morris et al., PRL 120, 152503 (2018)

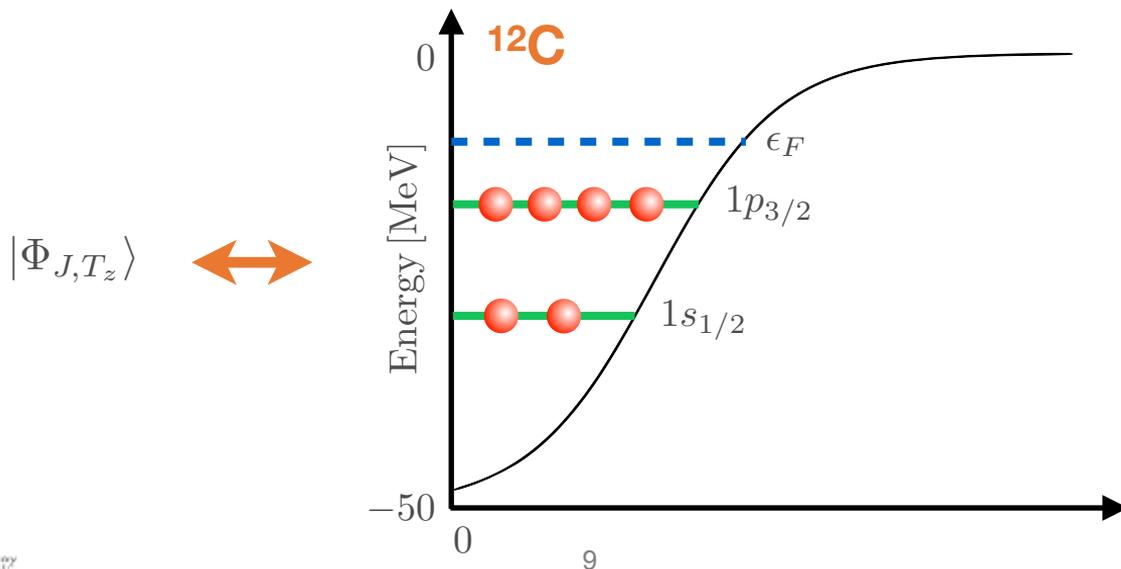


QUANTUM MONTE CARLO

The variational Monte Carlo wave function has correlations built in

$$|\Psi_T\rangle = \left(1 + \sum_{ijk} F_{ijk}\right) \left(\mathcal{S} \prod_{i<j} F_{ij}\right) |\Phi_{J,T_z}\rangle \iff E_T = \langle \Psi_T | H | \Psi_T \rangle \geq E_0$$

Mean-field component: Slater determinant of single-particle orbitals

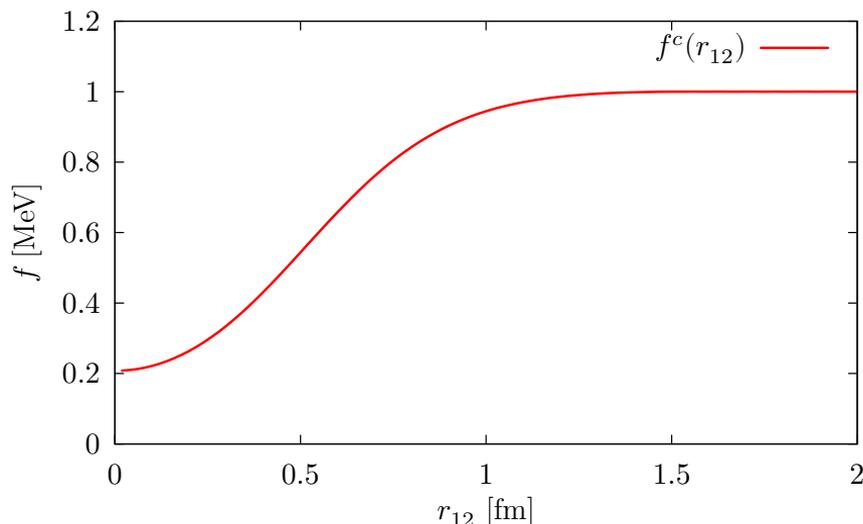
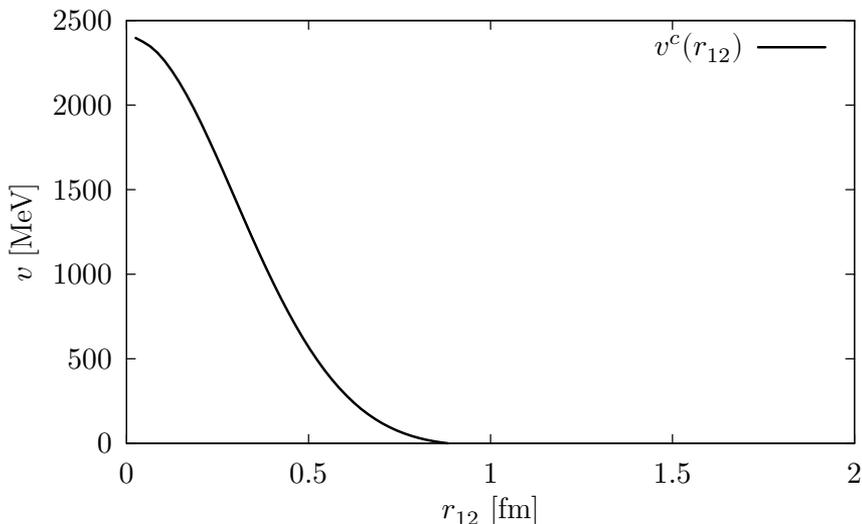


QUANTUM MONTE CARLO

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The correlations are consistent with the underlying nuclear interaction



QUANTUM MONTE CARLO

The trial wave function can be expanded in the set of the Hamiltonian eigenstates

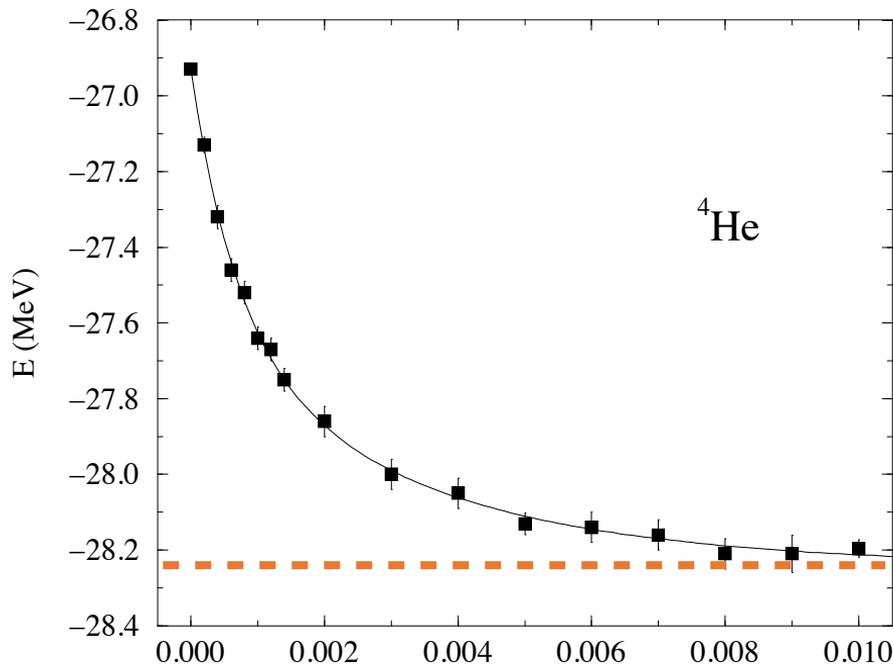
$$|\Psi_T\rangle = \sum_n c_n |\Psi_n\rangle$$

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

The GFMC projects out the lowest-energy state

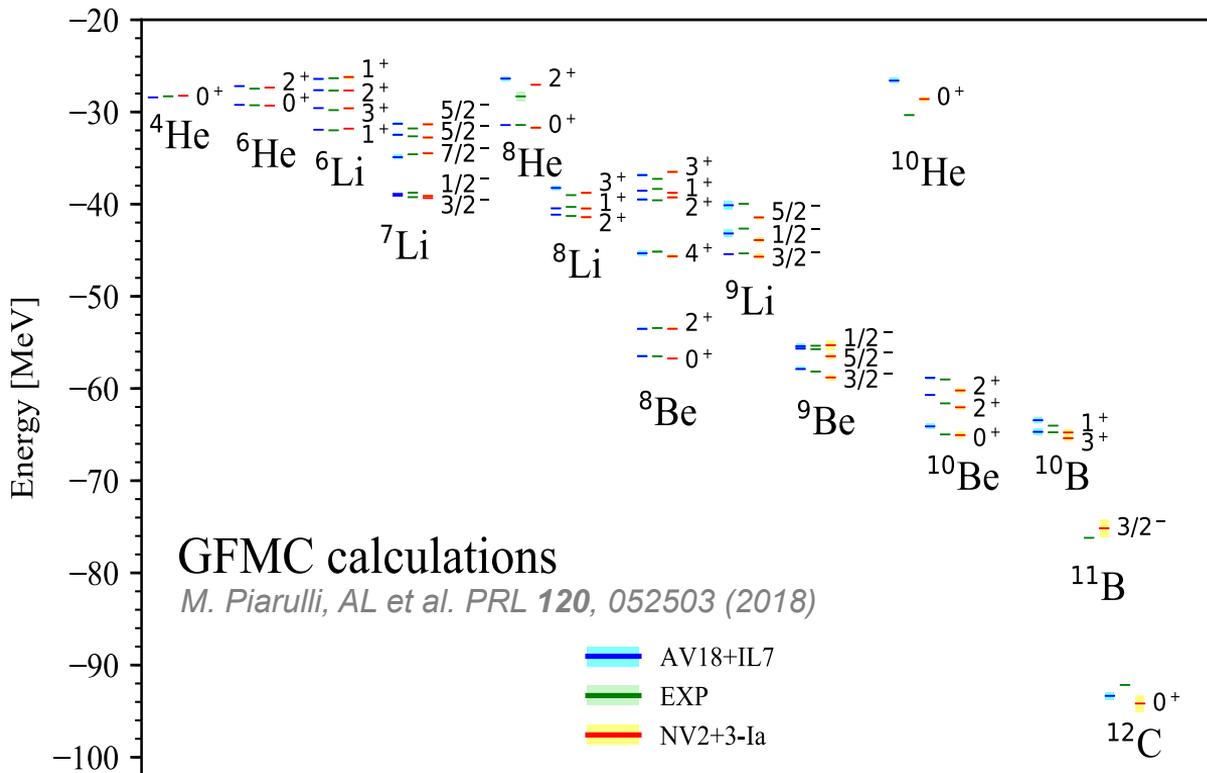
$$\lim_{\tau \rightarrow \infty} e^{-(H-E_0)\tau} |\Psi_T\rangle = c_0 |\Psi_0\rangle$$

B. Pudliner et al., PRC 56, 1720 (1997)

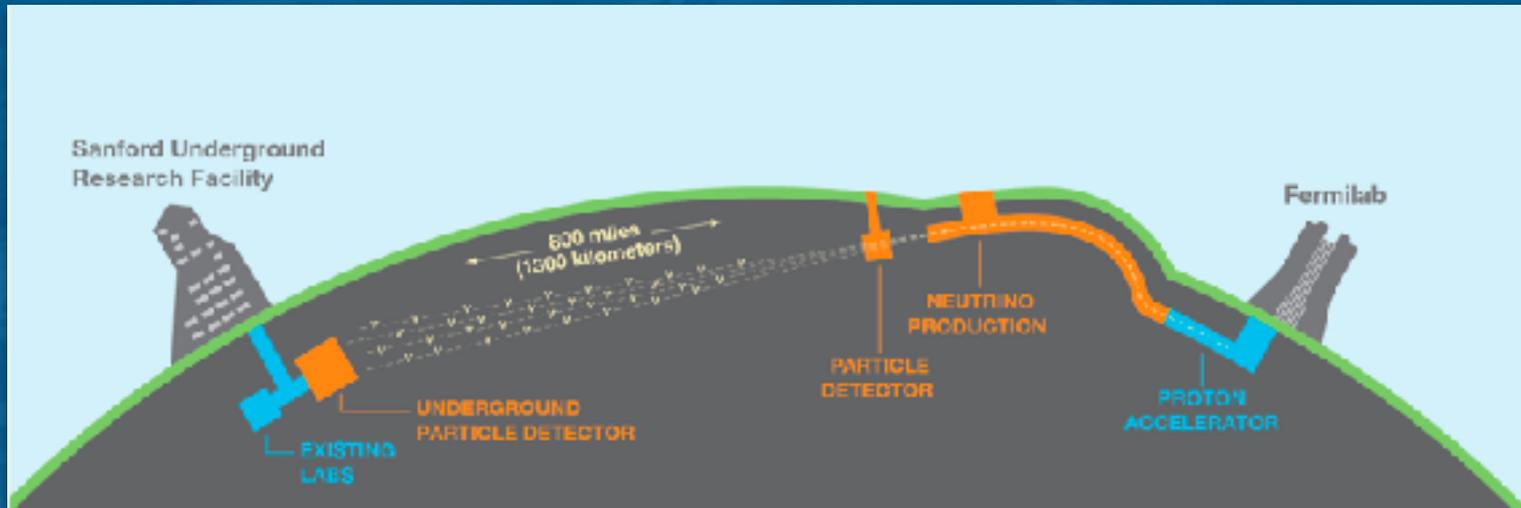


QUANTUM MONTE CARLO

GFMC solves the spectrum of light nuclei with percent-level accuracy

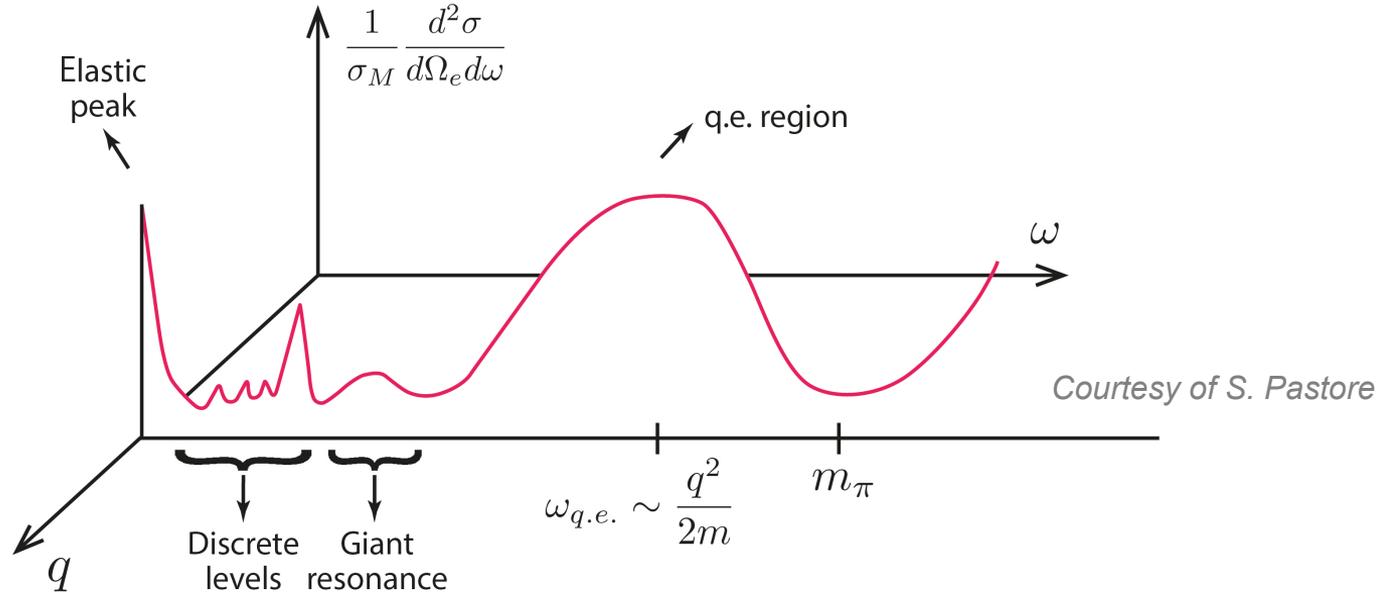


NEUTRINO-NUCLEUS SCATTERING



NEUTRINO-NUCLEUS SCATTERING

The inclusive cross section is characterized by a variety of reaction mechanisms



The response functions contain all nuclear-dynamics information

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle \Psi_0 | J_\alpha^\dagger(\mathbf{q}) | \Psi_f \rangle \langle \Psi_f | J_\beta(\mathbf{q}) | \Psi_0 \rangle \delta(\omega - E_f + E_0)$$

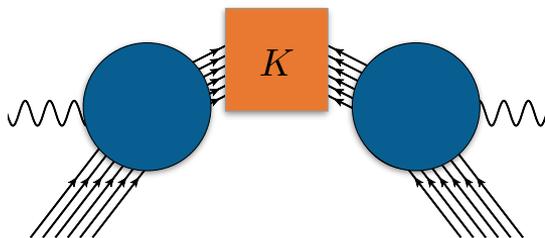
EUCLIDEAN RESPONSES

The integral transform of the response function is defined as

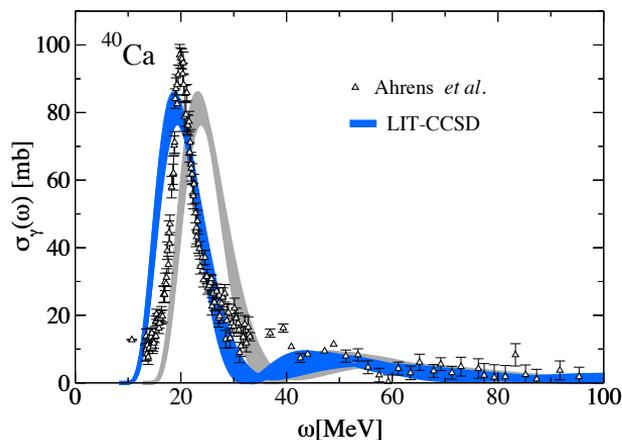
$$\begin{aligned} E_{\alpha\beta}(\sigma, \mathbf{q}) &\equiv \int d\omega K(\sigma, \omega) R_{\alpha\beta}(\omega, \mathbf{q}) \\ &= \sum_f \int d\omega K(\sigma, \omega) \langle \Psi_0 | J_\alpha^\dagger(\mathbf{q}) | \Psi_f \rangle \langle \Psi_f | J_\beta(\mathbf{q}) | \Psi_0 \rangle \delta(\omega - E_f + E_0) \end{aligned}$$

Using the completeness of the final states, it is expressed as a ground-state expectation value

$$E_{\alpha\beta}(\sigma, \mathbf{q}) = \langle \Psi_0 | J_\alpha^\dagger(\mathbf{q}) K(\sigma, H - E_0) J_\beta(\mathbf{q}) | \Psi_0 \rangle$$



Examples include the Lorentz and the Gauss integral transforms



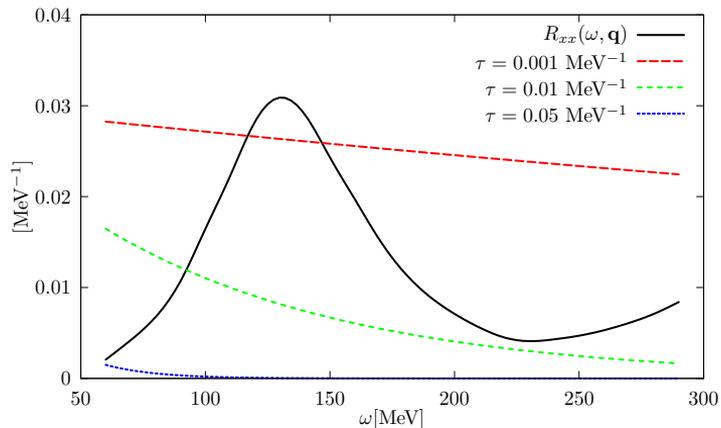
S. Bacca et al., PRC 6, 064619 (2014)

EUCLIDEAN RESPONSES

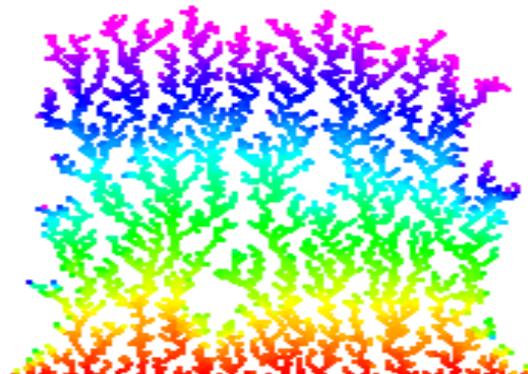
Another type of integral transform is the Laplace transform

$$E_{\alpha\beta}(\tau, \mathbf{q}) \equiv \int d\omega e^{-\omega\tau} R_{\alpha\beta}(\omega, \mathbf{q})$$

At finite imaginary time the contributions from large energy transfer are quickly suppressed



The system is first heated up by the transition operator. Its cooling determines the Euclidean response of the system



$$E_{\alpha\beta}(\tau, \mathbf{q}) = \langle \Psi_0 | J_{\alpha}^{\dagger}(\mathbf{q}) e^{-(H-E_0)\tau} J_{\beta}(\mathbf{q}) | \Psi_0 \rangle$$

$$\sum_f |\Psi_f\rangle \langle \Psi_f|$$

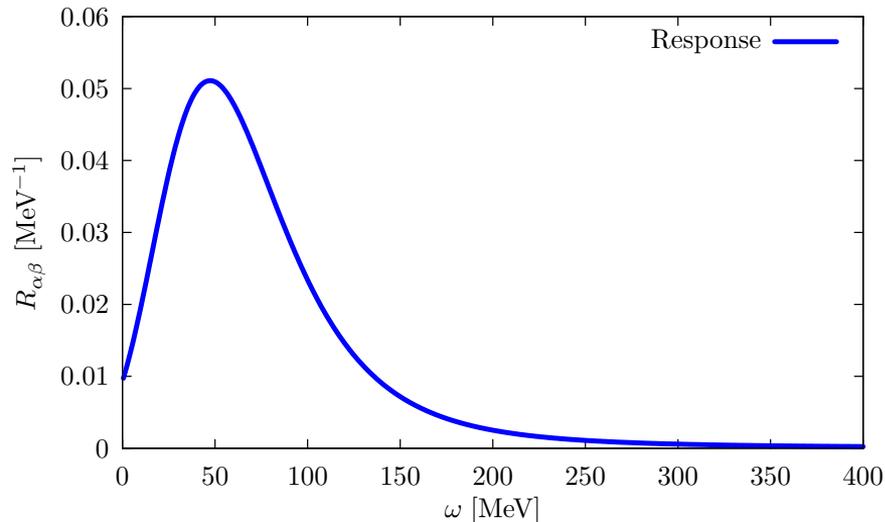
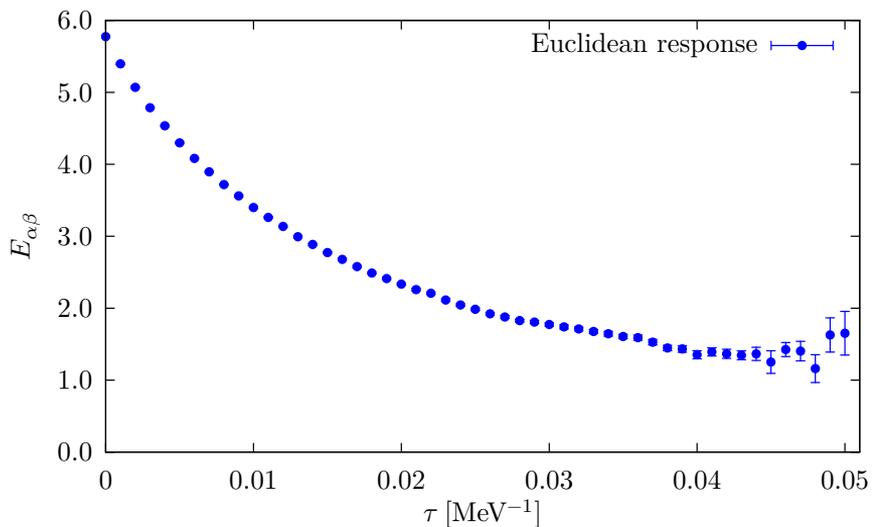
EUCLIDEAN RESPONSES

Inverting the Euclidean response is an ill posed problem: any set of observations is limited and noisy and the situation is even worse since the kernel is a smoothing operator.

$$E_{\alpha\beta}(\tau, \mathbf{q})$$

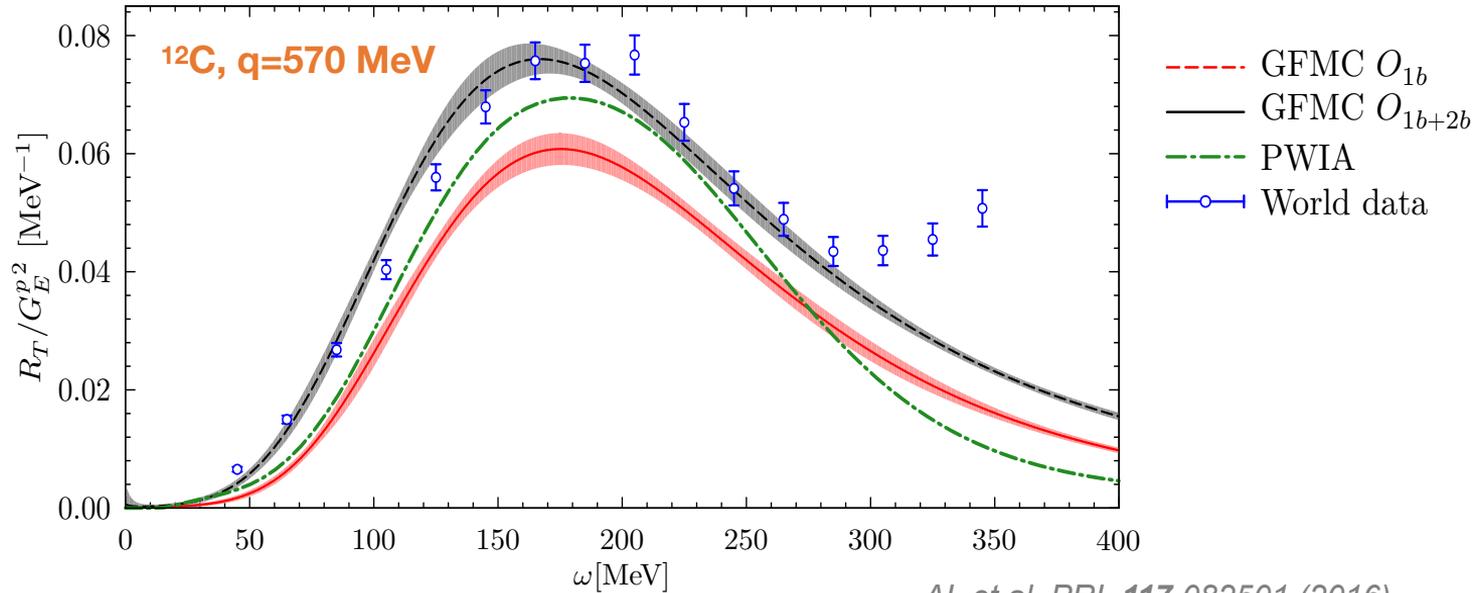


$$R_{\alpha\beta}(\omega, \mathbf{q})$$



We find Maximum-entropy techniques to be reliable enough for quasi-elastic responses

VALIDATION WITH ELECTRON SCATTERING



AL et al. PRL 117 082501 (2016)

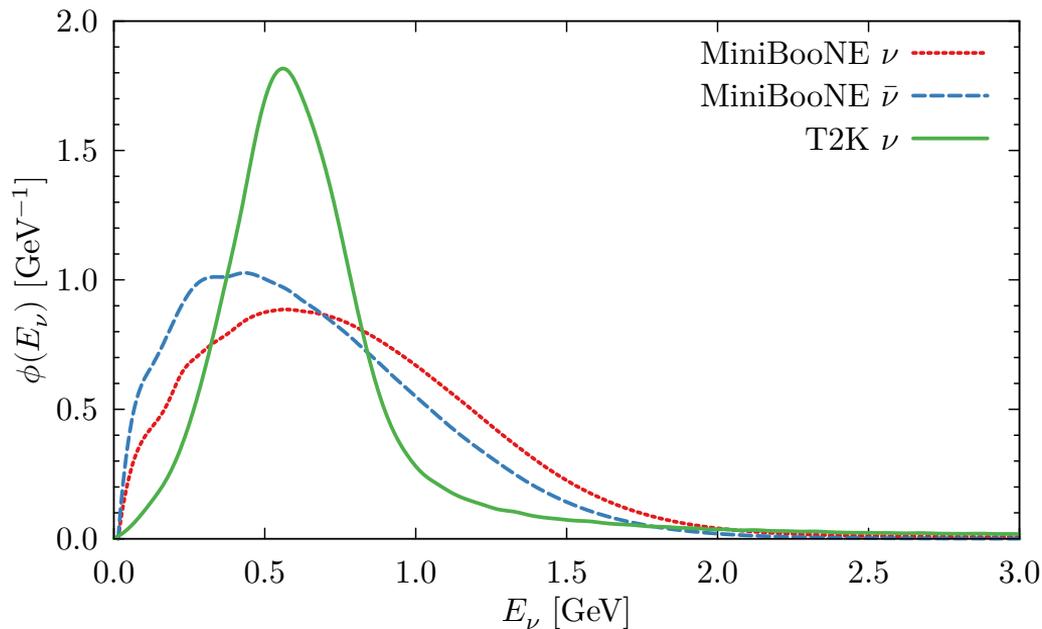
Two-body currents generate additional strength in over the whole quasi-elastic region

Correlations redistribute strength from the quasi-elastic peak to high-energy transfer regions

^{12}C CHARGED-CURRENT CROSS SECTIONS

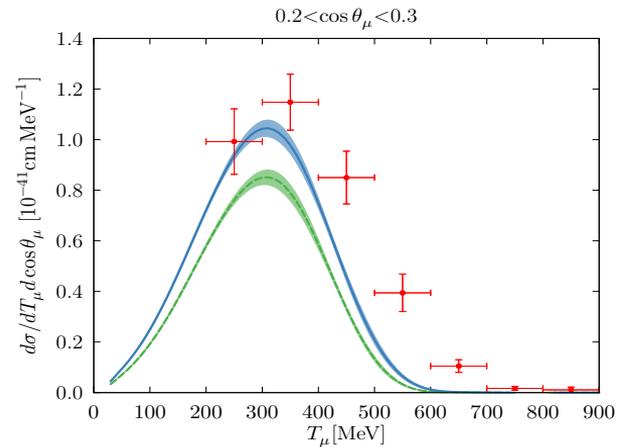
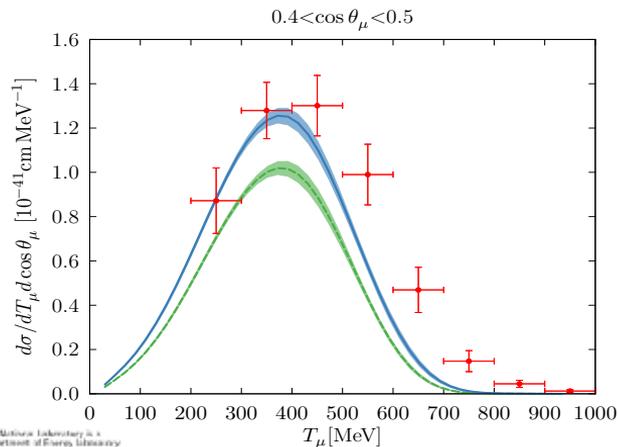
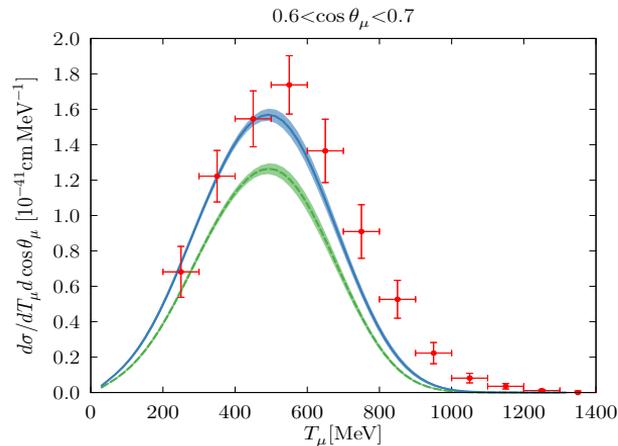
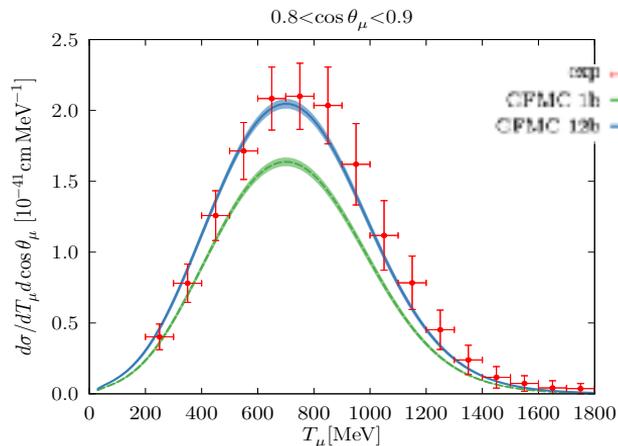
To obtain the inclusive cross section, we fold the MiniBooNE and T2K fluxes

$$\left\langle \frac{d\sigma}{dT_\mu d\cos\theta_\mu} \right\rangle = \int dE_\nu \phi(E_\nu) \frac{d\sigma(E_\nu)}{dT_\mu d\cos\theta_\mu},$$



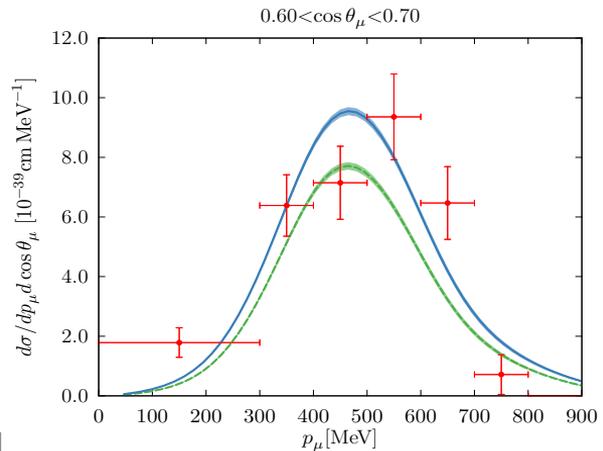
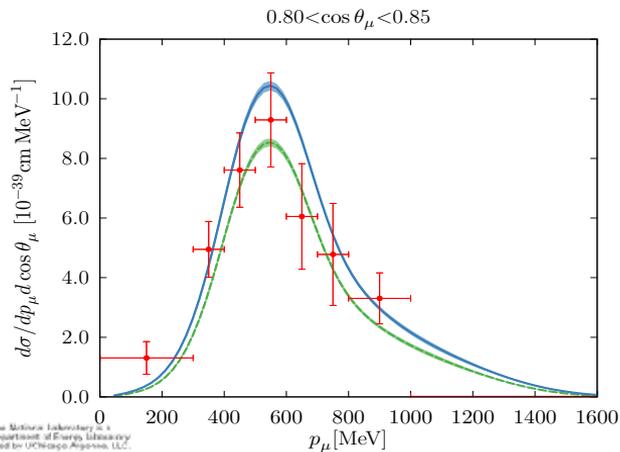
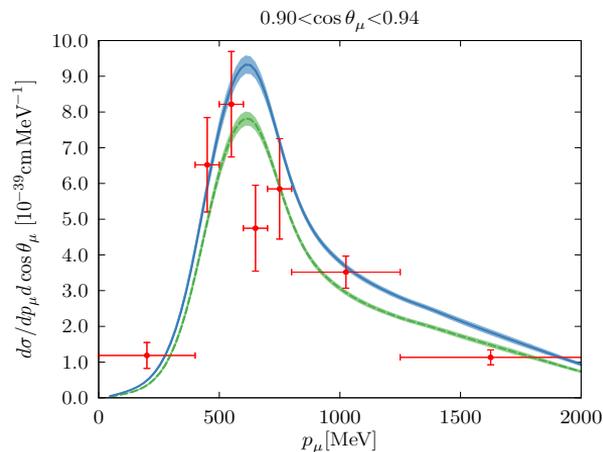
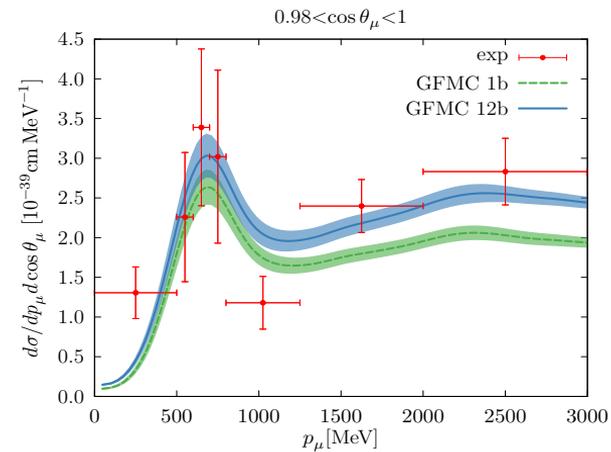
MINIBOONE CROSS SECTIONS

AL et al., PRX 10, 031068 (2020)

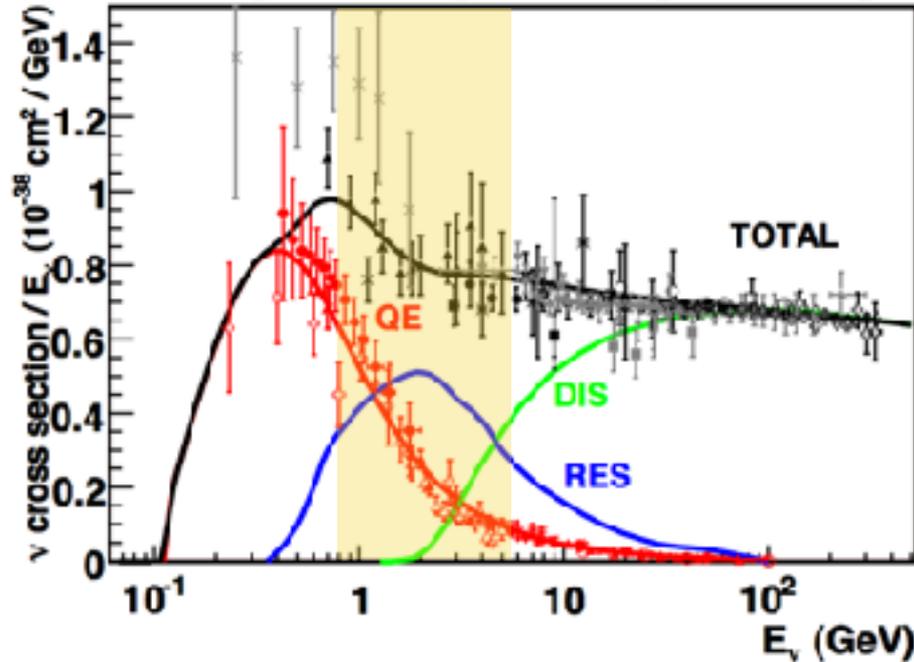
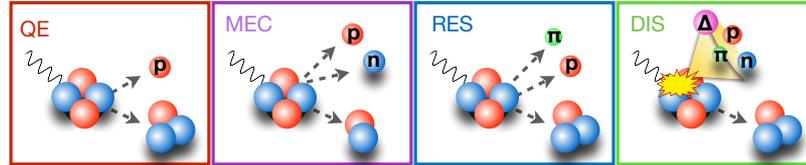


T2K CROSS SECTIONS

AL et al., PRX 10, 031068 (2020)



ADDRESSING DUNE'S PHYSICS



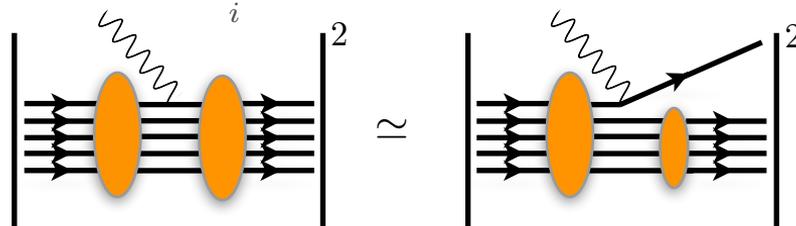
FACTORIZATION SCHEME

At large momentum transfer, the scattering reduces to the sum of individual terms

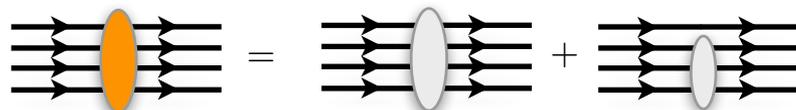
$$J^\mu \rightarrow \sum_i j_i^\mu \quad |\psi_f^A\rangle \rightarrow |p\rangle \otimes |\psi_f^{A-1}\rangle \quad E_f = E_f^{A-1} + e(\mathbf{p})$$

The incoherent contribution of the one-body response reads

$$R_{\alpha\beta} \simeq \int \frac{d^3k}{(2\pi)^3} dE P_h(\mathbf{k}, E) \sum_i \langle k | j_\alpha^{i\dagger} | k+q \rangle \langle k+q | j_\beta^i | k \rangle \delta(\omega + E - e(\mathbf{k} + \mathbf{q}))$$

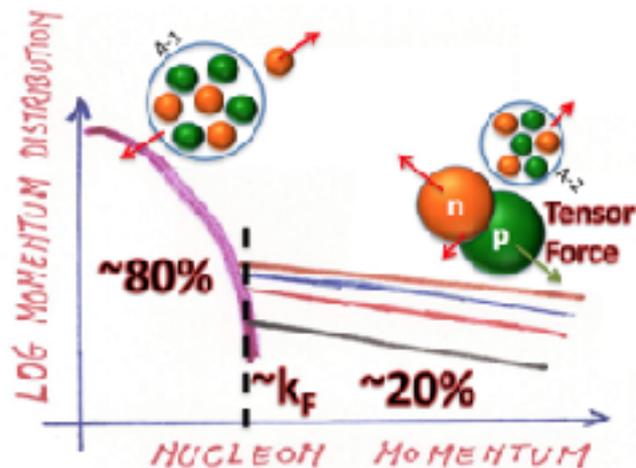


We include excitations of the A-1 final state with two nucleons in the continuum

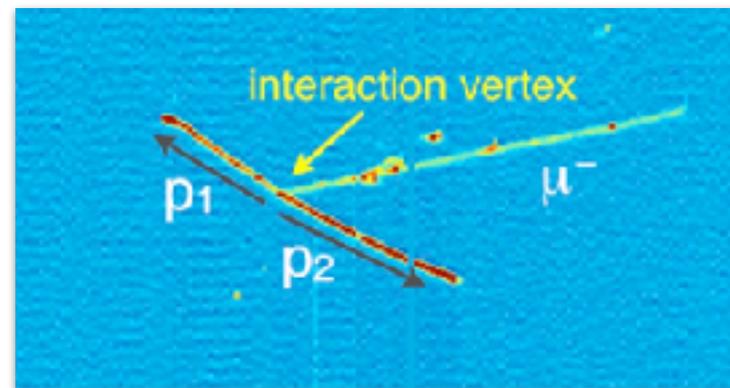


NUCLEAR CORRELATIONS IN LIQUID ARGON

Observed dominance of np over pp SRC pairs for a variety of nuclei \longrightarrow tensor interaction



O. Hen, et al. RMP 89, 4 (2017)



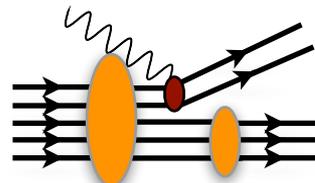
R. Acciari et al., PRD 90, 012008 (2014),
L.B. Weinstein et al., PRC 94, 045501 (2016)

Interplay with pion reabsorption and MEC: need for a unified description of the processes

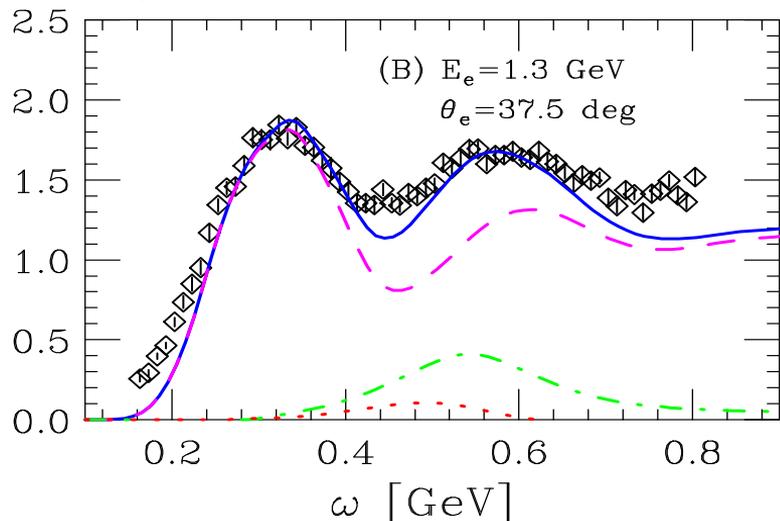
EXTENDED FACTORIZATION SCHEME

Using relativistic MEC requires extending the factorization scheme to two-nucleon emissions

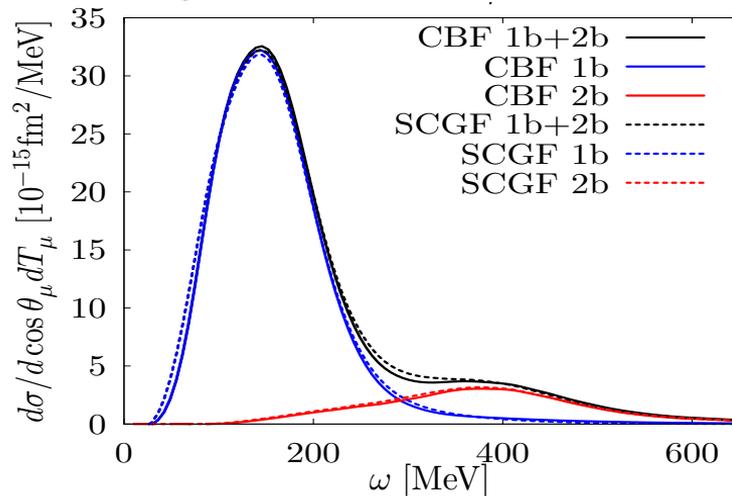
$$|\Psi_f^A\rangle \rightarrow |p_1 p_2\rangle \otimes |\Psi_f^{A-2}\rangle$$



We compute electron and neutrino inclusive cross sections using CBF and SCGF spectral functions



N. Rocco, et al. PRL. 116 192501 (2016)

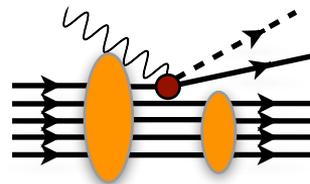


N. Rocco, et al. PRC 99 025502 (2019)

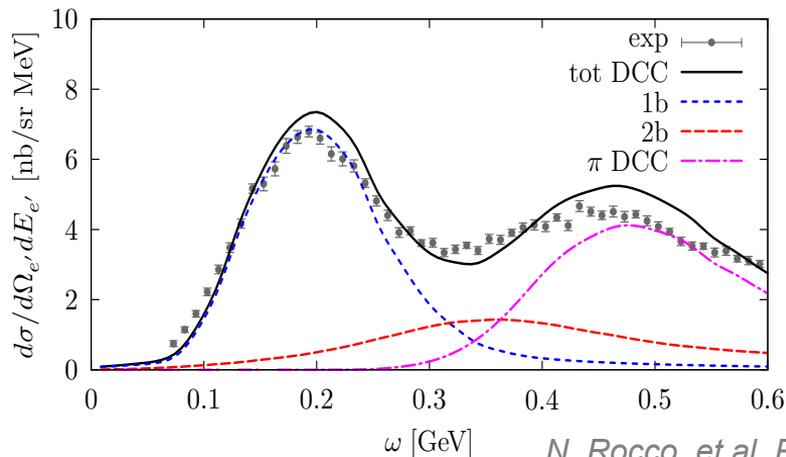
EXTENDED FACTORIZATION SCHEME

The factorization scheme can be further extended to include real pions in the final state

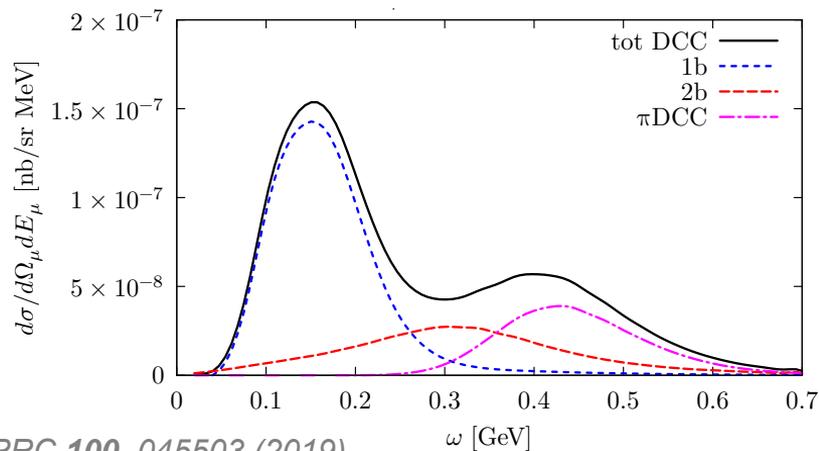
$$|\Psi_f^A\rangle \rightarrow |p_1, p_\pi\rangle \otimes |\Psi_f^{A-1}\rangle$$



The DCC model, suitable to accurately describe single-nucleon pion-production, is folded with a realistic spectral function



N. Rocco, et al. PRC 100, 045503 (2019)



SUMMARY AND OUTLOOK

Lepton-nucleus scattering from quantum Monte Carlo

- Validated our approach on electron- ^{12}C scattering (and muon-capture rates)
- Two-body currents enhance electromagnetic and charged-current responses
- Good agreement with MiniBooNE and T2K inclusive data  First ab-initio results!

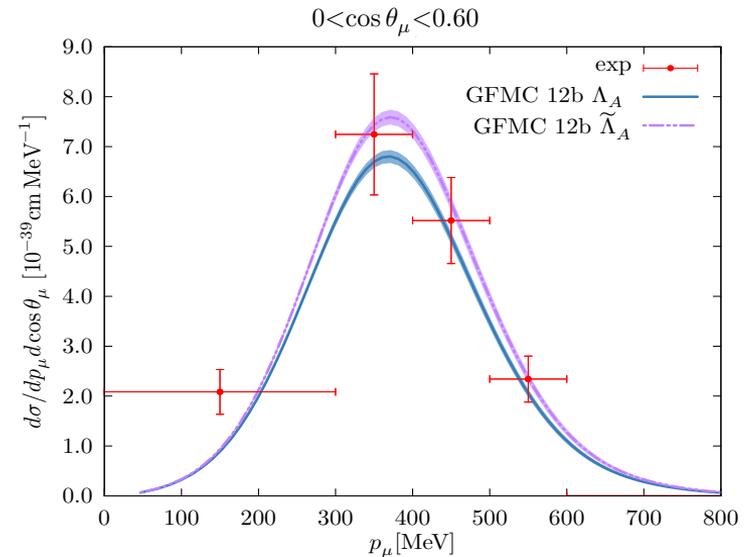
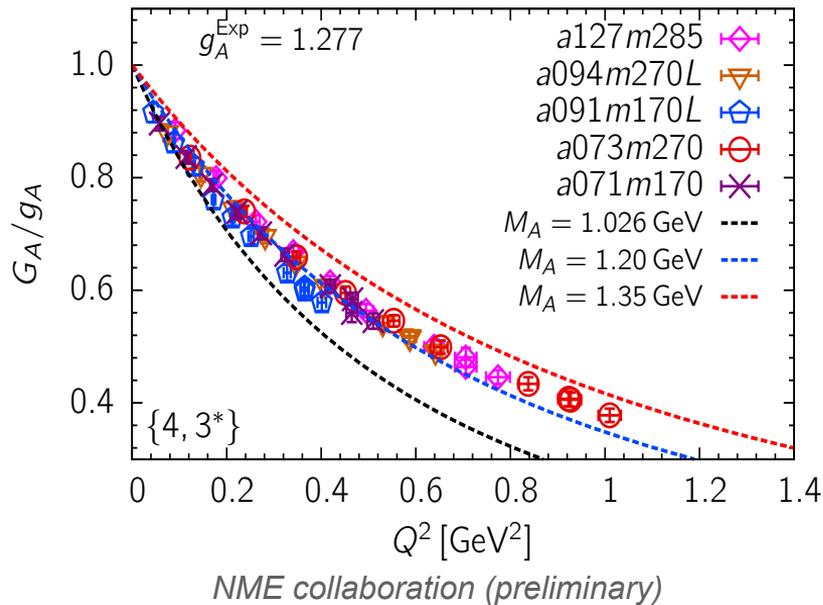
Extended factorization scheme

- Two-body currents and pion-production are essential to reproduce electron-scattering data
- Need to treat two-pion production and deep-inelastic scattering regions

SOME PERSPECTIVES

The success of the neutrino-oscillation program relies on accurate estimates of neutrino-nucleus interactions.

- Need for prompt comparisons between nuclear-theory prediction and experiments

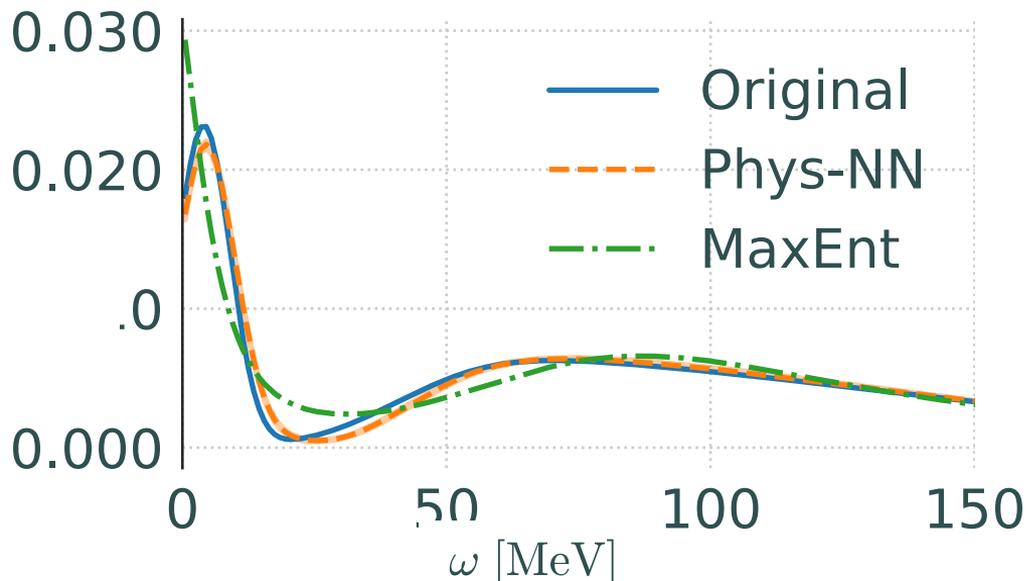


AL et al., *Phys. Rev. X* **10**, 031068 (2020)

SOME PERSPECTIVES

The success of the neutrino-oscillation program relies on accurate estimates of neutrino-nucleus interactions.

- Extend nuclear quantum Monte Carlo calculations to ^{16}O and ^{40}Ar nuclei



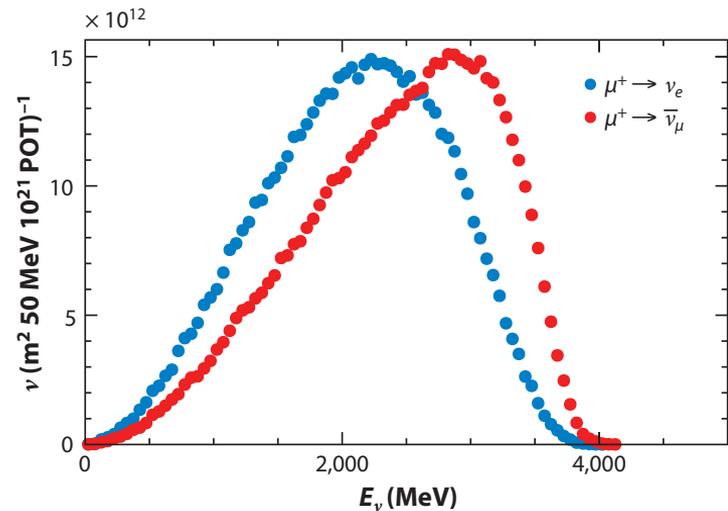
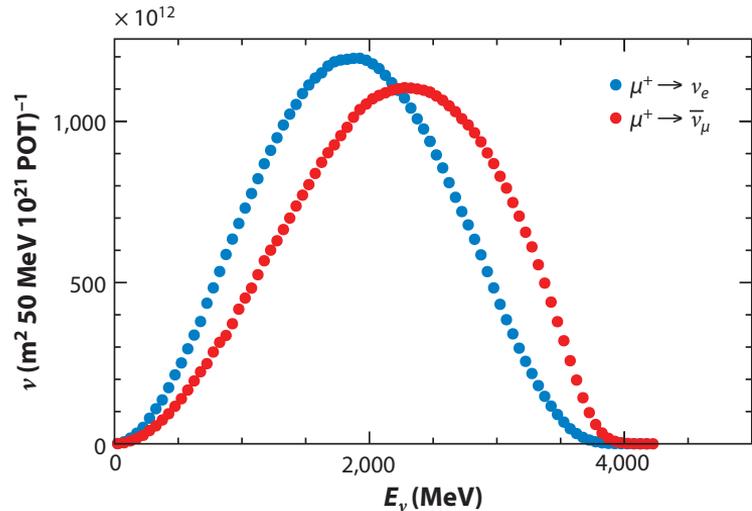
K. Raghavan, AL et al., ArXiv 2010.12703, PRC in press

SOME PERSPECTIVES

Achieving a robust description of all reaction mechanisms at play in the broad energy regime relevant for DUNE is a formidable challenge

Oscillation experiments can provide useful insights on nuclear dynamics.

NuStorm, providing extremely well-known fluxes is ideally suited to provide the necessary constraint on nuclear models so essential to the success of neutrino oscillation experiments



D. Adey, et al., Annu. Rev. Nucl. Part. Sci. 65, 145 (2015)