Factorization and resummation at subleading powers

Sebastian Jaskiewicz

IPPP Internal Seminar
November 20th, 2020
Durham

Threshold factorization of the Drell-Yan process at next-to-leading power

Martin Beneke, Alessandro Broggio, Sebastian Jaskiewicz and Leonardo Vernazza

JHEP, 2020(7):78   arXiv:1912.01585

Leading-logarithmic threshold resummation of the Drell-Yan process at next-to-leading power

Martin Beneke, Alessandro Broggio, Mathias Garny, Sebastian Jaskiewicz, Robert Szafron, Leonardo Vernazza and Jian Wang

JHEP, 2019(3):43   arXiv:1809.10631

Leading-logarithmic threshold resummation of Higgs production in gluon fusion at next-to-leading power

Martin Beneke, Mathias Garny, Sebastian Jaskiewicz, Robert Szafron, Leonardo Vernazza and Jian Wang

JHEP, 2020(1):94   arXiv:1910.12685

Large-x resummation of off-diagonal deep-inelastic parton scattering from d-dimensional refactorization

Martin Beneke, Mathias Garny, Sebastian Jaskiewicz, Robert Szafron, Leonardo Vernazza and Jian Wang

JHEP, 2020(10):196   arXiv: 2008.04943

Outline

- Motivations - an overview and introduction to SCET formalism
- The Drell-Yan process - review of factorization at leading power within the position space SCET framework.
- The Drell-Yan process - new features at next-to-leading power
  - Emergence of collinear functions
  - Generalized soft functions
- Factorization formula at next-to-leading power - starting point for resummation
- Resummation at next-to-leading power
  - Resummation of leading logarithms
  - Issues beyond leading logarithmic accuracy
- Bonus: Threshold resummation of Higgs production via gluon fusion
- Dealing with divergent convolutions: d-dimensional consistency relations and refactorization.
- Summary and outlook

Power expansion for observables

Schematic form for production cross-sections near threshold, $z \to 1$:

$$\hat{\sigma}(z) = \sum_{n=0}^{\infty} \alpha_s^n \left[c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+ + d_{nm} \ln^m(1-z) \right) + \dots \right]$$

LO	1		
NLO	αL^2	αL	α
NNLO	$\alpha^2 L^4$	$\alpha^2 L^3$	$\alpha^2 L^2$
NNNLO	$\alpha^3 L^6$	$\alpha^2 L^5$	$\alpha^2 L^4$
...	...	...	...

Higgs boson pair production at NNLO with top quark mass effects[M.Grazzini, G.Heinrich, S.Jones, S.Kallweit, M.Kerner, J.Lindert, J.Mazzitelli, 1803.02463]

First look at two-loop five-gluon scattering in QCD [S. Badger, C. Brønnum-Hansen, H. B. Hartanto, T. Peraro, 1712.02229]

Power expansion for observables

Schematic form for production cross-sections near threshold, $z \to 1$:

$$\hat{\sigma}(z) = \sum_{n=0}^{\infty} \alpha_s^n \left[c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+ + \quad d_{nm} \ln^m(1-z) \quad \right) + \ldots \right]$$

- Leading power (LP) logarithms

  Threshold Drell-Yan (N^{3}LL) [T. Becher, M. Neubert, G. Xu, 0710.0680]

  Higgs production with jet veto (NNLL) [T. Becher, M. Neubert, 1205.3806] [C. Berger, C. Marcantonini, I. Stewart, F. Tackmann, W. Waalewijn, 1012.4480]

  Thrust distribution in e^+e^- collisions (N^{3}LL)

  [T. Becher, M. Schwartz, 0803.0342]

LO	1		
NLO	αL^2	αL	α
NNLO	$\alpha^2 L^4$	$\alpha^2 L^3$	$\alpha^2 L^2$
NNNLO	$\alpha^3 L^6$	$\alpha^2 L^5$	$\alpha^2 L^4$
...	...	...	...

Power expansion for observables

Schematic form for production cross-sections near threshold, $z \to 1$:

$$\hat{\sigma}(z) = \sum_{n=0}^{\infty} \alpha_s^n \left[c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+ + d_{nm} \ln^m(1-z) \right) + \dots \right]$$

- Leading power (LP) logarithms

  Threshold Drell-Yan ($\mathrm{N^3LL}$) [T. Becher, M. Neubert, G. Xu, 0710.0680]

  Higgs production with jet veto (NNLL) [T. Becher M. Neubert, 1205.3806] [C. Berger, C. Marcantonini, I. Stewart, F. Tackmann, W. Waalewijn, 1012.4480]

  Thrust distribution in e^+e^- collisions ($\mathrm{N^3LL}$)

  [T. Becher, M. Schwartz, 0803.0342]

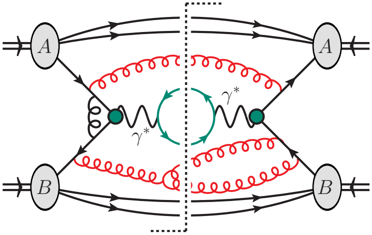
- Next-to-leading power (NLP) logarithms

  Subleading power resummed thrust spectrum for $H \to gg$ [I. Moult, I. Stewart, G. Vita, H. Zhu, 1804.04665]

  Drell-Yan production at threshold [M. Beneke, A.Broggio, M. Garny, **SJ**, R. Szafron, L. Vernazza, J.Wang, 1809.10631]

  Factorization at Subleading Power and Endpoint Divergences in $h \to \gamma\gamma$

  [Z.L. Liu, M.Neubert, 1912.08818]



Power expansion for observables

Schematic form for production cross-sections near threshold, $z \to 1$:

$$\hat{\sigma}(z) = \sum_{n=0}^{\infty} \alpha_s^n \left[c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+ + d_{nm} \ln^m(1-z) \right) + \dots \right]$$

- Leading power (LP) logarithms

  Threshold Drell-Yan ($\mathrm{N^3LL}$) [T. Becher, M. Neubert, G. Xu, 0710.0680]

  Higgs production with jet veto (NNLL) [T. Becher, M. Neubert, 1205.3806] [C. Berger, C. Marcantonini, I. Stewart, F. Tackmann, W. Waalewijn, 1012.4480]

  Thrust distribution in e^+e^- collisions ($\mathrm{N^3LL}$)

  [T. Becher, M. Schwartz, 0803.0342]

- Next-to-leading power (NLP) logarithms

  Subleading power resummed thrust spectrum for $H \to gg$ [I. Moult, I. Stewart, G. Vita, H. Zhu, 1804.04665]

  Drell-Yan production at threshold [M. Beneke, A.Broggio, M. Garny, **SJ**, R. Szafron, L. Vernazza, J.Wang, 1809.10631]

  Factorization at Subleading Power and Endpoint Divergences in $h \to \gamma\gamma$

  [Z.L. Liu, M.Neubert, 1912.08818]

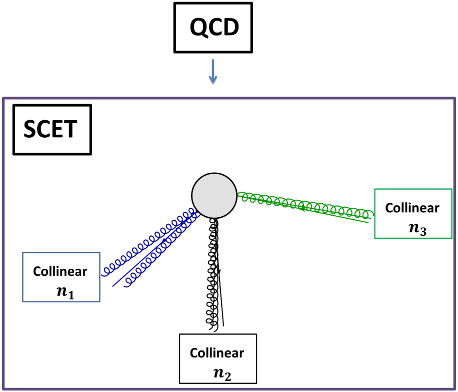
More on NLP:

- Violation of the Kluberg-Stern-Zuber theorem in SCET [M. Beneke, M. Garny, R. Szafron, J. Wang, 1907.05463]
- Towards all-order factorization of QED amplitudes at NLP [E. Laenen, J. Sinninghe Damste, L. Vernazza, W. Waalewijn, L. Zoppi, 2008.01736]
- Power corrections for N-jettiness subtractions at $\mathcal{O}(\alpha_s)$ [M. Ebert, I. Moult, I. Stewart, F. Tackmann, G. Vita, H. Zhu, 1807.10764]
- Light Quark Mediated Higgs Boson Threshold Production in the NLL Approximation [C. Anastasiou, A. Penin, 2004.03602]

Formalism of Soft Collinear Effective Field Theory (SCET)

SCET formalism

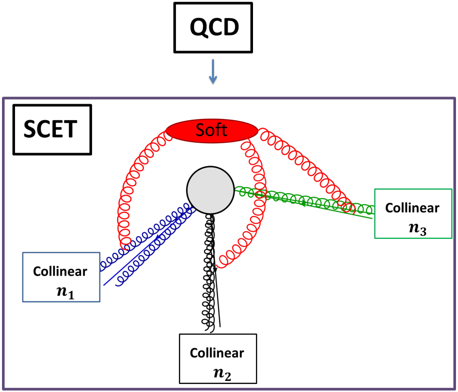
Soft collinear effective theory is contained within QCD. It is an EFT which describes energetic particles. It is an expansion of QCD.



- Process specific description, with collinear sectors formed by energetic particles. These can only interact with each other, and not between different sectors.

SCET formalism

Soft collinear effective theory is contained within QCD. It is an EFT which describes energetic particles. It is an expansion of QCD.



- Process specific description, with collinear sectors formed by energetic particles. These can only interact with each other, and not between different sectors.
- Interactions between sectors are mediated by the soft degrees of freedom.
- Every interaction is well defined in terms of power counting - this allows for systematic expansion.

The Drell-Yan process at threshold

$$A(p_A)B(p_B) \to \gamma^*(Q) + X \to l^+l^- + X$$

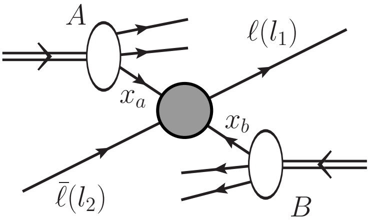
$$p^\mu = n_+p\,\frac{n_-^\mu}{2} + n_-p\,\frac{n_+^\mu}{2} + p_\perp^\mu$$

$$z = \frac{Q^2}{\hat{s}} \to 1 \qquad \lambda = \sqrt{(1-z)}$$

$$\begin{aligned} p_c &= (n_+p_c, n_-p_c, p_{c\perp}) \sim Q(1, \lambda^2, \lambda) \\ p_{\bar{c}} &= (n_+p_{\bar{c}}, n_-p_{\bar{c}}, p_{\bar{c}\perp}) \sim Q(\lambda^2, 1, \lambda) \\ p_s &= (n_+p_s, n_-p_s, p_{s\perp}) \sim Q(\lambda^2, \lambda^2, \lambda^2) \end{aligned}$$

$$Q^2\lambda^2 = Q^2(1-z) \gg \Lambda^2_{\mathrm{QCD}}$$

$$p_{c-\mathrm{PDF}} \sim (Q, \Lambda^2/Q, \Lambda)$$



The Drell-Yan process at threshold

$$A(p_A)B(p_B) \to \gamma^*(Q) + X \to l^+l^- + X$$

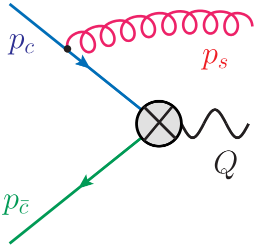
$$p^\mu = n_+p\,\frac{n_-^\mu}{2} + n_-p\,\frac{n_+^\mu}{2} + p_\perp^\mu$$

$$z = \frac{Q^2}{\hat{s}} \to 1 \qquad \lambda = \sqrt{(1-z)}$$

$$\begin{aligned} p_c &= (n_+p_c, n_-p_c, p_{c\perp}) \sim Q(1, \lambda^2, \lambda) \\ p_{\bar{c}} &= (n_+p_{\bar{c}}, n_-p_{\bar{c}}, p_{\bar{c}\perp}) \sim Q(\lambda^2, 1, \lambda) \\ p_s &= (n_+p_s, n_-p_s, p_{s\perp}) \sim Q(\lambda^2, \lambda^2, \lambda^2) \end{aligned}$$

$$Q^2\lambda^2 = Q^2(1-z) \gg \Lambda_{\mathrm{QCD}}^2$$

$$p_{c-\mathrm{PDF}} \sim (Q, \Lambda^2/Q, \Lambda)$$



The Drell-Yan process at threshold

$$A(p_A)B(p_B) \to \gamma^*(Q) + X \to l^+l^- + X$$

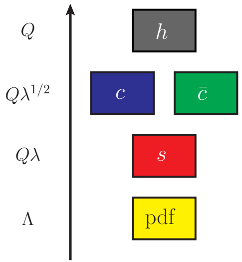
$$p^\mu = n_+p\,\frac{n_-^\mu}{2} + n_-p\,\frac{n_+^\mu}{2} + p_\perp^\mu$$

$$z = \frac{Q^2}{\hat{s}} \to 1 \qquad \lambda = \sqrt{(1-z)}$$

$$\begin{aligned} p_c &= (n_+p_c, n_-p_c, p_{c\perp}) \sim Q(1,\lambda^2,\lambda) \\ p_{\bar{c}} &= (n_+p_{\bar{c}}, n_-p_{\bar{c}}, p_{\bar{c}\perp}) \sim Q(\lambda^2,1,\lambda) \\ p_s &= (n_+p_s, n_-p_s, p_{s\perp}) \sim Q(\lambda^2,\lambda^2,\lambda^2) \end{aligned}$$

$$Q^2\lambda^2 = Q^2(1-z) \gg \Lambda^2_{\mathrm{QCD}}$$

$$p_{c-\mathrm{PDF}} \sim (Q, \Lambda^2/Q, \Lambda)$$



SCET formalism: Lagrangian

In this talk we employ position-space SCET formalism

[M. Beneke, A. Chapovsky, M. Diehl, Th. Feldmann, hep-ph/0206152]

$$\mathcal{L}_{\mathrm{SCET}} = \sum_{i=1}^{N} \mathcal{L}_{c_i} + \mathcal{L}_{\mathrm{soft}}$$

where each of the Lagrangians belonging to a collinear direction is expanded in powers of the small parameter $\lambda = \sqrt{1-z}$:

$$\mathcal{L}_{c_i} = \underbrace{\mathcal{L}_{c_i}^{(0)}}_{\mathrm{LP}} + \underbrace{\mathcal{L}_{c_i}^{(1)}}_{\mathcal{O}(\lambda^1)} + \underbrace{\mathcal{L}_{c_i}^{(2)}}_{\mathcal{O}(\lambda^2)} + \ldots$$

SCET formalism: Lagrangian

In this talk we employ position-space SCET formalism

[M. Beneke, A. Chapovsky, M. Diehl, Th. Feldmann, hep-ph/0206152]

$$\mathcal{L}_{\mathrm{SCET}} = \sum_{i=1}^{N} \mathcal{L}_{c_i} + \mathcal{L}_{\mathrm{soft}}$$

where each of the Lagrangians belonging to a collinear direction is expanded in powers of the small parameter $\lambda = \sqrt{1-z}$:

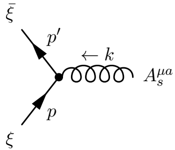
$$\mathcal{L}_{c_i} = \underbrace{\mathcal{L}^{(0)}_{c_i}}_{\mathrm{LP}} + \underbrace{\mathcal{L}^{(1)}_{c_i}}_{\mathcal{O}(\lambda^1)} + \underbrace{\mathcal{L}^{(2)}_{c_i}}_{\mathcal{O}(\lambda^2)} + \ldots$$

Separate collinear sectors interact only through soft gluon interactions. Focusing in LP term:

$$\mathcal{L}^{(0)}_{c} = \bar{\xi}\left(in_- D_c + g\, n_- A_s(x_-) + i\not{D}_{\perp c}\frac{1}{in_+ D_c}\, i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\,\xi + \mathcal{L}^{(0)}_{c,\mathrm{YM}}$$

with $in_- D_c = in_-\partial + g\, n_- A_c(x)$, $x^\mu_- = (n_+ x)\dfrac{n^\mu_-}{2}$.

The soft interaction with each collinear field at LP is given by the standard eikonal vertex



$$i g_s t^a \frac{\not{n}_+}{2} n_{-\mu} \qquad \mathcal{O}(\lambda^0)$$

SCET formalism: Lagrangian

In this talk we employ position-space SCET formalism

[M. Beneke, A. Chapovsky, M. Diehl, Th. Feldmann, hep-ph/0206152]

$$\mathcal{L}_{\mathrm{SCET}} = \sum_{i=1}^{N} \mathcal{L}_{c_i} + \mathcal{L}_{\mathrm{soft}}$$

where each of the Lagrangians belonging to a collinear direction is expanded in powers of the small parameter $\lambda = \sqrt{1-z}$:

$$\mathcal{L}_{c_i} = \underbrace{\mathcal{L}_{c_i}^{(0)}}_{\mathrm{LP}} + \underbrace{\mathcal{L}_{c_i}^{(1)}}_{\mathcal{O}(\lambda^1)} + \underbrace{\mathcal{L}_{c_i}^{(2)}}_{\mathcal{O}(\lambda^2)} + \ldots$$

Separate collinear sectors interact only through soft gluon interactions. Focusing in LP term:

$$\mathcal{L}_c^{(0)} = \bar{\xi}\left(in_-D_c + g\, n_-A_s(x_-) + i\not{D}_{\perp c}\frac{1}{in_+D_c}\, i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\,\xi + \mathcal{L}_{c,\mathrm{YM}}^{(0)}$$

with $in_-D_c = in_-\partial + g\, n_-A_c(x)$, $x_-^{\mu} = (n_+x)\dfrac{n_-^{\mu}}{2}$.

The decoupling transformation, $\chi_c^{(0)} = Y_+^{\dagger}(0)\chi_c$, separates soft and collinear sectors at LP

$$\mathcal{L}_{c+s} \to \bar{\chi}^{(0)}\frac{\not{n}_+}{2}\left(n_-\mathcal{A}_c + n_-\partial\right)\chi^{(0)}(x)$$

[C. Bauer, D. Pirjol, and I. Stewart, hep-ph/0109045]

where

$$Y_{\pm}(x) = \mathbf{P}\exp\left[ig_s\int_{-\infty}^{0} ds\, n_{\mp}A_s(x+sn_{\mp})\right]$$

SCET formalism: Lagrangian

The structure of the SCET Lagrangian beyond LP is more intricate

[M. Beneke, Th. Feldmann, hep-ph/0211358]

$$\mathcal{L}_c^{(1)} = \bar{\xi}\left(x_\perp^\mu n_-^\nu \, W_c \, g F^{\mathrm{s}}_{\mu\nu} W_c^\dagger\right)\frac{\not{n}_+}{2}\,\xi + \mathcal{L}^{(1)}_{\mathrm{YM}} + \left(\bar{q}\, W_c^\dagger \, i \not{D}_\perp \, \xi + \mathrm{h.c.}\right)$$

$$\begin{aligned}\mathcal{L}_\xi^{(2)} = {} & \frac{1}{2}\,\bar{\xi}\left((n_- x)\, n_+^\mu n_-^\nu \, W_c \, g F^{\mathrm{s}}_{\mu\nu} W_c^\dagger + x_\perp^\mu x_{\perp\rho} n_-^\nu W_c \left[D_{\mathrm{s}}^\rho, g F^{\mathrm{s}}_{\mu\nu}\right] W_c^\dagger\right)\frac{\not{n}_+}{2}\,\xi \\ & + \frac{1}{2}\,\bar{\xi}\left(i\not{D}_{\perp c}\,\frac{1}{i n_+ D_c}\, x_\perp^\mu \gamma_\perp^\nu \, W_c \, g F^{\mathrm{s}}_{\mu\nu} W_c^\dagger + x_\perp^\mu \gamma_\perp^\nu \, W_c \, g F^{\mathrm{s}}_{\mu\nu} W_c^\dagger \, \frac{1}{i n_+ D_c}\, i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\,\xi\end{aligned}$$

SCET formalism: Lagrangian

The structure of the SCET Lagrangian beyond LP is more intricate

[M. Beneke, Th. Feldmann, hep-ph/0211358]

$$\mathcal{L}_c^{(1)} = \bar{\xi}\left(x_\perp^\mu n_-^\nu\, W_c\, gF^{\mathrm{s}}_{\mu\nu} W_c^\dagger\right)\frac{\not{n}_+}{2}\,\xi + \mathcal{L}^{(1)}_{\mathrm{YM}} + \left(\bar{q}\, W_c^\dagger\, i\not{D}_\perp\,\xi + \mathrm{h.c.}\right)$$

$$\begin{aligned}\mathcal{L}_\xi^{(2)} &= \frac{1}{2}\,\bar{\xi}\left((n_-x)\, n_+^\mu n_-^\nu\, W_c\, gF^{\mathrm{s}}_{\mu\nu} W_c^\dagger + x_\perp^\mu x_{\perp\rho} n_-^\nu W_c\left[D_{\mathrm{s}}^\rho, gF^{\mathrm{s}}_{\mu\nu}\right] W_c^\dagger\right)\frac{\not{n}_+}{2}\,\xi \\ &+ \frac{1}{2}\,\bar{\xi}\left(i\not{D}_{\perp c}\,\frac{1}{in_+D_c}\, x_\perp^\mu \gamma_\perp^\nu\, W_c\, gF^{\mathrm{s}}_{\mu\nu} W_c^\dagger + x_\perp^\mu \gamma_\perp^\nu\, W_c\, gF^{\mathrm{s}}_{\mu\nu} W_c^\dagger\,\frac{1}{in_+D_c}\, i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\,\xi\end{aligned}$$

- Importantly, there are no purely collinear interactions at subleading powers. In each vertex there is at least one soft field.
- coordinate space arguments appear in the Lagrangian due to multipole expansion of the soft modes:
  $\phi_s(x)\phi_c(x) = (\phi_s(x_-) + \underbrace{x_\perp \cdot \partial_\perp \phi_s(x_-)}_{\mathcal{O}(\lambda)} + \dots)\phi_c(x)$
- For Feynman rules see [M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

SCET formalism: N-jet operator basis

Generic N-jet operator has the form:
[M. Beneke, M. Garny, R. Szafron, J. Wang, 1712.04416, 1712.07462, 1808.04742, 1907.05463]

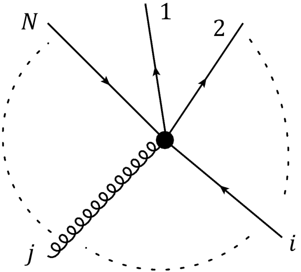
$$J = \int \prod_{i=1}^{N} \prod_{k_i=1}^{n_i} dt_{ik_i}\ C(\{t_{ik_i}\}) \prod_{i=1}^{N}\ J_i(t_{i_1}, t_{i_2}, ..., t_{i_{n_i}})$$

where the Js are constructed by multiplying collinear gauge invariant building blocks in the same direction (up to $\mathcal{O}(\lambda^2)$)

$$\chi_i(t_i n_{i+}) \equiv W_i^\dagger \xi_i \qquad\qquad \mathcal{A}^\mu_{i\perp}(t_i n_{i+}) \equiv W_i^\dagger [iD^\mu_{\perp i} W_i]$$

by acting on these with derivatives $i\partial^\mu_{\perp i} \sim \lambda$, and insertions of subleading SCET Lagrangian in a time-ordered product with lower power current.

Generic leading power N-jet operator:



SCET formalism: N-jet operator basis

Generic N-jet operator has the form:
[M. Beneke, M. Garny, R. Szafron, J. Wang, 1712.04416, 1712.07462, 1808.04742, 1907.05463]

$$J = \int \prod_{i=1}^{N} \prod_{k_i=1}^{n_i} dt_{ik_i} \; C(\{t_{ik_i}\}) \prod_{i=1}^{N} J_i(t_{i_1}, t_{i_2}, ..., t_{i_{n_i}})$$

where the Js are constructed by multiplying collinear gauge invariant building blocks in the same direction (up to $\mathcal{O}(\lambda^2)$)

$$\chi_i(t_i n_{i+}) \equiv W_i^\dagger \xi_i \qquad\qquad \mathcal{A}^\mu_{i\perp}(t_i n_{i+}) \equiv W_i^\dagger [iD^\mu_{\perp i} W_i]$$

by acting on these with derivatives $i\partial^\mu_{\perp i} \sim \lambda$, and insertions of subleading SCET Lagrangian in a time-ordered product with lower power current.

We adopt the notation: $J_i^{An}, J_i^{Bn}, J_i^{Cn}, J_i^{Tn}$ where:

- A, B, C... refers to number of fields in a given collinear direction
- n is the power of λ suppression (relative to $A0$) in a given sector.

at $\mathcal{O}(\lambda^2)$ for example we can construct $J_i^{A2}, J_i^{B2}, J_i^{C2}, J_i^{T2}$ respectively:

$$i\partial^\mu_{\perp i} i\partial^\nu_{\perp i}\chi_i, \quad \chi_i(t_{i_1}) i\partial^\nu_{\perp i}\mathcal{A}^\mu_{i\perp}(t_{i_2}), \quad \chi_i(t_{i_1})\mathcal{A}^\nu_{i\perp}(t_{i_2})\mathcal{A}^\mu_{i\perp}(t_{i_3}), \quad i\int d^4z\, \mathbf{T}\left[\chi_i(t_{i_1})\mathcal{L}^{(2)}(z)\right]$$

SCET formalism: N-jet operator basis

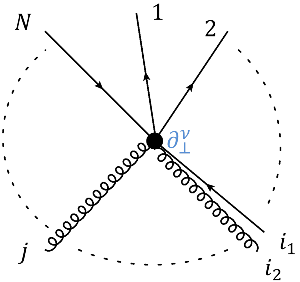
Generic N-jet operator has the form:
[M. Beneke, M. Garny, R. Szafron, J. Wang, 1712.04416, 1712.07462, 1808.04742, 1907.05463]

$$J = \int \prod_{i=1}^{N} \prod_{k_i=1}^{n_i} dt_{ik_i} \; C(\{t_{ik_i}\}) \prod_{i=1}^{N} \; J_i(t_{i_1}, t_{i_2}, ..., t_{i_{n_i}})$$

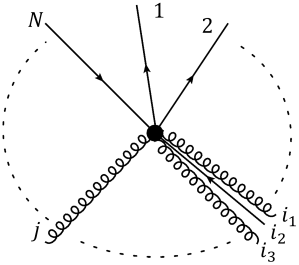
where the Js are constructed by multiplying collinear gauge invariant building blocks in the same direction (up to $\mathcal{O}(\lambda^2)$)

$$\chi_i(t_i n_{i+}) \equiv W_i^\dagger \xi_i \qquad\qquad \mathcal{A}^\mu_{i\perp}(t_i n_{i+}) \equiv W_i^\dagger [iD^\mu_{\perp i} W_i]$$

by acting on these with derivatives $i\partial^\mu_{\perp i} \sim \lambda$, and insertions of subleading SCET Lagrangian in a time-ordered product with lower power current.



J_i^{B2}



J_i^{C2}

SCET formalism: N-jet operator basis

Generic N-jet operator has the form:
[M. Beneke, M. Garny, R. Szafron, J. Wang, 1712.04416, 1712.07462, 1808.04742, 1907.05463]

$$J = \int \prod_{i=1}^{N} \prod_{k_i=1}^{n_i} dt_{ik_i} \; C(\{t_{ik_i}\}) \prod_{i=1}^{N} J_i(t_{i_1}, t_{i_2}, ..., t_{i_{n_i}})$$

where the Js are constructed by multiplying collinear gauge invariant building blocks in the same direction (up to $\mathcal{O}(\lambda^2)$)

$$\chi_i(t_i n_{i+}) \equiv W_i^\dagger \xi_i \qquad\qquad \mathcal{A}^\mu_{i\perp}(t_i n_{i+}) \equiv W_i^\dagger [iD^\mu_{\perp i} W_i]$$

by acting on these with derivatives $i\partial^\mu_{\perp i} \sim \lambda$, and insertions of subleading SCET Lagrangian in a time-ordered product with lower power current.

We adopt the notation: $J_i^{An}, J_i^{Bn}, J_i^{Cn}, J_i^{Tn}$ where:

- A, B, C... refers to number of fields in a given collinear direction
- n is the power of λ suppression (relative to $A0$) in a given sector.

at $\mathcal{O}(\lambda^2)$ for example we can construct $J_i^{A2}, J_i^{B2}, J_i^{C2}, J_i^{T2}$ respectively:

$$i\partial^\mu_{\perp i} i\partial^\nu_{\perp i}\chi_i, \quad \chi_i(t_{i_1}) i\partial^\nu_{\perp i}\mathcal{A}^\mu_{i\perp}(t_{i_2}), \quad \chi_i(t_{i_1})\mathcal{A}^\nu_{i\perp}(t_{i_2})\mathcal{A}^\mu_{i\perp}(t_{i_3}), \quad i\int d^4z\, \mathbf{T}\left[\chi_i(t_{i_1})\mathcal{L}^{(2)}(z)\right]$$

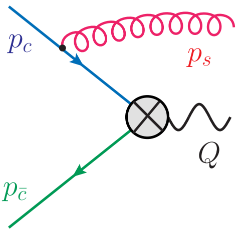
And a $\mathcal{O}(\lambda^2)$ 3-jet operator could be

$$J_1^{A2} J_2^{A0} J_3^{A0}, \qquad\qquad J_1^{A1} J_2^{A0} J_3^{B1}, \; ...$$

Drell-Yan process: leading power

The Drell-Yan process - Leading power amplitude

$$\bar{\psi}\gamma_\mu\psi = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t}\,)\, J_\mu^{A0}(t,\bar{t}\,)$$



$$J_\mu^{A0}(t,\bar{t}\,) = \bar{\chi}_{\bar{c}}(\bar{t}n_-)\,\gamma_{\perp\mu}\,\chi_c(tn_+)$$

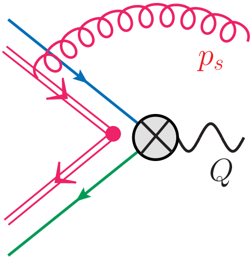
The Drell-Yan process - Leading power amplitude

$$\bar{\psi}\gamma_\mu\psi = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t}\,)\, J^{A0}_\mu(t,\bar{t}\,)$$

Use decoupled fields $\chi_c^{(0)} = Y_+^\dagger(0)\chi_c$.

Leading power current becomes

$$J^{A0}_\mu(t,\bar{t}\,) = \bar{\chi}_{\bar{c}}^{(0)}(\bar{t}n_-)Y_-^\dagger(0)\gamma_{\perp\mu}Y_+(0)\chi_c^{(0)}(tn_+)$$



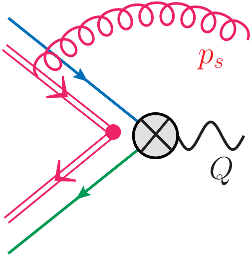
The Drell-Yan process - Leading power amplitude

$$\bar{\psi}\gamma_\mu\psi = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t}\,)\, J_\mu^{A0}(t,\bar{t}\,)$$

Use decoupled fields $\chi_c^{(0)} = Y_+^\dagger(0)\chi_c$.

Leading power current becomes

$$J_\mu^{A0}(t,\bar{t}\,) = \bar{\chi}_{\bar{c}}^{(0)}(\bar{t}n_-)Y_-^\dagger(0)\gamma_{\perp\mu}Y_+(0)\chi_c^{(0)}(tn_+)$$



Consider the matrix element:

$$\langle X|\bar{\psi}\gamma^\mu\psi(0)|A(p_A)B(p_B)\rangle \quad = \quad \int \frac{d(n_+p_a)}{2\pi}\frac{d(n_-p_b)}{2\pi}C^{A0}(n_+p_a,-n_-p_b)$$

$$\times\langle X_{\bar{c}}^{\mathrm{PDF}}|\hat{\bar{\chi}}_{\bar{c}}^{\mathrm{PDF}}(n_-p_b)|B(p_B)\rangle\gamma_\perp^\mu\langle X_c^{\mathrm{PDF}}|\hat{\chi}_c^{\mathrm{PDF}}(n_+p_a)|A(p_A)\rangle$$

$$\times\langle X_s|\mathbf{T}\left[Y_-^\dagger(0)Y_+(0)\right]|0\rangle$$

The states factorize: $\langle X| = \langle X_{\bar{c}}^{\mathrm{PDF}}|\,\langle X_c^{\mathrm{PDF}}|\,\langle X_s|$. The threshold collinear mode does not appear. Only the PDF collinear mode with scaling

$$p_{c-\mathrm{PDF}} \sim (Q,\Lambda_{\mathrm{QCD}}^2/Q,\Lambda_{\mathrm{QCD}})$$

The Drell-Yan proccess - Leading power cross-section

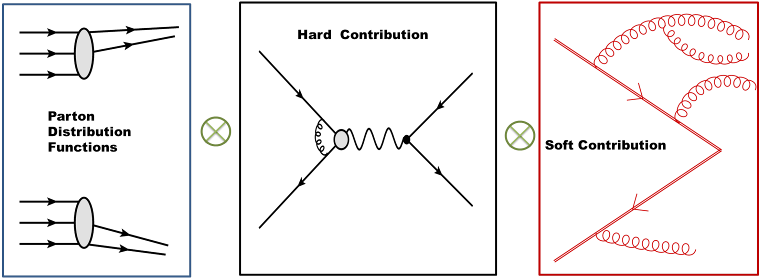
$$\frac{d\sigma_{\mathrm{DY}}}{dQ^2} = \frac{4\pi\alpha_{\mathrm{em}}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b \, f_{a/A}(x_a) f_{b/B}(x_b) \, \hat{\sigma}_{ab}^{\mathrm{LP}}(z)$$

where

[G. P. Korchemsky G. Marchesini, 1993]

[S. Moch, A. Vogt, hep-ph/0508265] [T. Becher, M. Neubert, G. Xu, 0710.0680]

$$\hat{\sigma}^{\mathrm{LP}}(z) = |C(Q^2)|^2 \, Q \, S_{\mathrm{DY}}(Q(1-z))$$



The Drell-Yan proccess - Leading power cross-section

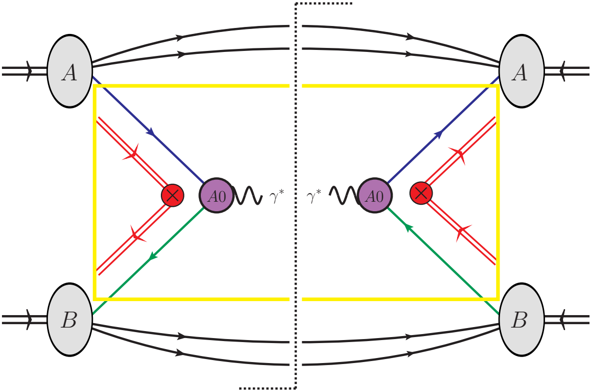
$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b \, f_{a/A}(x_a) f_{b/B}(x_b) \, \hat{\sigma}_{ab}^{\rm LP}(z)$$

where

[G. P. Korchemsky G. Marchesini, 1993]

[S. Moch, A. Vogt, hep-ph/0508265] [T. Becher, M. Neubert, G. Xu, 0710.0680]

$$\hat{\sigma}^{\rm LP}(z) = |C(Q^2)|^2 \; Q \; S_{\rm DY}(Q(1-z))$$



The Drell-Yan proccess - Leading power cross-section

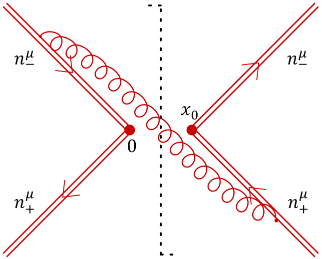
$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b\, f_{a/A}(x_a) f_{b/B}(x_b)\, \hat{\sigma}_{ab}^{\rm LP}(z)$$

where [G. P. Korchemsky G. Marchesini, 1993]

[S. Moch, A. Vogt, hep-ph/0508265] [T. Becher, M. Neubert, G. Xu, 0710.0680]

$$\hat{\sigma}^{\,\rm LP}(z) = |C(Q^2)|^2\; Q\; S_{\rm DY}(Q(1-z))$$

$$S_{\rm DY}(\Omega) = \int \frac{dx^0}{4\pi}\, e^{ix^0\Omega/2}\, \frac{1}{N_c}\, {\rm Tr}\, \langle 0|\bar{\mathbf{T}}(Y_+^\dagger(x^0) Y_-(x^0))\, \mathbf{T}(Y_-^\dagger(0) Y_+(0))|0\rangle$$



Drell-Yan process at next-to-leading power

Factorization formula at NLP

First let us compare leading power and *next*-to-leading power cross-sections schematically:

$$\frac{d\sigma_{\mathrm{DY}}}{dQ^2} = \frac{4\pi\alpha_{\mathrm{em}}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b\, f_{a/A}(x_a) f_{b/B}(x_b) \left(\hat{\sigma}_{ab}^{\,\mathrm{LP}}(z) + \hat{\sigma}_{ab}^{\,\mathrm{NLP}}(z) + \ldots \right)$$

We have discussed the LP piece

$$\hat{\sigma}^{\,\mathrm{LP}}(z) = |C(Q^2)|^2 \; Q \; S_{\mathrm{DY}}(\Omega)$$

Factorization formula at NLP

First let us compare leading power and *next*-to-leading power cross-sections schematically:

$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b\, f_{a/A}(x_a) f_{b/B}(x_b) \left(\hat{\sigma}_{ab}^{\rm LP}(z) + \hat{\sigma}_{ab}^{\rm NLP}(z) + \ldots \right)$$

We have discussed the LP piece

$$\hat{\sigma}^{\rm LP}(z) = |C(Q^2)|^2 \; Q \; S_{\rm DY}(\Omega)$$

and as will be shown the NLP is given by

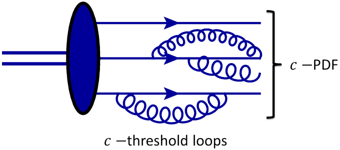
$$\hat{\sigma}^{\rm NLP} = \sum_{\rm terms} [C \otimes J \otimes \bar{J}\,]^2 \otimes S$$

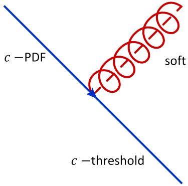
- C is the hard Wilson matching coefficient
- S is the generalized soft function
- J is the collinear function

Let us now motivate the emergence of this structure at next-to-leading power.

Collinear functions at LP and NLP

- The collinear function at LP is unity because of decoupling transformation. The threshold collinear modes can trivially be identified with c-PDF modes, $\chi_c \to \chi_c^{\mathrm{PDF}}$.

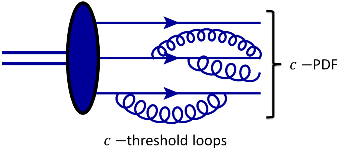


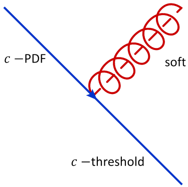


$$\left(J^{T2}_{A0,2\xi}(t)\right)^{\mu} = i\int d^4z\, \mathbf{T}\left[J^{\mu}_{A0}(t)\,\mathcal{L}^{(2)}_{2\xi}(z)\right]$$

Collinear functions at LP and NLP

- The collinear function at LP is unity because of decoupling transformation. The threshold collinear modes can trivially be identified with c-PDF modes, $\chi_c \to \chi_c^{\rm PDF}$.
- This is no longer true at NLP. Consider an example of subleading SCET Lagrangian: $\mathcal{L}_{2\xi}^{(2)} = \frac{1}{2}\bar{\chi}_c^{(0)} z_\perp^\mu z_\perp^\rho \left[i\partial_\rho i n_-\partial \mathcal{B}_\mu^+(z_-)\right] \frac{\not{n}_+}{2}\chi_c^{(0)}, \;\; \mathcal{B}_\pm^\mu = Y_\pm^\dagger \left[iD_s^\mu Y_\pm\right]$
  [M. Beneke and Th. Feldmann, hep-ph/0211358]





$$\left(J_{A0,2\xi}^{T2}(t)\right)^\mu = i\int d^4z\,\mathbf{T}\left[J_{A0}^\mu(t)\,\mathcal{L}_{2\xi}^{(2)}(z)\right]$$

Collinear functions emergent at NLP

- PDF collinear modes <u>can</u> be radiated into the final state
  Modes: $p_c \sim Q(1, \lambda^2, \lambda)$ and $p_{c-\mathrm{PDF}} \sim (Q, \Lambda^2_{\mathrm{QCD}}/Q, \Lambda_{\mathrm{QCD}})$

Collinear functions emergent at NLP

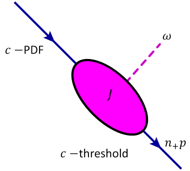
- PDF collinear modes can be radiated into the final state
  Modes: $p_c \sim Q(1, \lambda^2, \lambda)$ and $p_{c-\text{PDF}} \sim (Q, \Lambda^2_{\text{QCD}}/Q, \Lambda_{\text{QCD}})$

- Hence we define the matching equation which gives a SCET definition of what is known as the "radiative jet function"
  [V. Del Duca, 1990]
  see also[D.Bonocore, E.Laenen, L.Magnea, S.Melville, L.Vernazza, C.D.White,1503.05156]

- $$i \int d^4 z\, \mathbf{T}\Big[\chi_{c,\gamma f}\,(tn_+)\ \mathcal{L}^{(2)}(z)\Big]$$
  $$= 2\pi \sum_i \int du \int \frac{d(n_+ z)}{2}\ \tilde{J}_{i;\gamma\beta,\mu,fbd}\left(t, u; \frac{n_+ z}{2}\right) \chi^{\text{PDF}}_{c,\beta b}(un_+)\ \mathfrak{s}_{i;\mu,d}(z_-)$$
  $$\mathfrak{s}_i(z_-) \in \left\{ \frac{i\partial^\mu_\perp}{in_-\partial}\mathcal{B}^+_{\mu\perp}(z_-)\,,\ \frac{1}{(in_-\partial)}\left[\mathcal{B}^+_{\mu\perp}(z_-), \mathcal{B}^+_{\nu\perp}(z_-)\right], \dots \right\}$$

[M. Beneke, A. Broggio, M. Garny, **SJ**, R. Szafron, L. Vernazza, J. Wang, 1809.10631]

[M. Beneke, A. Broggio, **SJ**, L. Vernazza, 1912.01585]



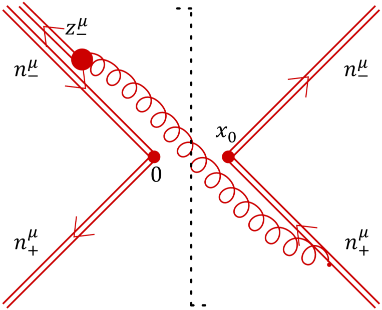
Generalized soft functions

The necessary presence of the time-ordered products at NLP gives rise to the concept of generalized soft functions with explicit gauge fields. Schematically we have

$$S(\Omega,\omega) = \int \frac{dx^0}{4\pi} e^{ix^0\Omega/2} \int \frac{d(n_+z)}{4\pi}\, e^{-i\omega(n_+z)/2}$$
$$\times \,\mathrm{Tr}\, \langle 0 | \bar{\mathbf{T}} \left[Y_+^\dagger(x^0) Y_-(x^0) \right] \mathbf{T} \left[Y_-^\dagger(0) Y_+(0) \times \mathcal{L}_s^{(n)}(z_-) \right] |0\rangle$$

$\mathcal{L}_s^{(n)}(z_-)$ contains $\mathcal{B}^+_{\perp\nu}(z_-)$ fields, $\mathcal{B}^\mu_\pm = Y_\pm^\dagger \left[iD_s^\mu Y_\pm \right]$, not made of Wilson lines only. More details on generalized soft functions later in the talk.

[M. Beneke , F. Campanario, T. Mannel, B.D. Pecjak, hep-ph/0411395]



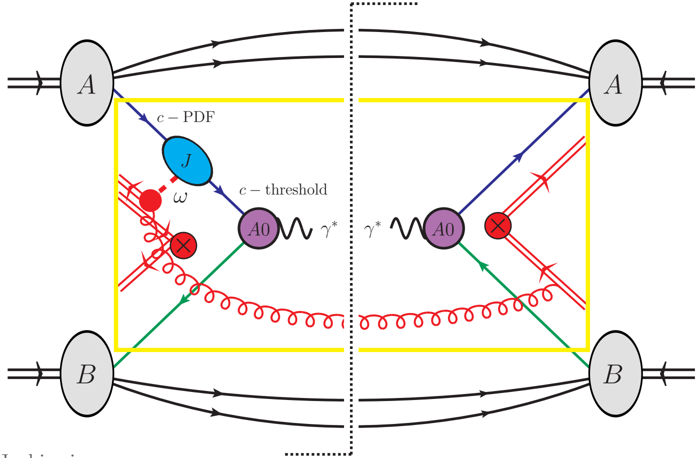
Factorization formula at NLP

Factorization formula at NLP

Defining $\Delta = \hat{\sigma}/z$, the final result is [M.Beneke, A.Broggio, **SJ**, L.Vernazza, 1912.01585]

$$\Delta^{dyn}_{\rm NLP}(z) = -2\,Q\left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\gamma^{\rho}_{\perp}\right]_{\beta\gamma}\int d(n_+p)\;C^{A0}\left(n_+p, x_b n_- p_B\right)$$

$$\times\, C^{*A0}\left(x_a\, n_+p_A,\, x_b n_- p_B\right)\sum_{i=1}^{5}\int\{d\omega_j\}\;J_i\left(n_+p, x_a\, n_+p_A;\{\omega_j\}\right)S_i(\Omega;\{\omega_j\}) + \text{h.c.}$$



Factorization formula at NLP

Defining $\Delta = \hat{\sigma}/z$, the final result is [M.Beneke, A.Broggio, **SJ**, L.Vernazza, 1912.01585]

$$\Delta^{dyn}_{\mathrm{NLP}}(z) = -2\;Q\left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\gamma^{\rho}_{\perp}\right]_{\beta\gamma}\int d(n_+p)\;C^{A0}\,(n_+p, x_b n_-p_B)$$

$$\times\, C^{*A0}\,(x_a\,n_+p_A,\, x_b n_-p_B)\sum_{i=1}^{5}\int\{d\omega_j\}\;J_i\,(n_+p, x_a\,n_+p_A;\{\omega_j\})\,S_i(\Omega;\{\omega_j\}) + \text{h.c.}$$

where the *generalised* soft functions have the structure:

$$\widetilde{S}_i\,(x;\{\omega_j\}) = \int\{dz_{j-}\}\;e^{-i\omega_j z_{j-}}\times\frac{1}{N_c}\mathrm{Tr}\langle 0|\bar{\mathbf{T}}\left(\left[Y_+^{\dagger}\,Y_-\right](x)\right)\mathbf{T}\left(\left[Y_-^{\dagger}\,Y_+\right](0)\,\mathfrak{s}_i\,(\{z_{j-}\})\right)|0\rangle$$

with

$$\mathfrak{s}_i(\{z_{j-}\})\;\in\;\Bigg\{\frac{i\partial^{\mu}_{\perp}}{in_-\partial}\mathcal{B}^{+}_{\mu\perp}(z_{1-})\;,\;\frac{1}{(in_-\partial)^2}\left[\mathcal{B}^{+\,\mu}{}_{\perp}(z_{1-}),\left[in_-\partial\mathcal{B}^{+}_{\mu\perp}(z_{1-})\right]\right],$$

$$\frac{1}{(in_-\partial)}\left[\mathcal{B}^{+}_{\mu\perp}(z_{1-}),\mathcal{B}^{+}_{\nu\perp}(z_{1-})\right],\;\frac{1}{(in_-\partial)}\mathcal{B}^{+}_{\mu\perp}(z_{1-})\mathcal{B}^{+}_{\nu\perp}(z_{2-}),\;\frac{1}{(in_-\partial)^2}q_{+\sigma}(z_{1-})\bar{q}_{+\lambda}(z_{2-})\Bigg\}$$

For comparison, LP result is:

$$\Delta_{\mathrm{LP}}(z) = |C(Q^2)|^2\;Q\;S_{\mathrm{DY}}(Q(1-z))$$

Factorization formula at NLP

Defining $\Delta = \hat{\sigma}/z$, the final result is [M.Beneke, A.Broggio, **SJ**, L.Vernazza, 1912.01585]

$$\Delta^{dyn}_{\rm NLP}(z) = -2\,Q\,\left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\,\gamma^{\rho}_{\perp}\right]_{\beta\gamma}\int d(n_+p)\;C^{\,A0}\,(n_+p, x_b n_- p_B)$$

$$\times\, C^{*A0}\,(\,x_a\,n_+p_A,\,x_b n_- p_B)\sum_{i=1}^{5}\int\{d\omega_j\}\;J_i\,(n_+p, x_a\,n_+p_A;\{\omega_j\})\,S_i(\Omega;\{\omega_j\}) + \text{h.c.}$$

where the *generalised* soft functions have the structure:

$$\widetilde{S}_{\,i}\,(x;\{\omega_j\}) = \int\{dz_{j-}\}\;e^{-i\omega_j z_{j-}}\times\frac{1}{N_c}{\rm Tr}\langle 0|\bar{\mathbf{T}}\left(\left[Y^{\dagger}_{+}\,Y_{-}\right](x)\right)\mathbf{T}\left(\left[Y^{\dagger}_{-}\,Y_{+}\right](0)\,\mathfrak{s}_i\,(\,\{z_{j-}\})\right)|0\rangle$$

with

$$\mathfrak{s}_i(\{z_{j-}\})\quad\in\;\Big\{\;\frac{i\partial^{\mu}_{\perp}}{in_-\partial}\mathcal{B}^{+}_{\mu\perp}(z_{1-})\;,\;\frac{1}{(in_-\partial)^2}\left[\mathcal{B}^{+\,\mu\perp}(z_{1-}),\left[in_-\partial\mathcal{B}^{+}_{\mu\perp}(z_{1-})\right]\right],$$

$$\frac{1}{(in_-\partial)}\left[\mathcal{B}^{+}_{\mu\perp}(z_{1-}),\mathcal{B}^{+}_{\nu\perp}(z_{1-})\right],\;\frac{1}{(in_-\partial)}\mathcal{B}^{+}_{\mu\perp}(z_{1-})\mathcal{B}^{+}_{\nu\perp}(z_{2-}),\;\frac{1}{(in_-\partial)^2}q_{+\sigma}(z_{1-})\bar{q}_{+\lambda}(z_{2-})\Big\}$$

Which terms contribute to the leading logarithms?

Specializing to leading logarithms: factorization and resummation

Leading logarithmic factorization formula

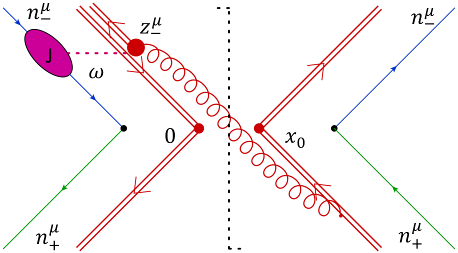
$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^4}\sum_{a,b}\int_0^1 dx_a dx_b\, f_{a/A}(x_a) f_{b/B}(x_b)\, z\, \Delta_{ab}(z)$$

Where [M. Beneke, A.Broggio, M. Garny, **SJ**, R. Szafron, L. Vernazza, J.Wang, 1809.10631]

$$\Delta(z) \quad = \quad H(\hat{s}) \times \; \frac{Q^2}{z}\int \frac{d^3\vec{q}}{(2\pi)^3\, 2\sqrt{Q^2+\vec{q}^{\,2}}}\, \frac{1}{2\pi}\int d^4x\, e^{i(x_a p_A + x_b p_B - q)\cdot x}$$

$$\times \left\{ \widetilde{S}_0(x) \; + 2\int d\omega \;\; J_1(x_a n_+ p_A;\omega)\, \widetilde{S}_{2\xi}(x,\omega) \;\; + \bar{c}\text{-term} \right\}$$

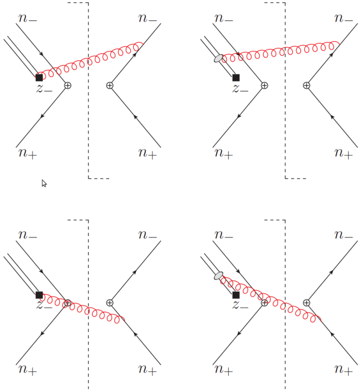
Only one new soft structure contributes! With a corresponding *tree-level* collinear function: $\mathfrak{s}_1(\{z_{j-}\}) = \frac{i\partial_\perp^\mu}{in_-\partial}\mathcal{B}^+_{\mu_\perp}(z_{1-})$



Soft functions

The generalised soft function at cross section level is

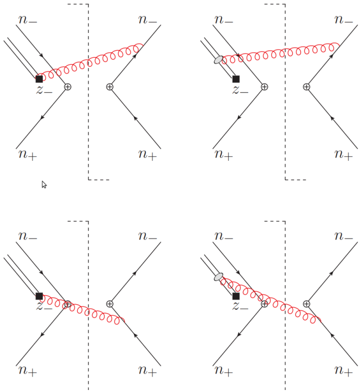
$$S_{2\xi}(\Omega,\omega) = \int \frac{dx^0}{4\pi} \int \frac{d(n_+z)}{4\pi}\, e^{ix^0\Omega/2 - i\omega(n_+z)/2}$$
$$\times \frac{1}{N_c}\,\mathrm{Tr}\,\langle 0|\bar{\mathbf{T}}\left[Y_+^\dagger(x^0)Y_-(x^0)\right]\mathbf{T}\left[Y_-^\dagger(0)Y_+(0)\frac{i\partial_\perp^\nu}{in_-\partial}\mathcal{B}^+_{\perp\nu}(z_-)\right]|0\rangle$$



Soft functions

The generalised soft function at cross section level is

$$S_{2\xi}(\Omega,\omega) = \frac{\alpha_s C_F}{2\pi}\left\{\theta(\Omega)\delta(\omega)\left(-\frac{1}{\epsilon}+\ln\frac{\Omega^2}{\mu^2}\right)+\ldots\right\}+\mathcal{O}(\alpha_s^2)$$



Soft function renormalization

The soft function starts at α_s order and is divergent. Natural question is how to renormalize this divergence? It is necessary to introduce a new object, with the same NLP power counting and a non-vanishing tree-level matrix element.

Soft function renormalization

The soft function starts at α_s order and is divergent. Natural question is how to renormalize this divergence? It is necessary to introduce a new object, with the same NLP power counting and a non-vanishing tree-level matrix element.

Object which satisfies the criteria is:

$$S_{x^0}(\Omega) = \int \frac{dx^0}{4\pi}\, e^{ix^0\Omega/2}\, \frac{-2i}{x^0 - i\epsilon}\, \frac{1}{N_c}\, \mathrm{Tr}\, \langle 0|\bar{\mathbf{T}}\left[Y_+^\dagger(x^0)Y_-(x^0)\right]\mathbf{T}\left[Y_-^\dagger(0)Y_+(0)\right]|0\rangle$$

$$S_{x^0}(\Omega) = \theta(\Omega) + \mathcal{O}(\alpha_s)$$

Similar to θ-functions appearing in [I. Moult, I. Stewart, G. Vita, H. Zhu,1804.04665]

Soft function renormalization

The soft function starts at α_s order and is divergent. Natural question is how to renormalize this divergence? It is necessary to introduce a new object, with the same NLP power counting and a non-vanishing tree-level matrix element.

Object which satisfies the criteria is:

$$S_{x^0}(\Omega) = \int \frac{dx^0}{4\pi}\, e^{ix^0\Omega/2}\, \frac{-2i}{x^0 - i\epsilon}\, \frac{1}{N_c}\, \mathrm{Tr}\, \langle 0|\bar{\mathbf{T}} \left[Y_+^\dagger(x^0)Y_-(x^0)\right] \mathbf{T} \left[Y_-^\dagger(0)Y_+(0)\right] |0\rangle$$

$$S_{x^0}(\Omega) = \theta(\Omega) + \mathcal{O}(\alpha_s)$$

Similar to θ-functions appearing in [I. Moult, I. Stewart, G. Vita, H. Zhu,1804.04665]

In momentum space, renormalization is a convolution in Ω and ω:

$$\begin{aligned} S_{2\xi}(\Omega,\omega)|_{\mathrm{ren}} &= \int d\Omega' \int d\omega'\, Z_{2\xi,2\xi}(\Omega,\omega;\Omega',\omega') S_{2\xi}(\Omega',\omega')|_{\mathrm{bare}} \\ &\quad + \int d\Omega'\, Z_{2\xi,x^0}(\Omega,\omega,\Omega') S_{x^0}(\Omega')|_{\mathrm{bare}} \end{aligned}$$

Leading logarithmic RG equation

$$\frac{d}{d\ln\mu}\begin{pmatrix} S_{2\xi}(\Omega,\omega) \\ S_{x_0}(\Omega) \end{pmatrix} = \frac{\alpha_s}{\pi}\begin{pmatrix} 4C_F\ln\dfrac{\mu}{\mu_s} & -C_F\delta(\omega) \\ 0 & 4C_F\ln\dfrac{\mu}{\mu_s} \end{pmatrix}\begin{pmatrix} S_{2\xi}(\Omega,\omega) \\ S_{x^0}(\Omega) \end{pmatrix}$$

where μ_s denotes a soft scale of order $Q(1-z)$ and the initial condition for

$S_{x^0}(\Omega)$ is $\theta(\Omega)$. The LL solution is

[M. Beneke, A.Broggio, M. Garny, **SJ**, R. Szafron, L. Vernazza, J.Wang, 1809.10631]

$$S_{2\xi}^{\mathrm{LL}}(\Omega,\omega,\mu) = \frac{\alpha_s C_F}{\pi}\ln\frac{\mu_s}{\mu}\exp\left[-4A(\mu_s,\mu)\right]\theta(\Omega)\delta(\omega)$$

and $A(\mu_s,\mu)$ is given by

$$A(\mu_s,\mu) = -\frac{\alpha_s C_F}{2\pi}\ln^2\frac{\mu}{\mu_s}$$

Leading logarithmic factorization formula

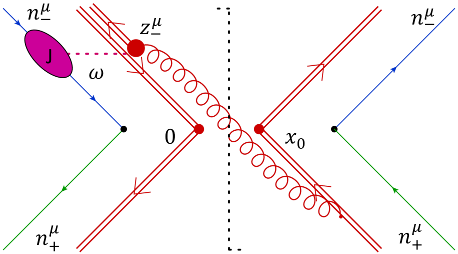
$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b \, f_{a/A}(x_a) f_{b/B}(x_b) \, z \, \Delta_{ab}(z)$$

Where [M. Beneke, A.Broggio, M. Garny, **SJ**, R. Szafron, L. Vernazza, J.Wang, 1809.10631]

$$\Delta(z) \quad = \quad H(\hat{s}) \times \frac{Q^2}{z} \int \frac{d^3\vec{q}}{(2\pi)^3 \, 2\sqrt{Q^2+\vec{q}^{\,2}}} \frac{1}{2\pi} \int d^4x \, e^{i(x_a p_A + x_b p_B - q)\cdot x}$$

$$\times \left\{ \widetilde{S}_0(x) \; + 2 \int d\omega \, \boxed{J_1(x_a n_+ p_A; \omega) \, \widetilde{S}_{2\xi}(x,\omega)} \; + \; \bar{c}\text{-term} \right\}$$

This piece we have just computed.



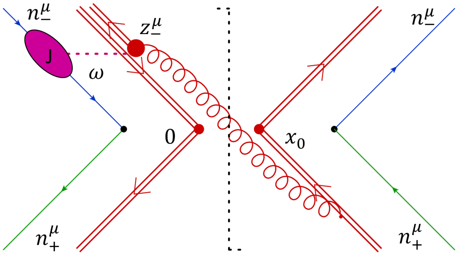
Leading logarithmic factorization formula

$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_cQ^4}\sum_{a,b}\int_0^1 dx_a dx_b\, f_{a/A}(x_a) f_{b/B}(x_b)\, z\, \Delta_{ab}(z)$$

Where [M. Beneke, A.Broggio, M. Garny, **SJ**, R. Szafron, L. Vernazza, J.Wang, 1809.10631]

$$\Delta(z) = H(\hat{s}) \times \boxed{\frac{Q^2}{z}\int \frac{d^3\vec{q}}{(2\pi)^3\, 2\sqrt{Q^2+\vec{q}^{\,2}}}\,\frac{1}{2\pi}\int d^4x\, e^{i(x_a p_A + x_b p_B - q)\cdot x}}$$
$$\times \left\{ \boxed{\widetilde{S}_0(x)} + 2\int d\omega \;\; J_1(x_a n_+ p_A;\omega)\, \widetilde{S}_{2\xi}(x,\omega) \;\; + \bar{c}\text{-term} \right\}$$

We must also consider kinematic corrections. In other words, LP soft function with NLP phase space.



Power corrections to the phase space

We investigate the kinematics, consider the centre of mass frame ($x_a\vec{p}_A + x_b\vec{p}_B = 0$) where the three-momentum of γ^* is balanced by soft radiation: $\vec{q} + \vec{p}_{X_s} = 0$

$$\left(x_a p_A + x_b p_B - q\right)^0 = p^0_{X_s} = \sqrt{\hat{s}} - \sqrt{Q^2 + \vec{q}^{\,2}} = \frac{Q}{2}\,(1-z) - \frac{\vec{q}^{\,2}}{2Q} + \frac{3}{8}\,Q\,(1-z)^2 + \mathcal{O}(\lambda^6)$$

$$\frac{Q^2}{z}\int \frac{d^3\vec{q}}{(2\pi)^3\, 2\sqrt{Q^2+\vec{q}^{2}}}\,\frac{1}{2\pi}\int d^4x\, e^{i(x_a p_A + x_b p_B - q)\cdot x}\,\widetilde{S}_0(x)$$

$$\to Q\int \frac{dx^0}{4\pi} e^{i\Omega x^0}\left(1 + \frac{i x_0 \partial^2_{\vec{x}}}{2Q} + \frac{3}{4} x_0 Q(1-z)^2 + (1-z) + \mathcal{O}(\lambda^4)\right)\,\widetilde{S}_0(x)|_{\vec{x}=0}$$

Power corrections to the phase space

We investigate the kinematics, consider the centre of mass frame ($x_a\vec{p}_A + x_b\vec{p}_B = 0$) where the three-momentum of γ^* is balanced by soft radiation: $\vec{q} + \vec{p}_{X_s} = 0$

$$\left(x_a p_A + x_b p_B - q\right)^0 = p^0_{X_s} = \sqrt{\hat{s}} - \sqrt{Q^2 + \vec{q}^{\,2}} = \frac{Q}{2}\,(1-z) - \frac{\vec{q}^{\,2}}{2Q} + \frac{3}{8}\,Q\,(1-z)^2 + \mathcal{O}(\lambda^6)$$

$$\frac{Q^2}{z}\int \frac{d^3\vec{q}}{(2\pi)^3\, 2\sqrt{Q^2+\vec{q}^2}}\,\frac{1}{2\pi}\int d^4x\, e^{i(x_a p_A + x_b p_B - q)\cdot x}\,\widetilde{S}_0(x)$$

$$\to Q\int\frac{dx^0}{4\pi}e^{i\Omega x^0}\left(1 + \frac{i x_0 \partial^2_{\vec{x}}}{2Q} + \frac{3}{4}x_0 Q(1-z)^2 + (1-z) + \mathcal{O}(\lambda^4)\right)\widetilde{S}_0(x)|_{\vec{x}=0}$$

$$\begin{aligned}
S_{K1}(\Omega) &= \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon} + 2\log\left(\frac{\mu}{\Omega}\right) - 2\right)\theta(\Omega)\\
S_{K2}(\Omega) &= \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{\epsilon} + 6\log\left(\frac{\mu}{\Omega}\right) + 6\right)\theta(\Omega)\\
S_{K3}(\Omega) &= \frac{\alpha_s C_F}{2\pi}\left(-\frac{4}{\epsilon} - 8\log\left(\frac{\mu}{\Omega}\right)\right)\theta(\Omega)
\end{aligned}$$

No LL due to kinematic correction!

Leading logarithmic results

Using soft funcion solution solution due to time-ordered product insertion along with a known hard function and *tree* level collinear function:

$$\Delta^{\text{LL}}_{\text{NLP}}(z) = -\exp\left[4A(\mu_h,\mu) - 4A(\mu_s,\mu)\right] \times 4\,\frac{\alpha_s C_F}{\pi}\ln\frac{\mu_s}{\mu}\,\theta(1-z)\,,$$

[M. Beneke, A.Broggio, M. Garny, **SJ**, R. Szafron, L. Vernazza, J.Wang, 1809.10631]

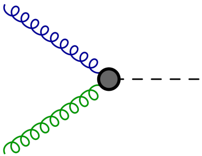
$$\begin{aligned}
\Delta^{\text{LL}}_{\text{NLP}}(z,\mu) = {} & -\theta(1-z)\Bigg\{4C_F\frac{\alpha_s}{\pi}\Big[\ln(1-z) - L_\mu\Big] \\
& + 8C_F^2\left(\frac{\alpha_s}{\pi}\right)^2\Big[\ln^3(1-z) - 3L_\mu\ln^2(1-z) + 2L_\mu^2\ln(1-z)\Big] \\
& + 8C_F^3\left(\frac{\alpha_s}{\pi}\right)^3\Big[\ln^5(1-z) - 5L_\mu\ln^4(1-z) + 8L_\mu^2\ln^3(1-z) - 4L_\mu^3\ln^2(1-z)\Big] \\
& + \frac{16}{3}C_F^4\left(\frac{\alpha_s}{\pi}\right)^4\Big[\ln^7(1-z) - 7L_\mu\ln^6(1-z) + 18L_\mu^2\ln^5(1-z) \\
& \qquad -20L_\mu^3\ln^4(1-z) + 8L_\mu^4\ln^3(1-z)\Big] \\
& + \frac{8}{3}C_F^5\left(\frac{\alpha_s}{\pi}\right)^5\Big[\ln^9(1-z) - 9L_\mu\ln^8(1-z) + 32L_\mu^2\ln^7(1-z) \\
& -56L_\mu^3\ln^6(1-z) + 48L_\mu^4\ln^5(1-z) - 16L_\mu^5\ln^4(1-z)\Big]\Bigg\} + \mathcal{O}(\alpha_s^6\times(\log)^{11})
\end{aligned}$$

where we define $L_\mu = \ln(\mu/Q)$. Comparison to [R. Hamberg, W. van Neerven, T. Matsuura, 1991] and [D. de Florian, J. Mazzitelli, S. Moch, A. Vogt, 1408.6277]

Bonus material:NLP resummation in Higgs production

Higgs production in gluon fusion

A related process is Higgs production in gluon fusion. We will highlight similarites and differences here.



$$\mathcal{L}_{\rm eff} = \frac{\alpha_s(\mu)}{12\pi}\, C_t(m_t,\mu)\, \frac{H}{v}\, F^A_{\mu\nu} F^{\mu\nu\, A}$$

notice it is a dimension 5 operator.

LP SCET current:

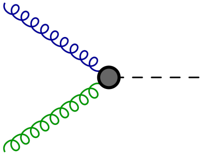
$$F^A_{\mu\nu} F^{\mu\nu\, A} \to 2 g_{\mu\nu}\, n_- \partial\, \mathcal{A}^{\nu\, A}_{\bar{c}\perp}\; n_+ \partial\, \mathcal{A}^{\mu\, A}_{c\perp}$$

Use adjoint Wilson lines:

$$\mathcal{Y}^{AB}_{\pm}(x) \quad = \quad \mathbf{P}\, \exp\left\{ g_s \int_{-\infty}^{0} ds\, f^{ABC}\, n_{\mp} A^C_s(x + s n_{\mp}) \right\}$$

Higgs production in gluon fusion

A related process is Higgs production in gluon fusion. We will highlight similarites and differences here.



$$\mathcal{L}_{\rm eff} = \frac{\alpha_s(\mu)}{12\pi}\, C_t(m_t,\mu)\, \frac{H}{v}\, F^A_{\mu\nu}F^{\mu\nu\, A}$$

notice it is a dimension 5 operator.

LP SCET current:

$$F^A_{\mu\nu}F^{\mu\nu\, A} \to 2 g_{\mu\nu}\, n_-\partial\, \mathcal{A}^{\nu\, A}_{\bar{c}\perp}\;\; n_+\partial\, \mathcal{A}^{\mu\, A}_{c\perp}$$

Use adjoint Wilson lines:

$$\mathcal{Y}^{AB}_{\pm}(x) \quad = \quad \mathbf{P}\, \exp\left\{ g_s \int_{-\infty}^{0} ds\, f^{ABC}\, n_{\mp} A^C_s(x+sn_{\mp}) \right\}$$

Important differences to DY

Since $\mathcal{L}_{\text{eff}}$ is a dimension 5 operator, in contrast to dimension 4 for DY, there is an extra factor of $\hat{s} = \frac{m_H^2}{z}$ in the cross section. Now the kinematic corrections do *not* cancel.

However, in Higgs production derivative Lagrangian terms contribute:

$$\begin{aligned}
\mathcal{L}^{(2)}_{1\text{YM}} &= -\frac{1}{2g_s^2}\text{tr}\Big(\big[n_+\partial\,\mathcal{A}^c_{\nu_\perp}\big]\Big[n_-x\,in_-\partial\,n_+\mathcal{B}^+,\,\mathcal{A}_c^{\nu_\perp}\Big]\Big),\\
\mathcal{L}^{(2)}_{2\text{YM}} &= -\frac{1}{2g_s^2}\text{tr}\Big(\big[n_+\partial\,\mathcal{A}^c_{\nu_\perp}\big]\Big[x_\perp^\rho\,x_{\perp\omega}\big[\partial^\omega,\,in_-\partial\,\mathcal{B}^+_\rho\big],\,\mathcal{A}_c^{\nu_\perp}\Big]\Big),
\end{aligned}$$

$$J^{DBC}_{\text{YM}\,\mu\rho}\,(n_+p, n_+p';\omega) \quad = \quad -2i\,T_R f^{DBC}\,g_{\perp\mu\rho}\Big[2\,-2\,(n_+p')\,\frac{\partial}{\partial n_+p}\Big]\delta(n_+p-n_+p')$$

Higgs production result

Now leading logarithms can be found in the kinematic correction, and a different prefactor for the Yang-Mills soft function

$$S_{\mathrm{K}}^{\mathrm{LL}}(\Omega, \omega, \mu) = 4\frac{\alpha_s C_A}{\pi} \ln \frac{\mu_s}{\mu} \exp\left[-4A(\mu_s, \mu)\right] \theta(\Omega)\delta(\omega)$$

$$S_{\mathrm{YM}}^{\mathrm{LL}}(\Omega, \omega, \mu) = -\frac{\alpha_s C_A}{\pi} \ln \frac{\mu_s}{\mu} \exp\left[-4A(\mu_s, \mu)\right] \theta(\Omega)\delta(\omega)$$

The two effects cancel each other!

Higgs production result

The end result is a simple replacement of $C_F \to C_A$:

$$\Delta^{\mathrm{LL}}_{\mathrm{NLP}}(z) = -\exp\left[4A(\mu_h,\mu) - 4A(\mu_s,\mu)\right] \times 4\frac{\alpha_s C_A}{\pi} \ln\frac{\mu_s}{\mu}\,\theta(1-z)\,,$$

Can be checked up to N^{3}LO with Higgs Boson Gluon Fusion Production Beyond Threshold in N3LO QCD [C. Anastasiou, C. Duhr, F. Dulat, E. Furlan, T. Gehrmann, F. Herzog, B. Mistlberger, 1411.3584] and to the fourth order in the coupling with [D. de Florian J. Mazzitelli, S. Moch, A. Vogt, 1408.6277]

σ (pb)	$\mu_h^2 = m_H^2$	$\mu_h^2 = -m_H^2$
$\sigma^{\mathrm{NNLL}}_{\mathrm{LP}}$	24.12	28.04
$\sigma^{\mathrm{LL}}_{\mathrm{NLP}}$	7.18	12.76

[M. Beneke, M. Garny, **SJ**, R. Szafron, L. Vernazza, J. Wang, 1910.12685].

Beyond leading logarithimic resummation

Factorization formula at NLP: ingredients needed beyond LL.

This is the next natural step.

Factorization formula we have written before:

$$\Delta^{dyn}_{\mathrm{NLP}}(z) = -2\, Q \left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\gamma^{\rho}_{\perp}\right]_{\beta\gamma}\int d(n_+p)\; C^{A0}\,(n_+p, x_b n_-p_B)$$

$$\times\, C^{*A0}\,(\,x_a\, n_+p_A,\, x_b n_-p_B)\sum_{i=1}^{5}\int\{d\omega_j\}\; J_i\,(n_+p, x_a\, n_+p_A; \{\omega_j\})\, S_i(\Omega; \{\omega_j\}) + \mathrm{h.c.}$$

where the *generalised* soft functions have the structure:

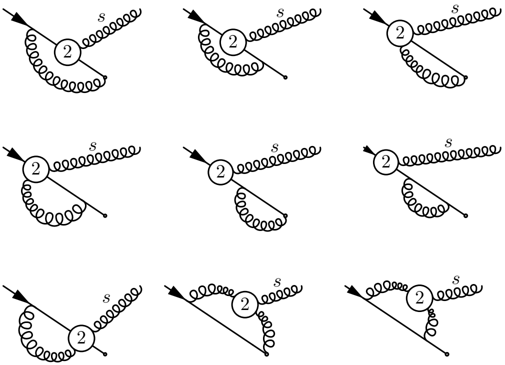
$$\widetilde{S}_{\,i}\,(x; \{\omega_j\}) = \int\{dz_{j-}\}\; e^{-i\omega_j z_{j-}} \times \frac{1}{N_c}\mathrm{Tr}\langle 0|\bar{\mathbf{T}}\left(\left[Y_+^\dagger\, Y_-\right](x)\right)\mathbf{T}\left(\left[Y_-^\dagger\, Y_+\right](0)\,\mathfrak{s}_i\,(\,\{z_{j-}\})\right)|0\rangle$$

with

$$\mathfrak{s}_i(\{z_{j-}\}) \quad\in\quad \Big\{\; \frac{i\partial^{\mu}_{\perp}}{in_-\partial}\mathcal{B}^{+}_{\mu_\perp}(z_{1-})\;,\; \frac{1}{(in_-\partial)^2}\left[\mathcal{B}^{+\,\mu_\perp}(z_{1-}),\left[in_-\partial\mathcal{B}^{+}_{\mu_\perp}(z_{1-})\right]\right],$$

$$\frac{1}{(in_-\partial)}\left[\mathcal{B}^{+}_{\mu_\perp}(z_{1-}),\mathcal{B}^{+}_{\nu_\perp}(z_{1-})\right],\;\frac{1}{(in_-\partial)}\mathcal{B}^{+}_{\mu_\perp}(z_{1-})\mathcal{B}^{+}_{\nu_\perp}(z_{2-}),\;\frac{1}{(in_-\partial)^2}q_{+\sigma}(z_{1-})\bar{q}_{+\lambda}(z_{2-})\Big\}$$

One-loop collinear function calculation



$$\langle g(k)_K | \mathcal{J}^{1g}_{\gamma f}(0) | q(p_A)_q \rangle \quad = \int dt\, dn_+p_1\, e^{itn_+p_1} \int \frac{dn_+p_a}{2\pi} du\, e^{i\, n_+p_a\, u} \int \frac{d\omega}{2\pi}\, dz_-\, e^{-i\,\omega\, z_-}$$

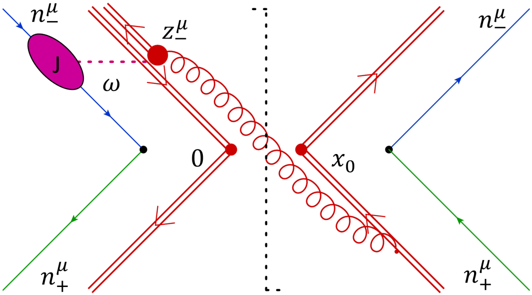
$$\times \int \frac{dn_+p}{2\pi}\, e^{-i\, n_+p\, t}\, J^A_{1;\gamma\beta,fb}\,(n_+p, n_+p_a; \omega)\, \langle 0 | \chi^{\mathrm{PDF}}_{c,\beta b}\,(un_+) | q(p_A)_q \rangle\, \langle g(k)_K | \mathfrak{s}_{1;A}(z_-)\, | 0 \rangle$$

One-loop collinear function result

The collinear function is calculated to be

$$
\begin{aligned}
J_1\left(n_+q, n_+p; \omega\right) \quad = \quad & -\frac{1}{n_+p}\delta(n_+q - n_+p) + 2\,\frac{\partial}{\partial n_+q}\delta(n_+q - n_+p) \\
& + \frac{\alpha_s}{4\pi}\,\frac{1}{(n_+p)}\left(\frac{n_+p\,\omega}{\mu^2}\right)^{-\epsilon}\,\frac{e^{\epsilon\,\gamma_E}\,\Gamma[1+\epsilon]\Gamma[1-\epsilon]^2}{(-1+\epsilon)(1+\epsilon)\Gamma[2-2\epsilon]} \\
\times \left(C_F\left(-\frac{4}{\epsilon} + 3 + 8\epsilon + \epsilon^2\right) - C_A\left(-5 + 8\epsilon + \epsilon^2\right)\right) & \delta(n_+q - n_+p) + \mathcal{O}(\alpha_s^2)
\end{aligned}
$$

[M.Beneke, A.Broggio, **SJ**, L.Vernazza, 1912.01585]



$$\mathfrak{s}_1 = \frac{i\partial_\perp^\mu}{in_-\partial}\mathcal{B}^+_{\mu\perp}(z_-)$$

How does the NLL behave?

Focus on one piece of the factorization formula

$$\int d\omega\, J_1^{(1)}(x_a n_+ p_A;\omega)\, \widetilde{S}_{2\xi}^{(1)}(x,\omega)$$

$$\begin{aligned} J_1^{(1)}(x_a\, n_+ p_A;\, \omega) \quad &= \quad \frac{\alpha_s}{4\pi}\frac{1}{(x_a\, n_+ p_A)}\left(\frac{(x_a\, n_+ p_A)\,\omega}{\mu^2}\right)^{-\epsilon}\frac{e^{\epsilon\,\gamma_E}\,\Gamma[1+\epsilon]\Gamma[1-\epsilon]^2}{(-1+\epsilon)(1+\epsilon)\Gamma[2-2\epsilon]} \\ &\times\left(C_F\left(-\frac{4}{\epsilon}+3+8\epsilon+\epsilon^2\right)-C_A\left(-5+8\epsilon+\epsilon^2\right)\right) \end{aligned}$$

$$S_{2\xi}\,(\Omega,\omega) = \frac{\alpha\, C_F}{2\pi}\frac{\mu^{2\epsilon}\, e^{\epsilon\gamma_E}}{\Gamma[1-\epsilon]}\,\frac{1}{\omega^{1+\epsilon}}\,\frac{1}{(\Omega-\omega)^{\epsilon}}\theta(\omega)\theta(\Omega-\omega)\;+\;\mathcal{O}(\alpha^2)$$

Performing the $d\omega$ convolution integral in d-dimensions, and only *after* expanding in ϵ gives the following...

How does the NLL behave?

Focus on one piece of the factorization formula

$$\int d\omega\, J_1^{(1)}(x_a n_+ p_A;\omega)\, \widetilde{S}_{2\xi}^{(1)}(x,\omega)$$

The factorization formula is valid for unrenormalized objects. Performing the convolution in d - dimensions reproduces fixed NNLO result:

[M.Beneke, A.Broggio, **SJ**, L.Vernazza, 1912.01585]

$$\Delta^{(2)}_{\mathrm{NLP-coll}} = \frac{\alpha_s^2}{(4\pi)^2}\Bigg(C_A C_F \left(\frac{20}{\epsilon} - 60\log(1-z) + 8 + \mathcal{O}(\epsilon^1)\right) \\ + C_F^2 \left(\frac{-16}{\epsilon^2} - \frac{20}{\epsilon} + \frac{48}{\epsilon}\log(1-z) + 60\log(1-z) - 72\log^2(1-z) + \mathcal{O}(\epsilon^1)\right)\Bigg)$$

after we set the scale to hard. In agreement with equation (4.22) of [D. Bonocore, E. Laenen, L. Magnea, S. Melville, L. Vernazza, C. White, 1503.05156]

Note that this goes beyond LL, here we have more information but only fixed order as opposed to resummed result as before. So, can we obtain a resummed result?

How does the NLL behave?

Focus on one piece of the factorization formula

$$\int d\omega\, J_1^{(1)}(x_a n_+ p_A;\omega)\, \widetilde{S}_{2\xi}^{(1)}(x,\omega)$$

For resummation, we treat the two objects independently, and expand in ϵ prior to performing the final convolution. However, there is a problem! At two loops:

$$J_1^{(1)}(x_a n_+ p_A;\omega) \sim \alpha_s \log(\omega)$$

and

$$S_{2\xi}\,(\Omega,\omega) \sim \alpha_s\, \delta(\omega) + \mathcal{O}(\alpha^2)$$

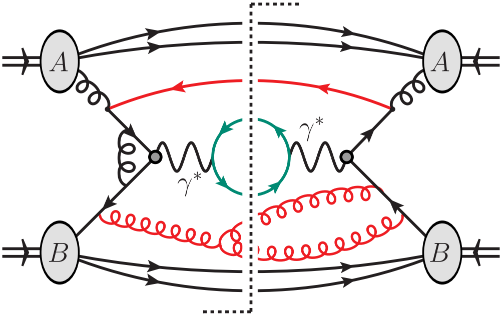
Clearly, an issue arises. The convolution $d\omega$ integral is now divergent. This prohibits the application of standard RG methods.

For LL resummation, only tree level collinear function is needed, as the soft function begins at one loop due to the explicit field insertions.

Endpoint divergences

Endpoint divergent convolutions

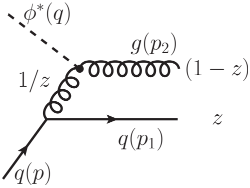
Divergent convolutions are appear already at leading logarithmic accuracy in the off-diagonal channels. For example $g\bar{q}$-channel of the Drell-Yan Process.



Factorization at Subleading Power and Endpoint Divergences in Soft-Collinear Effective Theory [Z. L. Liu, B. Mecaj, M.Neubert, X.Wang, 2009.04456]

Factorization at Subleading Power and Endpoint Divergences in $h \to \gamma\gamma$ Decay: II. Renormalization and Scale Evolution [Z. L. Liu, B. Mecaj, M.Neubert, X.Wang, 2009.06779]

Off-diagonal Deep Inelastic Scattering (DIS)



We consider DIS in $x = Q^2/2p \cdot q \to 1$

$$q(p) + \phi^*(q) \to X(p_X)$$

as it gives access to

$$P_{gq}^{\mathrm{LL}}(N) = \frac{1}{N} \frac{\alpha_s C_F}{\pi} \mathcal{B}_0(a), \qquad a = \frac{\alpha_s}{\pi}(C_F - C_A) \ln^2 N \,,$$

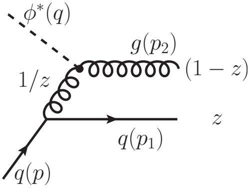
where

$$\mathcal{B}_0(x) = \sum_{n=0}^{\infty} \frac{B_n}{(n!)^2} x^n$$

with Bernoulli numbers $B_0 = 1$, $B_1 = -1/2$, $\ldots$.

[A. Vogt, 1005.1606] [A.A. Almasy, G. Soar A. Vogt, 1012.3352]
[A. Vogt, C. H. Kom, N. A. Lo Presti, G. Soar, A. A. Almasy, S. Moch, J. A. M. Vermaseren, K. Yeats, 1212.2932]

Off-diagonal Deep Inelastic Scattering (DIS)



We consider DIS in $x = Q^2/2p \cdot q \to 1$

$$q(p) + \phi^*(q) \to X(p_X)$$

as it gives access to

$$P_{gq}^{\mathrm{LL}}(N) = \frac{1}{N}\frac{\alpha_s C_F}{\pi}\,\mathcal{B}_0(a), \qquad a = \frac{\alpha_s}{\pi}(C_F - C_A)\ln^2 N\,,$$

where

$$\mathcal{B}_0(x) = \sum_{n=0}^{\infty}\frac{B_n}{(n!)^2}x^n$$

with Bernoulli numbers $B_0 = 1$, $B_1 = -1/2$, ...

[A. Vogt, 1005.1606] [A.A. Almasy, G. Soar A. Vogt, 1012.3352]
[A. Vogt, C. H. Kom, N. A. Lo Presti, G. Soar, A. A. Almasy, S. Moch, J. A. M. Vermaseren, K. Yeats, 1212.2932]

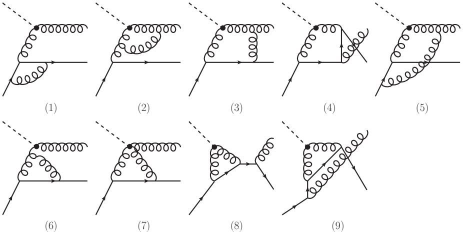
Some necessary definitions:

$$W_{\phi,i} = \frac{1}{8\pi Q^2}\int d^4x\, e^{iq\cdot x}\left\langle i(p)\middle|\left[G^A_{\mu\nu}G^{\mu\nu A}\right](x)\left[G^B_{\rho\sigma}G^{\rho\sigma B}\right](0)\middle|i(p)\right\rangle$$

$$W_{\phi,q}\big|_{q\phi^*\to qg} = \int_0^1 dz\left(\frac{\mu^2}{s_{qg}z\bar{z}}\right)^{\epsilon}\mathcal{P}_{qg}(s_{qg},z) \qquad z \equiv \frac{n_-p_1}{n_-p_1 + n_-p_2}$$

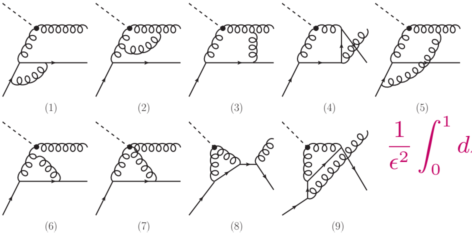
$$\mathcal{P}_{qg}(s_{qg},z) \equiv \frac{e^{\gamma_E\epsilon}Q^2}{16\pi^2\Gamma(1-\epsilon)}\frac{|\mathcal{M}_{q\phi^*\to qg}|^2}{|\mathcal{M}_0|^2} \qquad \mathcal{P}_{qg}(s_{qg},z)\big|_{\mathrm{tree}} = \frac{\alpha_s C_F}{2\pi}\frac{\bar{z}^2}{z}$$

Momentum distribution function



$$
\begin{aligned}
\mathcal{P}_{qg}(s_{qg},z)|_{1-\mathrm{loop}} \quad = \quad & \mathcal{P}_{qg}(s_{qg},z)|_{\mathrm{tree}} \frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(\mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^\epsilon + \mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^\epsilon\right. \\
& \left. + \mathbf{T}_1\cdot\mathbf{T}_2\left[\left(\frac{\mu^2}{Q^2}\right)^\epsilon - \left(\frac{\mu^2}{zQ^2}\right)^\epsilon + \left(\frac{\mu^2}{zs_{qg}}\right)^\epsilon\right]\right)
\end{aligned}
$$

Momentum distribution function

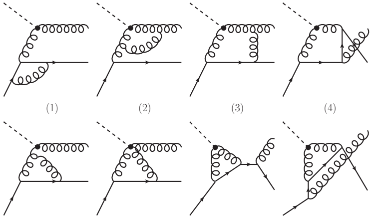


$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(1-z^{-\epsilon}\right) = -\frac{1}{2\epsilon^3}$$

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(\epsilon\ln z - \frac{\epsilon^2}{2!}\ln^2 z + \frac{\epsilon^2}{3!}\ln^3 z + \cdots\right)$$
$$= \quad -\frac{1}{\epsilon^3} + \frac{1}{\epsilon^3} - \frac{1}{\epsilon^3} + \cdots$$

$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\text{loop}} \quad = \quad \mathcal{P}_{qg}(s_{qg},z)|_{\text{tree}}\,\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(\mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^{\epsilon}\right.$$
$$\left. + \mathbf{T}_1\cdot\mathbf{T}_2\left[\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} - \left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \left(\frac{\mu^2}{zs_{qg}}\right)^{\epsilon}\right]\right)$$

Momentum distribution function



$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(1-z^{-\epsilon}\right)=-\frac{1}{2\epsilon^3}$$

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(\epsilon\ln z-\frac{\epsilon^2}{2!}\ln^2 z+\frac{\epsilon^2}{3!}\ln^3 z+\cdots\right)$$
$$=\quad -\frac{1}{\epsilon^3}+\frac{1}{\epsilon^3}-\frac{1}{\epsilon^3}+\cdots$$

We must keep the quantities dimensionally regularized!

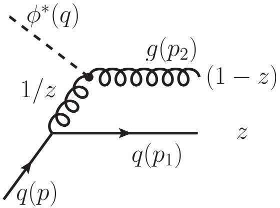
$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\text{loop}}\quad=\quad\mathcal{P}_{qg}(s_{qg},z)|_{\text{tree}}\,\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(\mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}+\mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^{\epsilon}\right.$$
$$\left.+\,\mathbf{T}_1\cdot\mathbf{T}_2\left[\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}-\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}+\left(\frac{\mu^2}{zs_{qg}}\right)^{\epsilon}\right]\right)$$

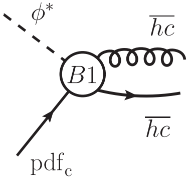
The EFT perspective

DIS factorization formula involves the scales:

- hard, $p^2 = Q^2$
- anti-hardcollinear, $p^2 = Q^2\lambda^2 = Q^2/N$
- collinear, $p^2 = \Lambda^2$
- softcollinear, $p^2 = \Lambda^2\lambda^2 = \Lambda^2/N$

where $\lambda = \sqrt{1-x}$. [T. Becher, M. Neubert, B. D. Pecjak, hep-ph/0607228]





The matching coefficient contains a $1/z$ divergence.

The EFT perspective

DIS factorization formula involves the scales:

- hard, $p^2 = Q^2$
- anti-hardcollinear, $p^2 = Q^2\lambda^2 = Q^2/N$
- collinear, $p^2 = \Lambda^2$
- softcollinear, $p^2 = \Lambda^2\lambda^2 = \Lambda^2/N$

where $\lambda = \sqrt{1-x}$. [T. Becher, M. Neubert, B. D. Pecjak, hep-ph/0607228]

Similarly to the conjectured Soft Quark Sudakov in [I. Moult, I.W. Stewart, G. Vita, H.X. Zhu, 1910.14038] we exponentiate

$$\mathcal{P}_{qg}(s_{qg}, z) = \frac{\alpha_s C_F}{2\pi}\frac{1}{z}\exp\left[\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(-C_A\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} + (C_A - C_F)\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}\right)\right]$$

Refactorization

The appearance of an endpoint divergence and the breakdown of standard SCET factorization points to the emergence of a new scale in the problem, which requires a refactorization of the B1-type SCET operator.

Refactorization

The appearance of an endpoint divergence and the breakdown of standard SCET factorization points to the emergence of a new scale in the problem, which requires a refactorization of the B1-type SCET operator.

Introduce a new power counting parameter z: $1 \gg z \gg \lambda$

Name	$(n_+l\,,\, l_\perp\,,\, n_-l)$	virtuality l^2
hard $[h]$	$Q(1\,,\,1\,,\,1)$	Q^2
z-hardcollinear $[z-hc]$	$Q(1\,,\,\sqrt{z}\,,\,z)$	$z\,Q^2$
z-anti-hardcollinear $[z-\overline{hc}]$	$Q(z\,,\,\sqrt{z}\,,\,1)$	$z\,Q^2$
z-soft $[z-s]$	$Q(z\,,\,z\,,\,z)$	$z^2\,Q^2$
z-anti-softcollinear $[z-\overline{sc}]$	$Q(\lambda^2\,,\,\sqrt{z}\,\lambda\,,\,z)$	$z\,\lambda^2\,Q^2$

Refactorization

The appearance of an endpoint divergence and the breakdown of standard SCET factorization points to the emergence of a new scale in the problem, which requires a refactorization of the B1-type SCET operator.

Introduce a new power counting parameter z: $1 \gg z \gg \lambda$

Name	$(n_+l\,,\, l_\perp\,,\, n_-l)$	virtuality l^2
hard $[h]$	$Q(1\,,\,1\,,\,1)$	Q^2
z-hardcollinear $[z-hc]$	$Q(1\,,\,\sqrt{z}\,,\,z)$	$z\,Q^2$
z-anti-hardcollinear $[z-\overline{hc}]$	$Q(z\,,\,\sqrt{z}\,,\,1)$	$z\,Q^2$
z-soft $[z-s]$	$Q(z\,,\,z\,,\,z)$	$z^2\,Q^2$
z-anti-softcollinear $[z-\overline{sc}]$	$Q(\lambda^2\,,\,\sqrt{z}\,\lambda\,,\,z)$	$z\,\lambda^2\,Q^2$

Performing a dedicated expansion-by-regions calculation, we find that large $\ln(z)$ contributions arise from hard and z-hardcollinear.

$$\int d^dx\, T\left\{J^{A0}, \mathcal{L}^{(1)}_{\xi q_{z-\overline{sc}}}(x)\right\} = D^{B1}(zQ^2,\mu^2)\, J^{B1}$$

Refactorization

The appearance of an endpoint divergence and the breakdown of standard SCET factorization points to the emergence of a new scale in the problem, which requires a refactorization of the B1-type SCET operator.

$$\left[D^{B1}\left(zQ^2,\mu^2\right)\right]_{\text{bare}} = D^{B1}\left(zQ^2,zQ^2\right)\exp\left[-\frac{\alpha_s}{2\pi}\left(C_F - C_A\right)\frac{1}{\epsilon^2}\left(\frac{zQ^2}{\mu^2}\right)^{-\epsilon}\right].$$

Summary

- Significant progress in understanding subleading power factorization theorems in the last years
- Achieved resummation at leading logarithmic accuracy
- Interesting conceptual challenges ahead. Important to understand from the point of view of gauge theories, as well as for delivering precise theoretical predictions.

Thank you

Auxiliary slides

The Drell-Yan process - Decoupling transformation

We define the decoupled field

$$\chi_c^{(0)}(tn_+) = Y_+^\dagger(0)\chi_c(tn_+)$$

[C. Bauer, D. Pirjol, and I. Stewart, 0109045]

where

$$Y_\pm\,(x) = \mathbf{P}\exp\left[ig_s\int_{-\infty}^{0} ds\, n_\mp A_s\,(x + sn_\mp)\right]$$

The Drell-Yan process - Decoupling transformation

We define the decoupled field

$$\chi_c^{(0)}(tn_+) = Y_+^\dagger(0)\chi_c(tn_+)$$

[C. Bauer, D. Pirjol, and I. Stewart, 0109045]

where

$$Y_\pm(x) = \mathbf{P}\exp\left[ig_s\int_{-\infty}^0 ds\, n_\mp A_s(x+sn_\mp)\right]$$

The LP quark Lagrangian is

$$\mathcal{L}_{\mathrm{LP}} = \bar{\chi}\left(in_-D + i\not{D}_{\perp c}\frac{1}{in_+D_c}i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\chi$$

[M. Beneke and Th. Feldmann, 0211358]

where

$$in_-D = in_-\partial + g\,n_-A_c(x) + g\,n_-A_s(x_-)$$

and after the decoupling transformation we have

$$\mathcal{L}_{c+s} \to \bar{\chi}^{(0)}\frac{\not{n}_+}{2}(n_-\mathcal{A}_c + n_-\partial)\,\chi^{(0)}(x)$$

The Drell-Yan process - Decoupling transformation

We define the decoupled field

$$\chi_c^{(0)}(tn_+) = Y_+^\dagger(0)\chi_c(tn_+)$$

[C. Bauer, D. Pirjol, and I. Stewart, 0109045]

where

$$Y_\pm(x) = \mathbf{P}\exp\left[ig_s\int_{-\infty}^{0} ds\, n_\mp A_s(x+sn_\mp)\right]$$

The LP quark Lagrangian is

$$\mathcal{L}_{\rm LP} = \bar\chi\left(in_-D + i\not{D}_{\perp c}\frac{1}{in_+D_c}\,i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\,\chi$$

[M. Beneke and Th. Feldmann, 0211358]

where

$$in_-D = in_-\partial + g\,n_-A_c(x) + g\,n_-A_s(x_-)$$

and after the decoupling transformation we have

$$\mathcal{L}_{c+s} \to \bar\chi^{(0)}\frac{\not{n}_+}{2}\left(n_-\mathcal{A}_c + n_-\partial\right)\chi^{(0)}(x)$$

From now on we use decoupled fields. Leading power current becomes

$$J_\rho^{A0}(t,\bar t) = \bar\chi_{\bar c}^{(0)}(\bar t n_-)Y_-^\dagger(0)\gamma_{\perp\rho}Y_+(0)\chi_c^{(0)}(tn_+)$$

Matching to quark current at NLP

N-jet operators are built out of following relevant building blocks.

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1712.04416, 1808.04742.]

(A1-type)

$\bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_\pm^\rho i\partial\!\!\!/_\perp]\chi_c(tn_+),\ \bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_\pm^\rho(-i)\overleftarrow{\partial\!\!\!/}_\perp]\chi_c(tn_+)$

(B1-type)

$\bar{\chi}_{\bar{c}}(\bar{t}n_-)\,[n_\pm^\rho \mathcal{A}\!\!\!/_{c\perp}(t_2n_+)]\chi_c(t_1n_+),\ \bar{\chi}_{\bar{c}}(\bar{t}_1n_-)[n_\pm^\rho \mathcal{A}\!\!\!/_{\bar{c}\perp}(\bar{t}_2n_-)]\,\chi_c(tn_+)$

With the the scaling

$$[n_\pm^\rho i\partial\!\!\!/_\perp]\,\chi_c(tn_+) \sim \lambda$$
$$[n_\pm^\rho \mathcal{A}\!\!\!/_{c\perp}(t_2n_+)]\,\chi_c(tn_+) \sim \lambda$$

relative to LP.

Definition of PDFs

$$\mathcal{A}_{\perp\mu} = Y_+^\dagger W_c^\dagger \, [i \, D_c \, W_c] \, Y_+$$

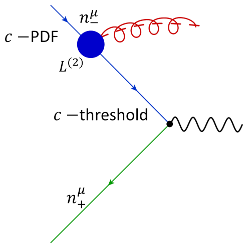
$$\langle A(p_A)|\bar{\chi}_{c,\alpha a}(x + u' n_+) \chi_{c,\beta b}(u n_+)|A(p_A)\rangle = \frac{\delta_{ba}}{N_c} \left(\frac{\not{n}_-}{4}\right)_{\beta\alpha} n_+ p_A$$

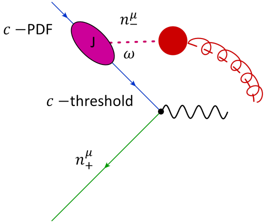
$$\times \int_0^1 dx_a \, f_{a/A}(x_a) \, e^{i(x + u' n_+ - u \, n_+)\cdot x_a p_A}$$

Collinear functions

Threshold collinear fields are matched to collinear-PDF fields

$$\int dt\, e^{i(n_+p)t}\, i\int d^4z\, e^{i\omega(n_+z)/2}\,\mathbf{T}\left[\chi_c(tn_+)\times\mathcal{L}_c^{(n)}(z)\right]$$
$$=\int d(n_+p')\int dt\, e^{i(n_+p')t}\, J(n_+p,n_+p';\omega)\,\chi_c^{\mathrm{PDF}}(tn_+)$$





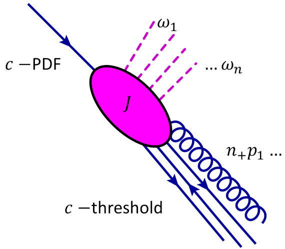
General collinear functions

- The discussed construction is actually general at subleading powers, not only next-to-leading power
- There can be many Lagrangian insertions at various positions each with its own ω_i conjugate to the large component of threshold collinear momentum

We can separate the Lagrangian insertions

$$\mathcal{L}_V^{(n)}(z) = \mathcal{L}_c^{(n)}(z) \otimes \mathcal{L}_s^{(n)}(z_-)$$

$$i^n \left(\prod_{j=1}^{n} \int d^4 z_j \right)$$
$$\times \mathbf{T}\left[\chi_c(t_1 n_+)\chi_c(t_2 n_+)... \times \mathcal{L}^{(n)}(z_1) \times ... \times \mathcal{L}^{(m)}(z_n)\right]$$
$$= 2\pi \sum_i \int du \left(\prod_{j=1}^{n} \int dz_{j-} \right) \tilde{J}_i\left(t_1, t_2 ..., u; z_{1-}, ..., z_{n-}\right)$$
$$\times\ \chi_c^{\mathrm{PDF}}(un_+)\ \mathfrak{s}_i(z_{1-}, ..., z_{n-})$$



Soft functions

We introduce the soft operator

$$\widetilde{\mathcal{S}}_{2\xi}\,(x, z_-) = \bar{\mathbf{T}}\left[Y_+^\dagger(x) Y_-(x)\right] \mathbf{T}\left[Y_-^\dagger(0) Y_+(0)\, \frac{i\partial_\perp^\nu}{in_-\partial} \mathcal{B}^+_{\perp\nu}(z_-)\right]$$

and the Fourier transform of its (colour-traced) vacuum matrix element

$$S_{2\xi}(\Omega, \omega) = \int \frac{dx^0}{4\pi} \int \frac{d(n_+z)}{4\pi}\, e^{ix^0\Omega/2 - i\omega(n_+z)/2}\, \frac{1}{N_c}\, \mathrm{Tr}\, \langle 0|\widetilde{\mathcal{S}}_{2\xi}(x^0, z_-)|0\rangle$$

Generalized soft functions

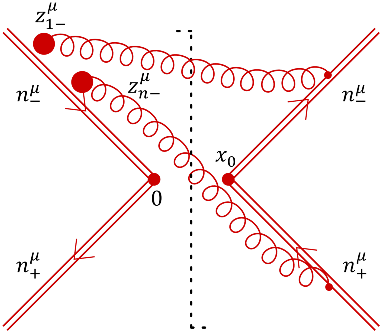
The necessary presence of the time-ordered products at NLP gives rise to the concept of generalized soft functions with explicit gauge fields. Schematically we have

$$S(\Omega,\omega) = \int \frac{dx^0}{4\pi} e^{ix^0\Omega/2} \left(\prod_{j=1}^{n} \int \frac{d(n_+ z_j)}{4\pi}\, e^{-i\omega_j (n_+ z_j)/2} \right)$$

$$\times \,\mathrm{Tr}\, \langle 0 | \bar{\mathbf{T}} \left[Y_+^\dagger(x^0) Y_-(x^0) \right] \mathbf{T} \left[Y_-^\dagger(0) Y_+(0) \times \mathcal{L}_s^{(n)}(z_{1-}) \times \ldots \times \mathcal{L}_s^{(n)}(z_{n-}) \right] |0\rangle$$

$\mathcal{L}_s^{(n)}(z_{j-})$ contains $\mathcal{B}^+_{\perp\nu}(z_{j-})$ fields, $\mathcal{B}^\mu_\pm = Y_\pm^\dagger \left[iD_s^\mu Y_\pm \right]$, not made of Wilson lines only. More details on generalized soft functions later in the talk.

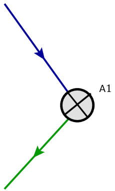
[M. Beneke , F. Campanario, T. Mannel, B.D. Pecjak, hep-ph/0411395]

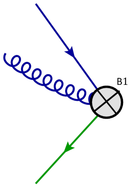


Possible contributing structures

First we check whether subleading power contributions start at order λ.

- Consider A1 and B1 type currents:
  A1-type: $\bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_\pm^\rho i\not{\partial}_\perp]\chi_c(tn_+)$ B1-type: $\bar{\chi}_{\bar{c}}(\bar{t}n_-)\,[n_\pm^\rho \not{\mathcal{A}}_{c\perp}(t_2n_+)]\chi_c(t_1n_+)$

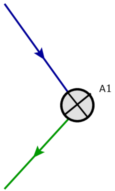


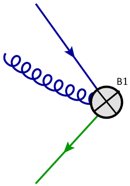


Possible contributing structures

First we check whether subleading power contributions start at order λ.

- Consider A1 and B1 type currents:
  A1-type: $\bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_\pm^\rho i\partial\!\!\!/_\perp]\chi_c(tn_+)$ B1-type: $\bar{\chi}_{\bar{c}}(\bar{t}n_-)\,[n_\pm^\rho \mathcal{A}\!\!\!/_{c\perp}(t_2 n_+)]\chi_c(t_1 n_+)$

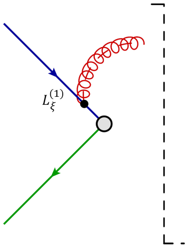




- Another possibility is a single power suppressed time-ordered product of the form $\left(J^{T1}_{A0,\xi}(s,t)\right)^\mu = i\int d^4x\,\mathbf{T}\left[J^\mu_{A0}(s,t)\,\mathcal{L}^{(1)}_\xi(x)\right]$

Only one possibility

$$\mathcal{L}^{(1)}_\xi \quad = \quad \bar{\chi}_c i x_\perp^\mu \left[in_-\partial\mathcal{B}^+_\mu\right]\frac{\not{n}_+}{2}\chi_c$$



Factorization formula at NLP

First step in derivation is to extend the matching equation of the DY to SCET current up to NLP accuracy:

$$\bar{\psi}\gamma_\rho\psi(0) = \sum_{m_1,m_2} \int \{dt_k\}\,\{d\bar{t}_{\bar{k}}\}\,\widetilde{C}^{\,m_1,m_2}\left(\{t_k\},\,\{\bar{t}_{\bar{k}}\}\right)\; J_\rho^{m_1,m_2}\left(\{t_k\},\,\{\bar{t}_{\bar{k}}\}\right)$$

$$J_\rho^{m_1,m_2}\left(\{t_k\},\,\{\bar{t}_{\bar{k}}\}\right) = J_{\bar{c}}^{m_1}\left(\{\bar{t}_{\bar{k}}\}\right)\,\Gamma_\rho^{m_1,m_2}\,J_c^{m_2}\left(\{t_k\}\right)$$

Contrast with LP where the current is simply given by:

$$J_\rho^{A0A0}(t,\bar{t}\,) = \bar{\chi}_{\bar{c}}(\bar{t}n_-)\gamma_{\perp\rho}\chi_c(tn_+)$$

Now must consider all possible sources of power suppression. In the presented formalism, this means including power suppressed currents, A1, B1, A2, B2, C2 and T1, T2 with all possible Lagrangian insertions for each direction. For example:

$$\begin{aligned} J_\rho^{A0A2}(t,\bar{t}\,) &= \bar{\chi}_{\bar{c}}(\bar{t}n_-)\gamma_{\perp\rho}i\partial_\perp^\mu i\partial_{\perp\mu}\chi_c(tn_+) \\ J_\rho^{A0B2}(t_1,t_2,\bar{t}\,) &= \bar{\chi}_{\bar{c}}(\bar{t}n_-)\gamma_{\perp\rho}\mathcal{A}_{c\perp}^{\mu}(t_2n_+)i\partial_\mu\chi_c(t_1n_+) \end{aligned}$$

Factorization formula at NLP

First step in derivation is to extend the matching equation of the DY to SCET current up to NLP accuracy:

$$\bar{\psi}\gamma_\rho\psi(0) = \sum_{m_1,m_2} \int \{dt_k\}\,\{d\bar{t}_{\bar{k}}\}\,\widetilde{C}^{\,m_1,m_2}\left(\{t_k\},\,\{\bar{t}_{\bar{k}}\}\right)\; J_\rho^{m_1,m_2}\left(\{t_k\},\,\{\bar{t}_{\bar{k}}\}\right)$$

$$J_\rho^{m_1,m_2}\left(\{t_k\},\,\{\bar{t}_{\bar{k}}\}\right) = J_{\bar{c}}^{m_1}\left(\{\bar{t}_{\bar{k}}\}\right)\,\Gamma_\rho^{m_1,m_2}\,J_c^{m_2}\left(\{t_k\}\right)$$

Note that currents without time-ordered product operators with a Lagrangian insertion can be discarded: lead to scaleless integrals just as at LP!

$$J_c^{T2}(t) = i\int d^4z\,\mathbf{T}\left[J_c^{A0}(t)\,\mathcal{L}^{(2)}(z)\right]$$

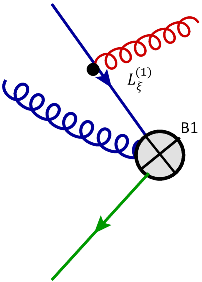
Now must consider all possible sources of power suppression. In the presented formalism, this means including power suppressed currents, ~~A1~~, ~~B1~~, ~~A2~~, ~~B2~~, ~~C2~~ and T1, T2 with all possible Lagrangian insertions for each direction. For example:

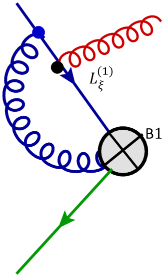
$$\begin{aligned} J_\rho^{A0A2}(t,\bar{t}\,) &= \bar{\chi}_{\bar{c}}(\bar{t}n_-)\gamma_{\perp\rho}i\partial_\perp^\mu i\partial_{\perp\mu}\chi_c(tn_+) \\ J_\rho^{A0B2}(t_1,t_2,\bar{t}\,) &= \bar{\chi}_{\bar{c}}(\bar{t}n_-)\gamma_{\perp\rho}\mathcal{A}_{c\perp}^\mu(t_2n_+)i\partial_\mu\chi_c(t_1n_+) \end{aligned}$$

Possible contributing structures

First subleading contributions are found at λ^2 order. This we call next-to-leading power.

- B1-type current, $\bar{\chi}_{\bar{c}}(\bar{t}n_-)\,[n_\pm^\rho \not{\mathcal{A}}_{c\perp}(t_2 n_+)]\chi_c(t_1 n_+)$, with Lagrangian insertion $\mathcal{L}_\xi^{(1)} = \bar{\chi}_c i x_\perp^\mu \left[i n_- \partial \mathcal{B}_\mu^+ \right] \frac{\not{n}_+}{2} \chi_c$

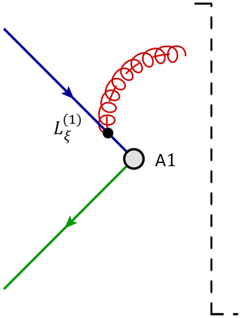




Possible contributing structures

First subleading contributions are found at λ^2 order. This we call next-to-leading power.

- A1-type current with $\mathcal{L}_\xi^{(1)} = \bar{\chi}_c i x_\perp^\mu \left[i n_- \partial \mathcal{B}_\mu^+ \right] \frac{\not{n}_+}{2} \chi_c$ insertion



Feynman rule for emission of a soft gluon from $\mathcal{B}_\mu^+$ is

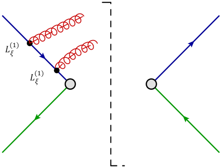
$$g\, T^A \left[-\frac{k_\perp^\mu n_{-\nu}}{(n_- k)} + g_\perp^{\mu\nu} \right] \epsilon_\nu^* \, e^{+ik\cdot z_-}$$

Considering the Lagrangian insertions

The following contributions start at $\mathcal{O}(\alpha^2)$

$$\left(J^{T2}_{A0,\xi}(s,t)\right)^\mu = i\int d^4x_1 i\int d^4x_2\, \mathbf{T}\left[J^\mu_{A0}(s,t)\,\mathcal{L}^{(1)}_\xi(x_1)\,\mathcal{L}^{(1)}_\xi(x_2)\right]$$

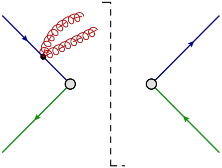
$$\mathcal{L}^{(1)}_\xi \quad = \quad \bar\chi_c i x^\mu_\perp \left[i n_-\partial \mathcal{B}^+_\mu\right] \frac{\not{n}_+}{2}\chi_c$$



$$\left(J^{T2}_{A0,V}(s,t)\right)^\mu = i\int d^4x\, \mathbf{T}\left[J^\mu_{A0}(s,t)\,\mathcal{L}^{(2)}_V(x)\right]$$

$$\mathcal{L}^{(2)}_{3\xi} \quad = \quad \frac{1}{2}\bar\chi_c x^\mu_\perp x^\rho_\perp \left[\mathcal{B}^+_\rho,\, i n_-\partial \mathcal{B}^+_\mu\right] \frac{\not{n}_+}{2}\chi_c$$

$$\mathcal{L}^{(2)}_{5\xi} \quad = \quad \frac{1}{2}\bar\chi_c \left(i\not{\partial}_\perp + \not{A}_{c\perp}\right) \frac{1}{i n_+\partial} i x^\mu_\perp \gamma^\nu_\perp \times \left[\mathcal{B}^+_\nu, \mathcal{B}^+_\mu\right] \frac{\not{n}_+}{2}\chi_c + \text{h.c.}$$

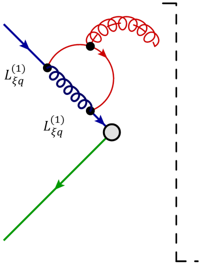


Considering the Lagrangian insertions

It is also possible to construct diagrams containing soft quarks

$$\left(J^{T2}_{A0,\xi q}(s,t)\right)^{\mu} = i\int d^4x_1 i\int d^4x_2\,\mathbf{T}\left[J^{\mu}_{A0}(s,t)\,\mathcal{L}^{(1)}_{\xi q}(x_1)\,\mathcal{L}^{(1)}_{\xi q}(x_2)\right]$$

$$\mathcal{L}^{(1)}_{\xi q} \quad = \quad \bar{q}_{+}\mathcal{A}_{c\perp}\chi_c + \text{h.c.}$$



These contributions also start at $\mathcal{O}(\alpha^2)$

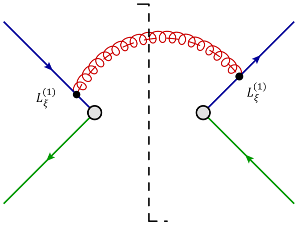
Considering the Lagrangian insertions

Previous arguments allow us also to drop following possible contributions

$$\left(J^{T1}_{A0,\xi}(s,t)\right)^{\mu} = i\int d^4x_1\,\mathbf{T}\left[J^{\mu}_{A0}(s,t)\,\mathcal{L}^{(1)}_{\xi}(x_1)\right]$$

$$\left(\bar{J}^{T1}_{A0,\xi}(\bar{s},\bar{t})\right)^{\mu} = (-i)\int d^4x_2\,\mathbf{T}\left[\bar{J}^{\mu}_{A0}(\bar{s},\bar{t})\,\mathcal{L}^{(1)}_{\xi}(x_2)\right]$$

$$\mathcal{L}^{(1)}_{\xi} \quad = \quad \bar{\chi}_c i x^{\mu}_{\perp}\left[i n_{-}\partial\mathcal{B}^{+}_{\mu}\right]\frac{\not{n}_{+}}{2}\chi_c$$

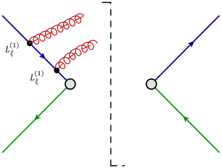


Considering the Lagrangian insertions

The following contributions start at $\mathcal{O}(\alpha^2)$

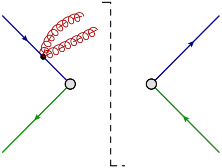
$$\left(J^{T2}_{A0,\xi}(s,t)\right)^{\mu} = i\int d^4x_1 i\int d^4x_2\,\mathbf{T}\left[J^{\mu}_{A0}(s,t)\,\mathcal{L}^{(1)}_{\xi}(x_1)\,\mathcal{L}^{(1)}_{\xi}(x_2)\right]$$

$$\mathcal{L}^{(1)}_{\xi} \quad = \quad \bar{\chi}_c i x^{\mu}_{\perp}\left[in_-\partial\mathcal{B}^{+}_{\mu}\right]\frac{\not{n}_+}{2}\chi_c$$



$$\left(J^{T2}_{A0,V}(s,t)\right)^{\mu} = i\int d^4x\,\mathbf{T}\left[J^{\mu}_{A0}(s,t)\,\mathcal{L}^{(2)}_{V}(x)\right]$$

$$\begin{aligned}
\mathcal{L}^{(2)}_{3\xi} \quad &= \quad \frac{1}{2}\bar{\chi}_c x^{\mu}_{\perp}x^{\rho}_{\perp}\left[\mathcal{B}^{+}_{\rho},\, in_-\partial\mathcal{B}^{+}_{\mu}\right]\frac{\not{n}_+}{2}\chi_c \\
\mathcal{L}^{(2)}_{5\xi} \quad &= \quad \frac{1}{2}\bar{\chi}_c\left(i\not{\partial}_{\perp}+\not{A}_{c\perp}\right)\frac{1}{in_+\partial}ix^{\mu}_{\perp}\gamma^{\nu}_{\perp} \\
&\quad\times\left[\mathcal{B}^{+}_{\nu},\mathcal{B}^{+}_{\mu}\right]\frac{\not{n}_+}{2}\chi_c + \text{h.c.}
\end{aligned}$$

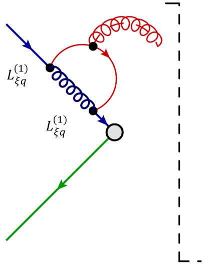


Considering the Lagrangian insertions

It is also possible to construct diagrams containing soft quarks

$$\left(J^{T2}_{A0,\xi q}(s,t)\right)^{\mu} = i\int d^4x_1 i\int d^4x_2\, \mathbf{T}\left[J^{\mu}_{A0}(s,t)\,\mathcal{L}^{(1)}_{\xi q}(x_1)\,\mathcal{L}^{(1)}_{\xi q}(x_2)\right]$$

$$\mathcal{L}^{(1)}_{\xi q} \quad = \quad \bar{q}_{+}\mathcal{A}_{c\perp}\chi_c + \text{h.c.}$$

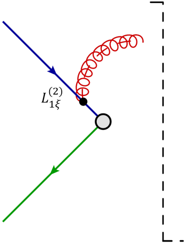


These contributions also start at $\mathcal{O}(\alpha^2)$

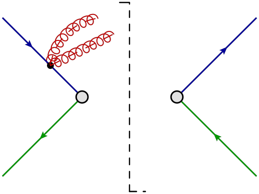
Considering the Lagrangian insertions

Two more possible contributions with following Lagrangian terms making up the time-ordered product

$$\mathcal{L}_{1\xi}^{(2)} = \frac{1}{2}\bar{\chi}_c i n_- x n_+^\mu \left[i n_- \partial \mathcal{B}_\mu^+ \right] \frac{\not{n}_+}{2}\chi_c$$



$$\mathcal{L}_{4\xi}^{(2)} = \frac{1}{2}\bar{\chi}_c \left(i\not{\partial}_\perp + \not{A}_{c\perp}\right) \frac{1}{i n_+ \partial} i x_\perp^\mu \gamma_\perp^\nu \times \left[i\partial_{\nu_\perp}\mathcal{B}_{\mu_\perp}^+ - i\partial_{\mu_\perp}\mathcal{B}_{\nu_\perp}^+ \right] \frac{\not{n}_+}{2}\chi_c + \text{h.c.}$$



Conclusion

We therefore find that for LL resummation at NLP in the quark-antiquark channel only the single time-ordered product contribution:

$$\left(J^{T2}_{A0,2\xi}(s,t)\right)^{\mu} = i\int d^4x\,\mathbf{T}\left[J^{\mu}_{A0}(s,t)\,\mathcal{L}^{(2)}_{2\xi}(x)\right]$$

To NLP LL accuracy the matching equation is then extended to

$$\bar{\psi}\gamma^{\mu}\psi(0) = \int dt\,d\bar{t}\,\widetilde{C}^{A0}(t,\bar{t}\,)\left[J^{\mu}_{A0}(t,\bar{t}\,) + \left(J^{T2}_{A0,2\xi}(t,\bar{t}\,)\right)^{\mu} + \;\bar{c}\text{-term}\right]$$

Again we consider

$$\langle X|\bar{\psi}\gamma^{\mu}\psi(0)|A(p_A)B(p_B)\rangle$$

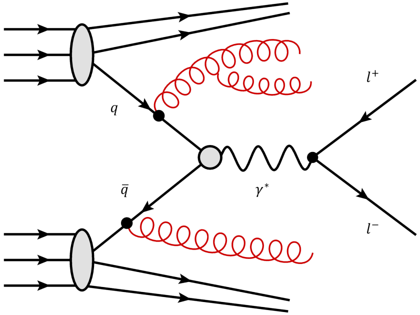
Factorization formula at NLP

Cross section is given by a combination of the lepton and hadronic tensor

$$d\sigma = \frac{4\pi\alpha_{\rm EM}^2}{3sq^2}\frac{d^4q}{(2\pi)^4}\left(-g^{\mu\rho}W_{\mu\rho}\right)$$

where the hadronic tensor is given by

$$\begin{aligned} g^{\mu\rho}W_{\mu\rho} &= \int d^4x e^{-iq\cdot x}\langle A(p_A)B(p_B)|J^{\dagger\,\rho}(x)J_\rho(0)|A(p_A)B(p_B)\rangle \\ &= \sum_X \langle A(p_A)B(p_B)|J^\dagger_\rho(0)|X\rangle\langle X|J^\rho(0)|A(p_A)B(p_B)\rangle \\ &\qquad \times(2\pi)^4\delta\left(p_A+p_B-q-p_{X_s}-p_{X_c^{\rm PDF}}-p_{X_{\bar c}^{\rm PDF}}\right) \end{aligned}$$



Factorization formula at NLP

Cross section is given by a combination of the lepton and hadronic tensor

$$d\sigma = \frac{4\pi\alpha_{\rm EM}^2}{3sq^2}\frac{d^4q}{(2\pi)^4}\left(-g^{\mu\rho}W_{\mu\rho}\right)$$

where the hadronic tensor is given by

$$\begin{aligned} g^{\mu\rho}W_{\mu\rho} &= \int d^4x e^{-iq\cdot x}\langle A(p_A)B(p_B)|J^{\dagger\,\rho}(x)J_\rho(0)|A(p_A)B(p_B)\rangle \\ &= \sum_X \langle A(p_A)B(p_B)|J^\dagger_\rho(0)|X\rangle\langle X|J^\rho(0)|A(p_A)B(p_B)\rangle \\ &\qquad\qquad \times(2\pi)^4\delta\left(p_A+p_B-q-p_{X_s}-p_{X_c^{\rm PDF}}-p_{X_{\bar c}^{\rm PDF}}\right) \end{aligned}$$

The recipe for derivation of the factoritation formula

- Consider the matrix element, with only soft radiation allowed due to threhold kinematics
- Consider all possible insertions of subleading power Lagrangian and perform the second matching of threshold collinear fields to PDF collinear fileds. Here we introduce the new objects: collinear functions

$$i\int d^4z\,\mathbf{T}\Big[\chi_{c,\gamma f}\left(tn_+\right)\,\mathcal{L}^{(2)}(z)\Big] = 2\pi\sum_i\int du\int\frac{d(n_+z)}{2}\,\tilde{J}_{i;\gamma\beta,\mu,fbd}\left(t,u;\frac{n_+z}{2}\right)\chi^{\rm PDF}_{c,\beta b}(un_+)\,\mathfrak{s}_{i;\mu,d}(z_-)$$

- Usual steps follow: square amplitude, sum over intermediate states → standard PDFs

Time-ordered products

$$\left(J^{Tm}_{W,V}(x)\right)^{\mu} = i\int d^4z\,\mathbf{T}\left[J^{\mu}_{W}(t)\,\mathcal{L}^{(n)}_{V}(x)\right]$$

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1712.04416, 1808.04742]

The NLP soft-collinear SCET quark-gluon interaction Lagrangian written in terms of building blocks $\mathcal{B}^{\mu}_{\pm} = Y^{\dagger}_{\pm}\left[iD^{\mu}_{s} Y_{\pm}\right]$ and $q^{\pm}(x_-) = Y^{\dagger}_{\pm}\, q_s(x_-)$ is

[M. Beneke, F. Campanario, T. Mannel, B.D. Pecjak , hep-ph/0411395]

$$\begin{aligned}
\mathcal{L}^{(1)}_{\xi} &= \bar{\chi}_c i x^{\mu}_{\perp}\left[in_-\partial\mathcal{B}^{+}_{\mu}(x_-)\right]\frac{\not{n}_+}{2}\chi_c \\
\mathcal{L}^{(2)}_{1\xi} &= \frac{1}{2}\bar{\chi}_c i n_- x n^{\mu}_{+}\left[in_-\partial\mathcal{B}^{+}_{\mu}(x_-)\right]\frac{\not{n}_+}{2}\chi_c \\
\mathcal{L}^{(2)}_{2\xi} &= \frac{1}{2}\bar{\chi}_c x^{\mu}_{\perp} x^{\rho}_{\perp}\left[i\partial_{\rho} i n_-\partial\mathcal{B}^{+}_{\mu}(x_-)\right]\frac{\not{n}_+}{2}\chi_c \\
\mathcal{L}^{(2)}_{3\xi} &= \frac{1}{2}\bar{\chi}_c x^{\mu}_{\perp} x^{\rho}_{\perp}\left[\mathcal{B}^{+}_{\rho}(x_-),\, in_-\partial\mathcal{B}^{+}_{\mu}(x_-)\right]\frac{\not{n}_+}{2}\chi_c \\
\mathcal{L}^{(2)}_{4\xi} &= \frac{1}{2}\bar{\chi}_c\left(i\not{\partial}_{\perp} + \not{A}_{c\perp}\right)\frac{1}{in_+\partial} i x^{\mu}_{\perp}\gamma^{\nu}_{\perp}\left[i\partial_{\nu}\mathcal{B}^{+}_{\mu}(x_-) - i\partial_{\mu}\mathcal{B}^{+}_{\nu}(x_-)\right]\frac{\not{n}_+}{2}\chi_c + \text{h.c.} \\
\mathcal{L}^{(2)}_{5\xi} &= \frac{1}{2}\bar{\chi}_c\left(i\not{\partial}_{\perp} + \not{A}_{c\perp}\right)\frac{1}{in_+\partial} i x^{\mu}_{\perp}\gamma^{\nu}_{\perp}\left[\mathcal{B}^{+}_{\nu}(x_-), \mathcal{B}^{+}_{\mu}(x_-)\right]\frac{\not{n}_+}{2}\chi_c + \text{h.c.} \\
\mathcal{L}^{(1)}_{\xi q} &= \bar{q}_{+}(x_-)\mathcal{A}_{c\perp}\chi_c + \text{h.c.}
\end{aligned}$$

 based on [M. Beneke and Th. Feldmann, hep-ph/0211358]

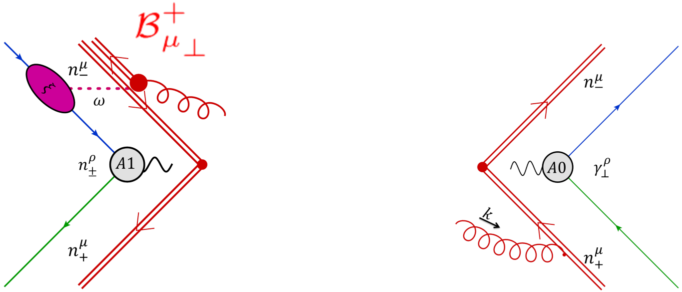
Factorization formula at NLP

We now take a closer look at the structure of the hadronic tensor

$$g^{\mu\rho}W_{\mu\rho} \quad = \quad \int d^4x e^{-iq\cdot x}\langle A(p_A)B(p_B)|J^{\dagger\,\rho}(x)J_\rho(0)|A(p_A)B(p_B)\rangle$$

$$\text{Consider :} \qquad J_c^{T2}(t) = i\int d^4z\,\mathbf{T}\left[J_c^{A1}(t)\,\mathcal{L}^{(1)}(z)\right]$$

$$J_\rho^{A0,A1}(t,\bar{t}) \quad = \quad \bar{\chi}_{\bar{c}}(\bar{t}n_-)\,n_{+\rho}\;i\partial\!\!\!/_\perp\chi_c(t\,n_+)$$



$$J_\rho^{A0,A0}(t,\bar{t}) \quad = \quad \bar{\chi}_{\bar{c}}(\bar{t}n_-)\,\gamma_{\perp\rho}\;\chi_c(t\,n_+)$$

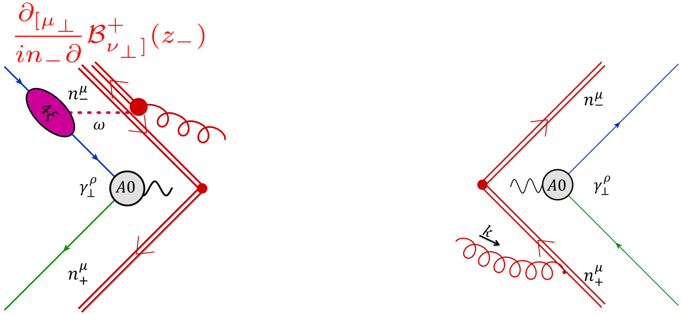
Contributions from power suppressed currents can start contributing at NNLP!
Only the LP J^{A0A0} and insertions of the Lagrangian needed up to NLP.

Factorization formula at NLP

We now take a closer look at the structure of the hadronic tensor

$$g^{\mu\rho}W_{\mu\rho} \quad = \quad \int d^4x e^{-iq\cdot x}\langle A(p_A)B(p_B)|J^{\dagger\,\rho}(x)J_\rho(0)|A(p_A)B(p_B)\rangle$$

$$\text{Consider :} \qquad J_c^{T2}(t) = i\int d^4z\,\mathbf{T}\left[J_c^{A0}(t)\,\mathcal{L}^{(2)}(z)\right]$$



$$\widetilde{S}_{4\xi;\mu\nu}(x^0;\omega) \;=\; \int dz_-\,e^{-i\omega\,z_-}\,\frac{1}{N_c}\mathrm{Tr}\langle 0|\bar{\mathbf{T}}\left[Y_+^\dagger Y_-\right](x^0)\,\mathbf{T}\left(\left[Y_-^\dagger Y_+\right](0)\,\frac{i\partial_{[\mu_\perp}}{in_-\partial}\mathcal{B}^+_{\nu_\perp]}(z_-)\right)|0\rangle$$

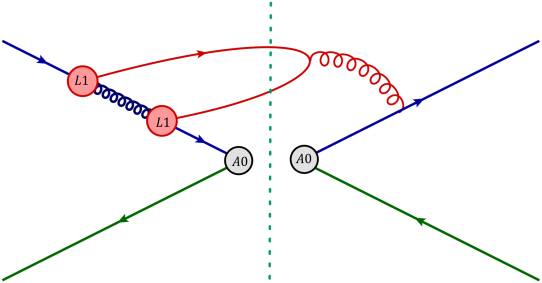
Factorization formula at NLP

We now take a closer look at the structure of the hadronic tensor

$$g^{\mu\rho}W_{\mu\rho} = \int d^4x e^{-iq\cdot x}\langle A(p_A)B(p_B)|J^{\dagger\,\rho}(x)J_\rho(0)|A(p_A)B(p_B)\rangle$$

Consider : $J_c^{T2}(t) = i^2 \int d^4z_1\, d^4z_2\, \mathbf{T}\left[J_c^{A0}(t)\,\mathcal{L}_{q\xi}^{(1)}(z_1)\mathcal{L}_{q\xi}^{(1)}(z_2)\right]$

$$\mathcal{L}_{\xi q}^{(1)}(z) = \bar{q}_+(z_-)\,\mathcal{A}_{c\perp}(z)\chi_c(z) + \text{h.c.}$$



Power corrections due to soft quark contributions studied in B-physics [M. Beneke, F. Campanario, T. Mannel, B. Pecjak, hep-ph/0411395]

Factorization formula at NLP: leading logarithmic accuracy

Defining $\Delta = \hat{\sigma}/z$, we arrive at the final result:

$$\Delta^{dyn}_{\rm NLP}(z) = -2\,Q\left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\gamma_\perp^\rho\right]_{\beta\gamma}\int d(n_+p)\;C^{A0}\,(n_+p, x_b n_- p_B)$$

$$\times\, C^{*A0}\,(x_a\, n_+p_A, x_b n_- p_B)\sum_{i=1}^{5}\int\{d\omega_j\}\; J_i\,(n_+p, x_a\, n_+p_A; \{\omega_j\})\, S_i(\Omega;\{\omega_j\}) + \text{h.c.}$$

where the *generalised* soft functions have the structure:

$$\widetilde{S}_i\,(x;\{\omega_j\}) = \int\{dz_{j-}\}\; e^{-i\omega_j z_{j-}} \times \frac{1}{N_c}\mathrm{Tr}\langle 0|\bar{\mathbf{T}}\left(\left[Y_+^\dagger\, Y_-\right](x)\right)\mathbf{T}\left(\left[Y_-^\dagger\, Y_+\right](0)\,\mathfrak{s}_i\,(\{z_{j-}\})\right)|0\rangle$$

with

$$\mathfrak{s}_i(\{z_{j-}\}) \;\in\; \left\{\; \frac{i\partial_\perp^\mu}{in_-\partial}\mathcal{B}^+_{\mu\perp}(z_{1-})\;,\; \frac{1}{(in_-\partial)^2}\left[\mathcal{B}^{+\,\mu\perp}(z_{1-}), \left[in_-\partial\mathcal{B}^+_{\mu\perp}(z_{1-})\right]\right],\right.$$

$$\left.\frac{1}{(in_-\partial)}\left[\mathcal{B}^+_{\mu\perp}(z_{1-}), \mathcal{B}^+_{\nu\perp}(z_{1-})\right], \frac{1}{(in_-\partial)}\mathcal{B}^+_{\mu\perp}(z_{1-})\mathcal{B}^+_{\nu\perp}(z_{2-}), \frac{1}{(in_-\partial)^2}q_{+\sigma}(z_{1-})\bar{q}_{+\lambda}(z_{2-})\right\}$$

Which terms contribute to the leading logarithms?

- NLP LL series is given by the terms $\alpha_s^n \ln^{2n-1}(1-z)$.
  $\to \alpha_s \ln(1-z) + \alpha_s^2 \ln^3(1-z) + \alpha_s^3 \ln^5(1-z) + \ldots$
- NLP LL can be generated at one loop only if one-loop soft function contains a $\alpha_s \ln(1-z)$ term AND $\sum_{\text{terms}}[C\otimes J\otimes \bar{J}]^2$ starts at tree-level.

Factorization formula at NLP: leading logarithmic accuracy

Defining $\Delta = \hat{\sigma}/z$, we arrive at the final result:

$$\Delta^{dyn}_{\rm NLP}(z) = -2\,Q \left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\gamma_\perp^\rho\right]_{\beta\gamma}\int d(n_+p)\; C^{A0}\,(n_+p, x_b n_- p_B)$$

$$\times\, C^{*A0}\,(x_a\,n_+p_A,\, x_b n_- p_B)\sum_{i=1}^{5}\int\{d\omega_j\}\; J_i\,(n_+p, x_a\,n_+p_A; \{\omega_j\})\, S_i(\Omega;\{\omega_j\}) + \text{h.c.}$$

where the *generalised* soft functions have the structure:

$$\widetilde{S}_i\,(x;\{\omega_j\}) = \int\{dz_{j-}\}\; e^{-i\omega_j z_{j-}} \times \frac{1}{N_c}\mathrm{Tr}\langle 0|\bar{\mathbf{T}}\left(\left[Y_+^\dagger\, Y_-\right](x)\right)\mathbf{T}\left(\left[Y_-^\dagger\, Y_+\right](0)\,\mathfrak{s}_i\,(\{z_{j-}\})\right)|0\rangle$$

with

$$\mathfrak{s}_i(\{z_{j-}\}) \quad\in\quad \left\{\boxed{\frac{i\partial_\perp^\mu}{in_-\partial}\mathcal{B}^+_{\mu_\perp}(z_{1-})}, \frac{1}{(in_-\partial)^2}\left[\mathcal{B}^{+\,\mu_\perp}(z_{1-}), \left[in_-\partial\mathcal{B}^+_{\mu_\perp}(z_{1-})\right]\right],\right.$$

$$\left.\frac{1}{(in_-\partial)}\left[\mathcal{B}^+_{\mu_\perp}(z_{1-}), \mathcal{B}^+_{\nu_\perp}(z_{1-})\right], \frac{1}{(in_-\partial)}\mathcal{B}^+_{\mu_\perp}(z_{1-})\mathcal{B}^+_{\nu_\perp}(z_{2-}), \frac{1}{(in_-\partial)^2}q_{+\sigma}(z_{1-})\bar{q}_{+\lambda}(z_{2-})\right\}$$

Only one new soft structure contributes! With a corresponding *tree-level* collinear function.

Consider again: Factorization formula at NLP

Following this line of reasoning, we can eliminate most of the possible new contributions coming from Lagrangian insertions at LL accuracy.

$$\Delta^{dyn}_{\rm NLP}(z) = -2\, Q \left[\left(\frac{\not{n}_-}{4} \right) \gamma_{\perp\rho} \left(\frac{\not{n}_+}{4} \right) \gamma_\perp^\rho \right]_{\beta\gamma} \int d(n_+p)\; C^{\,A0}\,(n_+p, x_b n_- p_B)$$

$$\times\, C^{*A0}\,(\,x_a\, n_+p_A,\, x_b n_- p_B) \sum_{i=1}^{5} \int \{d\omega_j\}\; J_i\,(n_+p, x_a\, n_+p_A; \{\omega_j\})\, S_i(\Omega; \{\omega_j\}) + \text{h.c.}$$

where the *generalised* soft functions have the structure:

$$\widetilde{S}_{\,i}\,(x; \{\omega_j\}) = \int \{dz_{j-}\}\; e^{-i\omega_j z_{j-}} \times \frac{1}{N_c} {\rm Tr}\langle 0|\bar{\mathbf{T}}\left(\left[Y_+^\dagger\, Y_-\right](x)\right) \mathbf{T}\left(\left[Y_-^\dagger\, Y_+\right](0)\, \mathfrak{s}_i\,(\,\{z_{j-}\})\right)|0\rangle$$

with

$$\mathfrak{s}_i(\{z_{j-}\}) \;\; \in \;\; \Big\{\; \frac{i\partial_\perp^\mu}{in_-\partial} \mathcal{B}^+_{\mu_\perp}(z_{1-})\;,\; \frac{1}{(in_-\partial)^2}\left[\mathcal{B}^{+\,\mu_\perp}(z_{1-}), \left[in_-\partial \mathcal{B}^+_{\mu_\perp}(z_{1-})\right]\right],$$

$$\frac{1}{(in_-\partial)}\left[\mathcal{B}^+_{\mu_\perp}(z_{1-}), \mathcal{B}^+_{\nu_\perp}(z_{1-})\right], \frac{1}{(in_-\partial)}\mathcal{B}^+_{\mu_\perp}(z_{1-})\mathcal{B}^+_{\nu_\perp}(z_{2-}), \frac{1}{(in_-\partial)^2} q_{+\sigma}(z_{1-})\bar{q}_{+\lambda}(z_{2-})\Big\}$$

Consider again: Factorization formula at NLP

Following this line of reasoning, we can eliminate most of the possible new contributions coming from Lagrangian insertions at LL accuracy.

$$\Delta^{dyn}_{\rm NLP}(z) = -2\,Q \left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\gamma^{\rho}_{\perp}\right]_{\beta\gamma}\int d(n_+p)\; C^{A0}\,(n_+p, x_b n_- p_B)$$

$$\times\, C^{*A0}\,(x_a\, n_+p_A,\, x_b n_- p_B)\sum_{i=1}^{5}\int \{d\omega_j\}\; J_i\,(n_+p, x_a\, n_+p_A; \{\omega_j\})\; S_i(\Omega; \{\omega_j\}) + \text{h.c.}$$

where the *generalised* soft functions have the structure:

$$\widetilde{S}_i\,(x;\{\omega_j\}) = \int \{dz_{j-}\}\; e^{-i\omega_j z_{j-}} \times \frac{1}{N_c}\mathrm{Tr}\langle 0|\bar{\mathbf{T}}\left(\left[Y_+^\dagger\, Y_-\right](x)\right)\mathbf{T}\left(\left[Y_-^\dagger\, Y_+\right](0)\,\mathfrak{s}_i\,(\{z_{j-}\})\right)|0\rangle$$

with

$$\mathfrak{s}_i(\{z_{j-}\}) \;\in\; \left\{\boxed{\frac{i\partial^\mu_\perp}{in_-\partial}\mathcal{B}^+_{\mu_\perp}(z_{1-})},\; \frac{1}{(in_-\partial)^2}\left[\mathcal{B}^{+\,\mu_\perp}(z_{1-}), \left[in_-\partial\mathcal{B}^+_{\mu_\perp}(z_{1-})\right]\right],\right.$$

$$\left.\frac{1}{(in_-\partial)}\left[\mathcal{B}^+_{\mu_\perp}(z_{1-}), \mathcal{B}^+_{\nu_\perp}(z_{1-})\right], \frac{1}{(in_-\partial)}\mathcal{B}^+_{\mu_\perp}(z_{1-})\mathcal{B}^+_{\nu_\perp}(z_{2-}), \frac{1}{(in_-\partial)^2}q_{+\sigma}(z_{1-})\bar{q}_{+\lambda}(z_{2-})\right\}$$

Only one new soft structure contributes! With a corresponding *tree-level* collinear function.

A power suppressed amplitude

$$\bar{\psi}\gamma^{\mu}\psi(0) = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t}) \left[J^{\mu}_{A0}(t,\bar{t}) + \boxed{ i\int d^4x\, \mathbf{T}\left[J^{\mu}_{A0}(s,t)\, \mathcal{L}^{(2)}_{2\xi}(x) \right] } + \ \bar{c}\text{-term} \right]$$

$$\begin{aligned} \langle X|\bar{\psi}\gamma^{\mu}\psi(0)|A(p_A)B(p_B)\rangle &= \int \frac{d(n_+p)}{2\pi}\frac{d(n_-\bar{p})}{2\pi} \int dt\, d\bar{t}\, e^{it\, n_+p} e^{i\bar{t}n_-\bar{p}} C^{A0}(n_+p, n_-\bar{p}) \\ &\times \langle X|\mathbf{T}\Bigg[\underbrace{\bar{\chi}_{\bar{c}}(\bar{t}n_-)Y_-^{\dagger}(0)\gamma^{\mu}_{\perp}\, Y_+(0)\chi_c\,(tn_+)}_{J^{\mu}_{A0}(t,\bar{t})}\, i\int d^4z\, \bar{\chi}_{c,e}\,(z)\, \frac{1}{2} z^{\nu}_{\perp} z^{\rho}_{\perp} \\ &\times \left[\boxed{\left(\frac{in_-\partial_z}{in_-\partial_z}\right)} (in_-\partial_z) i\partial^{\rho}_{\perp} \mathcal{B}^{+}_{\perp\nu,ed}\,(z_-) \right] \frac{\not{n}_+}{2}\chi_{c,d}\,(z) \Bigg] |A(p_A)B(p_B)\rangle \end{aligned}$$

A power suppressed amplitude

The states factorize as at leading power: $\langle X| = \langle X_{\bar{c}}^{\rm PDF}|\,\langle X_c^{\rm PDF}|\,\langle X_s|$ as they are eigenstates of the LP Lagrangian

$$\langle X|\bar{\psi}\gamma^\mu\psi(0)|A(p_A)B(p_B)\rangle = \int \frac{d(n_+p)}{2\pi}\frac{d(n_-\bar{p})}{2\pi}\int dt\, d\bar{t}\, e^{it\, n_+p} e^{i\bar{t}n_-\bar{p}} C^{A0}(n_+p, n_-\bar{p})$$
$$\times\langle X_{\bar{c}}^{\rm PDF}|\bar{\chi}_{\bar{c},\alpha a}(\bar{t}n_-)|B(p_B)\rangle\gamma^\mu_{\perp,\alpha\gamma}$$
$$\times i\int d^4z\langle X_c^{\rm PDF}|\frac{1}{2}z_\perp^\nu z_\perp^\rho (in_-\partial_z)^2\,\mathbf{T}\left[\chi_{c,\gamma f}(tn_+)\,\bar{\chi}_{c,e}(z)\,\frac{\not{n}_+}{2}\chi_{c,d}(z)\right]|A(p_A)\rangle$$
$$\times\langle X_s|\mathbf{T}\left(\left[Y_-^\dagger(0)Y_+(0)\right]_{af}\frac{i\partial_\perp^\rho}{in_-\partial_z}\mathcal{B}^+_{\perp\nu,ed}(z_-)\right)|0\rangle$$

Amplitude with collinear function

$$
\begin{aligned}
\langle X|\bar{\psi}\gamma^{\mu}\psi(0)|A(p_A)B(p_B)\rangle = \int \frac{d(n_+p)}{2\pi}\frac{d(n_-\bar{p})}{2\pi}\int d(n_+p_a)d(n_-p_b) \\
\delta\left(n_-\bar{p}+(n_-p_b)\right)\, C^{A0}(n_+p,n_-\bar{p}) \\
\times\int\frac{d\omega}{2\pi}\, J^{\rho\nu}_{2\xi,\gamma\beta,fbed}\,(n_+p,n_+p_a;\omega)\langle X^{\mathrm{PDF}}_{\bar{c}}|\hat{\bar{\chi}}^{\mathrm{PDF}}_{\bar{c},\alpha a}(n_-p_b)|B(p_B)\rangle \\
\times\gamma^{\mu}_{\perp,\alpha\gamma}\langle X^{\mathrm{PDF}}_{c}|\hat{\chi}^{\mathrm{PDF}}_{c,\beta b}(n_+p_a)|A(p_A)\rangle \\
\times\int\frac{dn_+z}{2}e^{-i\omega\frac{n_+z}{2}}\langle X_s|\mathbf{T}\left(\left[Y^{\dagger}_{-}(0)Y_{+}(0)\right]_{af}\frac{i\partial^{\rho}_{\perp}}{in_-\partial}\mathcal{B}^{+}_{\perp\nu,ed}\,(z_-)\right)|0\rangle
\end{aligned}
$$

Computation of collinear function

The short-distance coefficient can be extracted by computing the partonic matrix element $\langle 0 | \mathcal{J}^{\rho\nu}_{\gamma,fed}\,(n_+q_a,\omega)\,|q(q)_q\rangle$. Running to collinear scale: only tree level collinear function is necessary.

Collinear function:

$$J^{\rho\nu}_{2\xi,\gamma\beta,fbed}\,(n_+q_a,(n_+q);\omega) = -\delta_{bd}\delta_{fe}\delta_{\beta\gamma}\delta\left((n_+q_a)-(n_+q)\right)\frac{g_\perp^{\nu\rho}}{(n_+q)}$$

LP + NLP amplitude

We are considering the matching up to NLP

$$\bar{\psi}\gamma^{\mu}\psi(0) = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t}) \left[J^{\mu}_{A0}(t,\bar{t}) + \left(J^{T2}_{A0,2\xi}(t,\bar{t})\right)^{\mu} + \ \bar{c}\text{-term}\right]$$

For which we obtained

$$\langle X|\bar{\psi}\gamma^{\mu}\psi(0)|A(p_A)B(p_B)\rangle = \int \frac{dn_+p_a}{2\pi}\frac{dn_-p_b}{2\pi}\, C^{A0}(n_+p_a, -n_-p_b)$$

$$\times \langle X_{\bar{c},\mathrm{PDF}}|\hat{\bar{\chi}}^{\mathrm{PDF}}_{\bar{c},\alpha a}(n_-p_b)|B(p_B)\rangle\, \gamma^{\mu}_{\perp\alpha\beta}\, \langle X_{c,\mathrm{PDF}}|\hat{\chi}^{\mathrm{PDF}}_{c,\beta b}(n_+p_a)|A(p_A)\rangle$$

$$\times \Bigg\{ \langle X_s|\mathbf{T}\left[Y_-^{\dagger}(0)Y_+(0)\right]_{ab}|0\rangle$$

$$+ \frac{1}{2}\int\frac{d\omega}{4\pi}\, J^{(O)}_{2\xi}(n_+p_a;\omega)\int d(n_+z)\, e^{-i\omega(n_+z)/2}$$

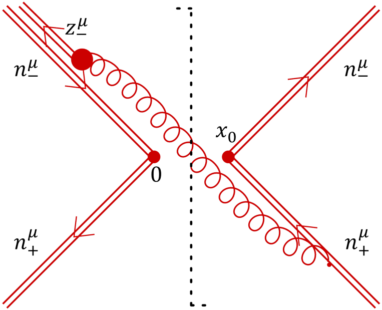
$$\times\langle X_s|\mathbf{T}\left(\left[Y_-^{\dagger}(0)Y_+(0)\right]_{af}\frac{i\partial^{\nu}_{\perp}}{in_-\partial_z}\mathcal{B}^{+}_{\perp\nu;fb}(z_-)\right)|0\rangle\Bigg\} + \bar{c}\text{-term}$$

Note that $J^{(\mathrm{O})}_{2\xi}(n_+p_a;\omega) = -\dfrac{2}{n_+p_a}$

Relevant soft function

The generalized soft function at cross section level here is

$$S_{2\xi}(\Omega,\omega) = \int \frac{dx^0}{4\pi} \int \frac{d(n_+z)}{4\pi}\, e^{ix^0\Omega/2 - i\omega(n_+z)/2}$$
$$\times \frac{1}{N_c}\,\mathrm{Tr}\,\langle 0|\bar{\mathbf{T}}\left[Y_+^\dagger(x^0)Y_-(x^0)\right]\mathbf{T}\left[Y_-^\dagger(0)Y_+(0)\frac{i\partial_\perp^\nu}{in_-\partial}\mathcal{B}^+_{\perp\nu}(z_-)\right]|0\rangle$$



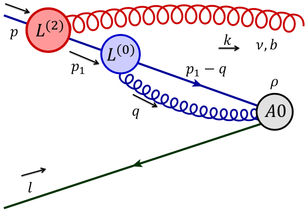
For details on renormalization of soft functions and resummation see Robert's talk.

Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.

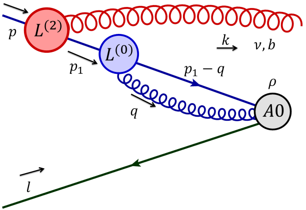
Power suppressed amplitude calculation

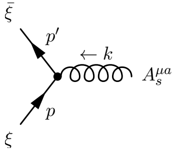
Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.



Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.



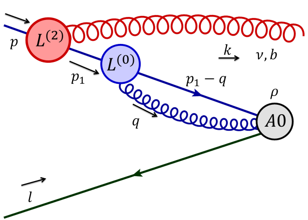


$$ig_s t^a \begin{cases} \dfrac{\not{n}_+}{2} n_{-\mu} & \mathcal{O}(\lambda^0) \\ \dfrac{\not{n}_+}{2} X_\perp^\rho n_-^\nu (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda) \\ S^{\rho\nu}(k,p,p') \dfrac{\not{n}_+}{2} (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2) \end{cases}$$

$$S^{\rho\nu}(k,p,p') \equiv \frac{1}{2}\left[(n_- X) n_+^\rho n_-^\nu \; + \; (k X_\perp) X_\perp^\rho n_-^\nu \; + \; X_\perp^\rho \left(\frac{\not{p}'_\perp}{n_+ p'} \gamma_\perp^\nu + \gamma_\perp^\nu \frac{\not{p}_\perp}{n_+ p} \right) \right]$$

Power suppressed amplitude calculation

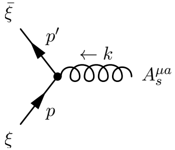
Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.



$$\mathcal{L}_{\xi}^{(2)} = \frac{1}{2}\bar{\chi}_c i\ (n_- x)\ n_+^{\mu}\left[in_-\partial\mathcal{B}_{\mu}^{+}(x_-)\right]\frac{\not{n}_+}{2}\chi_c + \ldots$$

[M. Beneke and Th. Feldmann, 0211358]

$$X^{\alpha} = -\frac{\partial}{\partial p_{1\alpha}}\left\{(2\pi)^4\delta^4(p-k_+-p_1)\right\}$$



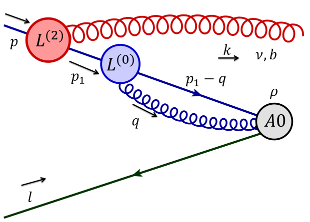
$$ig_s t^a\begin{cases}\dfrac{\not{n}_+}{2}n_{-\mu} & \mathcal{O}(\lambda^0)\\[2mm] \dfrac{\not{n}_+}{2}X_{\perp}^{\rho}n_-^{\nu}(k_\rho g_{\nu\mu}-k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda)\\[2mm] S^{\rho\nu}(k,p,p')\dfrac{\not{n}_+}{2}(k_\rho g_{\nu\mu}-k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2)\end{cases}$$

$$S^{\rho\nu}(k,p,p') \equiv \frac{1}{2}\left[\ (n_-X)n_+^{\rho}n_-^{\nu}\ +\ (kX_\perp)X_\perp^{\rho}n_-^{\nu}\ +\ X_\perp^{\rho}\left(\frac{\not{p}'_\perp}{n_+p'}\gamma_\perp^{\nu}+\gamma_\perp^{\nu}\frac{\not{p}_\perp}{n_+p}\right)\ \right]$$

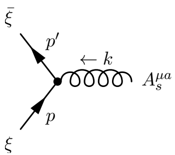
[M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.



$$\bar{\nu}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon} \frac{C_F t^b}{(n_+p)(n_-k)}$$
$$\times\Big\{ \left((n_+k)n_{-\nu} - (n_-k)n_{+\nu}\right) \left(\frac{2}{\epsilon} + \mathcal{O}(\epsilon^0)\right)$$
$$+ \left(\frac{k_\perp^2}{(n_-k)}n_{-\nu} - k_{\perp\nu}\right) \left(\frac{2}{\epsilon^2} + \frac{4}{\epsilon} + \mathcal{O}(\epsilon^0)\right)$$
$$+ \left[\gamma_{\perp\nu}\,,\, \not{k}_\perp\right] \left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon} + \mathcal{O}(\epsilon^0)\right)\Big\} u_c(p)$$



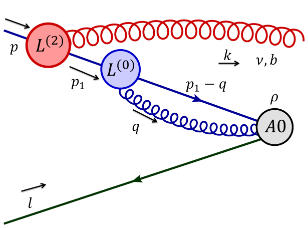
$$ig_s t^a \begin{cases} \frac{\not{n}_+}{2} n_{-\mu} & \mathcal{O}(\lambda^0) \\ \frac{\not{n}_+}{2} X_\perp^\rho n_-^\nu (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda) \\ S^{\rho\nu}(k,p,p')\frac{\not{n}_+}{2}(k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2) \end{cases}$$

$$S^{\rho\nu}(k,p,p') \equiv \frac{1}{2}\left[(n_-X)n_+^\rho n_-^\nu \;+\; (kX_\perp)X_\perp^\rho n_-^\nu \;+\; X_\perp^\rho\left(\frac{\not{p}'_\perp}{n_+p'}\gamma_\perp^\nu + \gamma_\perp^\nu \frac{\not{p}_\perp}{n_+p}\right)\right]$$

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.

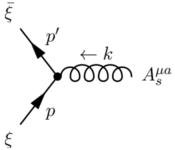


$$\bar{\nu}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\, g\, \alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\frac{C_F t^b}{(n_+p)(n_-k)}$$

$$\times\left\{\boxed{((n_+k)n_{-\nu} - (n_-k)n_{+\nu})}\left(\frac{2}{\epsilon}+\mathcal{O}(\epsilon^0)\right)\right.$$

$$+\left(\frac{k_\perp^2}{(n_-k)}n_{-\nu} - k_{\perp\nu}\right)\left(\frac{2}{\epsilon^2}+\frac{4}{\epsilon}+\mathcal{O}(\epsilon^0)\right)$$

$$\left.+\left[\gamma_{\perp\nu}\,,\,\not{k}_\perp\right]\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}+\mathcal{O}(\epsilon^0)\right)\right\}u_c(p)$$

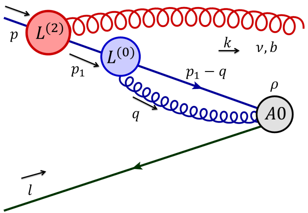


$$ig_s t^a\begin{cases} \frac{\not{n}_+}{2}n_{-\mu} & \mathcal{O}(\lambda^0) \\ \frac{\not{n}_+}{2}X_\perp^\rho n_-^\nu (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda) \\ S^{\rho\nu}(k,p,p')\frac{\not{n}_+}{2}(k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2) \end{cases}$$

$$S^{\rho\nu}(k,p,p') \equiv \frac{1}{2}\left[\boxed{(n_-X)n_+^\rho n_-^\nu} + (kX_\perp)X_\perp^\rho n_-^\nu + X_\perp^\rho\left(\frac{\not{p}'_\perp}{n_+p'}\gamma_\perp^\nu + \gamma_\perp^\nu\frac{\not{p}_\perp}{n_+p}\right)\right]$$

Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.

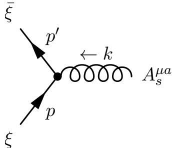


$$\bar{\nu}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\frac{C_F t^b}{(n_+p)(n_-k)}$$

$$\times\Bigg\{ \left((n_+k)n_{-\nu} - (n_-k)n_{+\nu}\right) \left(\frac{2}{\epsilon} + \mathcal{O}(\epsilon^0)\right)$$

$$+ \boxed{\left(\frac{k_\perp^2}{(n_-k)}n_{-\nu} - k_{\perp\nu}\right)} \left(\frac{2}{\epsilon^2} + \frac{4}{\epsilon} + \mathcal{O}(\epsilon^0)\right)$$

$$+ \left[\gamma_{\perp\nu}\,,\, \not{k}_\perp\right] \left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon} + \mathcal{O}(\epsilon^0)\right)\Bigg\} u_c(p)$$



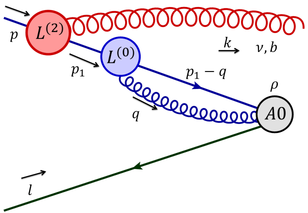
$$ig_s t^a \begin{cases} \frac{\not{n}_+}{2} n_{-\mu} & \mathcal{O}(\lambda^0) \\ \frac{\not{n}_+}{2} X_\perp^\rho n_-^\nu (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda) \\ S^{\rho\nu}(k,p,p')\frac{\not{n}_+}{2}(k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2) \end{cases}$$

$$S^{\rho\nu}(k,p,p') \equiv \frac{1}{2}\left[(n_-X)n_+^\rho n_-^\nu + \boxed{(kX_\perp)X_\perp^\rho n_-^\nu} + X_\perp^\rho\left(\frac{\not{p}'_\perp}{n_+p'}\gamma_\perp^\nu + \gamma_\perp^\nu\frac{\not{p}_\perp}{n_+p}\right)\right]$$

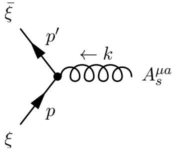
[M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.



$$\bar{\nu}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\frac{C_F t^b}{(n_+p)(n_-k)}$$
$$\times\Big\{ \left((n_+k)n_{-\nu} - (n_-k)n_{+\nu}\right)\left(\frac{2}{\epsilon} + \mathcal{O}(\epsilon^0)\right)$$
$$+\left(\frac{k_\perp^2}{(n_-k)}n_{-\nu} - k_{\perp\nu}\right)\left(\frac{2}{\epsilon^2} + \frac{4}{\epsilon} + \mathcal{O}(\epsilon^0)\right)$$
$$+\boxed{\left[\gamma_{\perp\nu}\,,\,\not{k}_\perp\right]}\left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon} + \mathcal{O}(\epsilon^0)\right)\Big\}u_c(p)$$



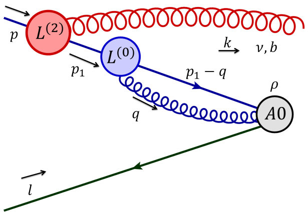
$$ig_s t^a \begin{cases} \dfrac{\not{n}_+}{2}n_{-\mu} & \mathcal{O}(\lambda^0) \\ \dfrac{\not{n}_+}{2}X_\perp^\rho n_-^\nu(k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda) \\ S^{\rho\nu}(k,p,p')\dfrac{\not{n}_+}{2}(k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2) \end{cases}$$

$$S^{\rho\nu}(k,p,p') \equiv \frac{1}{2}\left[(n_-X)n_+^\rho n_-^\nu \;+\; (kX_\perp)X_\perp^\rho n_-^\nu \;+\; \boxed{X_\perp^\rho\left(\frac{\not{p}'_\perp}{n_+p'}\gamma_\perp^\nu + \gamma_\perp^\nu\frac{\not{p}_\perp}{n_+p}\right)}\right]$$

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.



$$\bar{\nu}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\frac{C_F t^b}{(n_+p)(n_-k)}$$
$$\times\Bigg\{\boxed{((n_+k)n_{-\nu} - (n_-k)n_{+\nu})}\left(\frac{2}{\epsilon} + \mathcal{O}(\epsilon^0)\right)$$
$$+\boxed{\left(\frac{k_\perp^2}{(n_-k)}n_{-\nu} - k_{\perp\nu}\right)}\left(\frac{2}{\epsilon^2} + \frac{4}{\epsilon} + \mathcal{O}(\epsilon^0)\right)$$
$$+\boxed{\left[\gamma_{\perp\nu}\,,\,\not{k}_\perp\right]}\left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon} + \mathcal{O}(\epsilon^0)\right)\Bigg\}u_c(p)$$

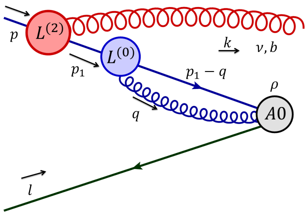
$$(n_+k)(n_-\epsilon^*) = 2\left(-\frac{(n_-k)(n_+\epsilon^*)}{2} - k_\perp\cdot\epsilon_\perp^*\right)$$

$$S^{\rho\nu}(k,p,p') \equiv \frac{1}{2}\left[\boxed{(n_-X)n_+^\rho n_-^\nu} + \boxed{(kX_\perp)X_\perp^\rho n_-^\nu} + \boxed{X_\perp^\rho\left(\frac{\not{p}'_\perp}{n_+p'}\gamma_\perp^\nu + \gamma_\perp^\nu\frac{\not{p}_\perp}{n_+p}\right)}\right]$$

 [M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.



$$\bar{\nu}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\frac{C_F t^b}{(n_+p)(n_-k)}$$
$$\times\left\{\boxed{\left(\frac{-2k_\perp^2}{n_-k}n_{-\nu}+2k_{\perp\nu}\right)}\left(\frac{2}{\epsilon}+\mathcal{O}(\epsilon^0)\right)\right.$$
$$+\boxed{\left(\frac{k_\perp^2}{(n_-k)}n_{-\nu}-k_{\perp\nu}\right)}\left(\frac{2}{\epsilon^2}+\frac{4}{\epsilon}+\mathcal{O}(\epsilon^0)\right)$$
$$\left.+\boxed{\left[\gamma_{\perp\nu}\,,\,\not{k}_\perp\right]}\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}+\mathcal{O}(\epsilon^0)\right)\right\}u_c(p)$$

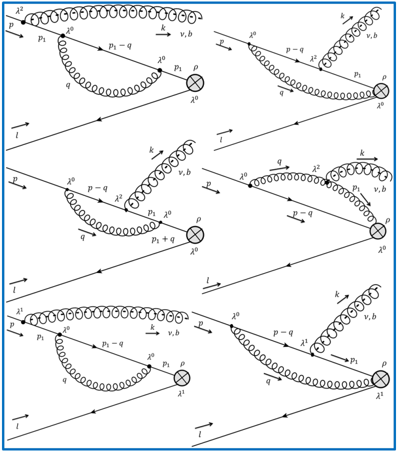
$$(n_+k)(n_-\epsilon^*) = 2\left(-\frac{(n_-k)(n_+\epsilon^*)}{2} - k_\perp\cdot\epsilon_\perp^*\right)$$

$$S^{\rho\nu}(k,p,p') \equiv \frac{1}{2}\left[\boxed{(n_-X)n_+^\rho n_-^\nu} + \boxed{(kX_\perp)X_\perp^\rho n_-^\nu} + \boxed{X_\perp^\rho\left(\frac{\not{p}'_\perp}{n_+p'}\gamma_\perp^\nu + \gamma_\perp^\nu\frac{\not{p}_\perp}{n_+p}\right)}\right]$$

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

Amplitude calculation: 1-real emission

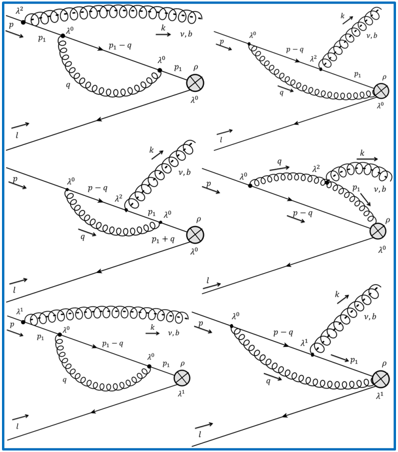
$$\mathcal{A} = \; C \otimes J_{2\xi} \otimes \langle X| \; \frac{\partial_{\perp}^{\mu}}{in_{-}\partial} \mathcal{B}^{+}_{\mu_{\perp}} \; |0\rangle + \; C \otimes J_{4\xi} \otimes \langle X| \; \frac{\partial_{[\mu_{\perp}}}{in_{-}\partial} \mathcal{B}^{+}_{\nu_{\perp}]} \; |0\rangle + \; C \otimes J_{\xi} \otimes \langle X| \; \mathcal{B}^{+}_{\mu_{\perp}} \; |0\rangle$$



1-loop collinear $\otimes$ 1-real soft emission

Amplitude calculation: 1-real emission

$$\mathcal{A} = C \otimes \boxed{J_{2\xi}} \otimes \langle X| \frac{\partial_\perp^\mu}{in_-\partial} \mathcal{B}^+_{\mu_\perp} |0\rangle + C \otimes \boxed{J_{4\xi}} \otimes \langle X| \frac{\partial_{[\mu_\perp}}{in_-\partial} \mathcal{B}^+_{\nu_\perp]} |0\rangle + C \otimes \boxed{J_\xi} \otimes \langle X| \mathcal{B}^+_{\mu_\perp} |0\rangle$$

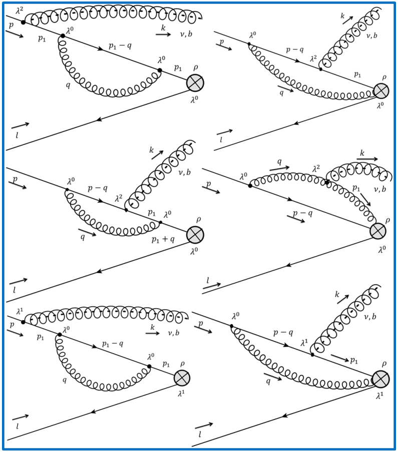


1-loop collinear $\otimes$ 1-real soft emission

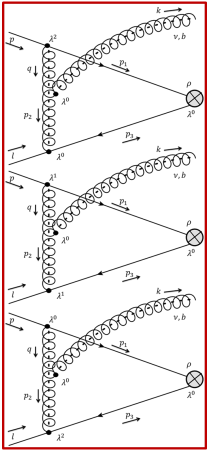
Extract 1-loop collinear functions

Amplitude calculation: 1-real emission

$$\mathcal{A} = C \otimes J_{2\xi} \otimes \langle X| \boxed{\frac{\partial_\perp^\mu}{in_-\partial}\mathcal{B}^+_{\mu_\perp}} |0\rangle + C \otimes J_{4\xi} \otimes \langle X| \boxed{\frac{\partial_{[\mu_\perp}}{in_-\partial}\mathcal{B}^+_{\nu_\perp]}} |0\rangle + C \otimes J_\xi \otimes \langle X| \boxed{\mathcal{B}^+_{\mu_\perp}} |0\rangle$$



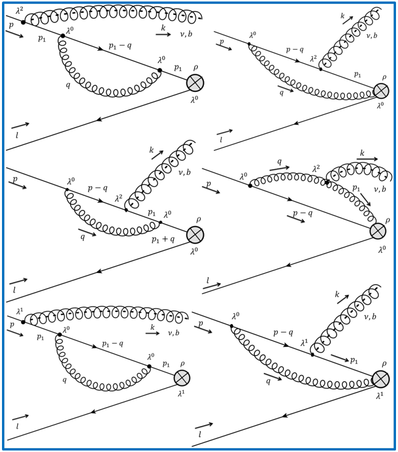
1-loop collinear $\otimes$ 1-real soft emission
Extract 1-loop collinear functions



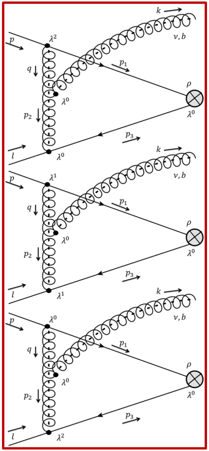
1-loop soft $\otimes$ 1-real soft emission

Amplitude calculation: 1-real emission

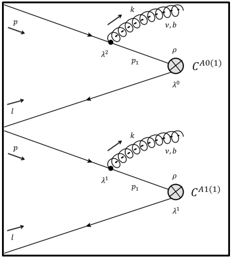
$$\mathcal{A} = \boxed{C} \otimes J_{2\xi} \otimes \langle X | \frac{\partial^{\mu}_{\perp}}{in_-\partial} \mathcal{B}^{+}_{\mu_\perp} |0\rangle + \boxed{C} \otimes J_{4\xi} \otimes \langle X | \frac{\partial_{[\mu_\perp}}{in_-\partial} \mathcal{B}^{+}_{\nu_\perp]} |0\rangle + \boxed{C} \otimes J_{\xi} \otimes \langle X | \mathcal{B}^{+}_{\mu_\perp} |0\rangle$$



1-loop collinear ⊗ 1-real soft emission
Extract 1-loop collinear functions



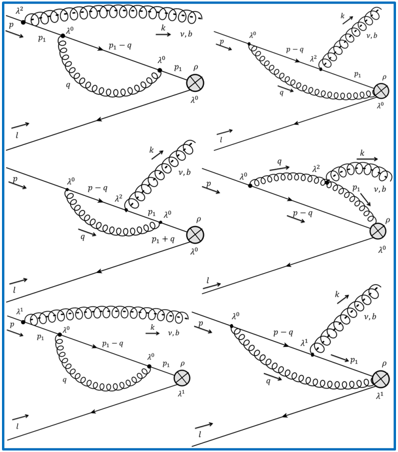
1-loop soft ⊗ 1-real soft emission



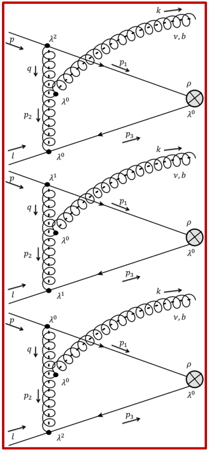
1-loop hard ⊗ 1-real soft emission

Amplitude calculation: 1-real emission

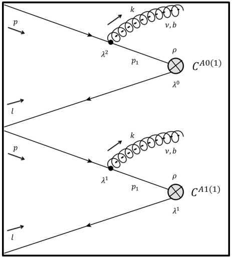
We find agreement with the method of regions expansion for the 1-real 1-virtual amplitude. For explicit results see next slides.



1-loop collinear $\otimes$ 1-real soft emission
Extract 1-loop collinear functions



1-loop soft $\otimes$ 1-real soft emission



1-loop hard $\otimes$ 1-real soft emission

Result for the power suppressed amplitude: C_F

$$
\begin{aligned}
i\, C_F\, \gamma_{\perp\rho}\, \frac{1}{(n_+p)(n_-k)} \Bigg(& (n_+k) n_{-\nu} \left(\frac{3}{\epsilon} + 2 - \frac{3}{2}\zeta(2)\epsilon + \left(-\zeta(2) - \frac{21\zeta(3)}{3} - 4 \right) \epsilon^2 \right) \\
& + (n_-k) n_{+\nu} \left(-\frac{1}{\epsilon} - 3 + (-6 + \frac{1}{2}\zeta(2))\epsilon + \left(\frac{3}{2}\zeta(2) - 12 + \frac{7\zeta(3)}{3} \right) \epsilon^2 \right) \\
& + k_{\perp\nu} \left(+ \frac{2}{\epsilon} - 1 + (-6 - \zeta(2))\epsilon + \left(+ \frac{\zeta(2)}{2} - \frac{14\zeta(3)}{3} - 16 \right) \epsilon^2 \right) \\
& + [\not{k}_\perp, \gamma_{\perp\nu}] \left(+ \frac{1}{2} + \epsilon + \left(-\frac{1}{4}\zeta(2) + 2 \right) \epsilon^2 \right) \Bigg)
\end{aligned}
$$

$$
i\, C_F\, n_{-\rho}\, \frac{1}{n_-l} \left(\gamma_{\perp\nu} - \frac{\not{k}_\perp n_{-\nu}}{(n_-k)} \right) \left(+1 + 4\epsilon - \frac{1}{2}(\zeta(2) - 20)\epsilon^2 \right)
$$

$$
i\, C_F\, n_{+\rho}\, \frac{1}{n_+p} \left(\gamma_{\perp\nu} - \frac{\not{k}_\perp n_{-\nu}}{(n_-k)} \right) \left(-1 - 4\epsilon + \frac{1}{2}(\zeta(2) - 20)\epsilon^2 \right)
$$

Result for the power suppressed amplitude: C_A

$$
\begin{aligned}
i\,C_A\,\gamma_{\perp\rho}\,\frac{1}{(n_+p)(n_-k)}\Bigg(&(n_+k)n_{-\nu}\Big(-\frac{1}{2\,\epsilon^2}-\frac{3}{2\,\epsilon}+\frac{1}{4}(\zeta(2)-18)\\
&+\frac{1}{12}(9\zeta(2)+14\zeta(3)-48)\epsilon+\frac{1}{32}(72\zeta(2)+112\zeta(3)+47\zeta(4)-288)\epsilon^2\Big)\\
&+(n_-k)n_{+\nu}\Big(-\frac{1}{2\epsilon^2}-\frac{3}{2\epsilon}+\frac{1}{4}(\zeta(2)+2)+\frac{1}{12}(9\zeta(2)+14\zeta(3)-24)\epsilon\\
&-\frac{1}{32}(8\zeta(2)-112\zeta(3)-47\zeta(4)+32)\epsilon^2\Big)\\
&+k_{\perp\nu}\Big(-\frac{1}{\epsilon^2}-\frac{3}{\epsilon}+\frac{1}{2}(\zeta(2)-8)\\
&+\left(\frac{3\zeta(2)}{2}+\frac{7\zeta(3)}{3}-6\right)\epsilon+\left(2\zeta(2)+7\zeta(3)+\frac{47\zeta(4)}{16}-10\right)\epsilon^2\Big)\\
&+[\not{k}_\perp,\gamma_{\perp\nu}]\left(\frac{1}{4}\left(-2-4\epsilon+(\zeta(2)-8)\epsilon^2\right)\right)\Bigg)
\end{aligned}
$$

$$
i\,C_A\,n_{-\rho}\,\frac{1}{n_-l}\left(\gamma_{\perp\nu}-\frac{\not{k}_\perp n_{-\nu}}{(n_-k)}\right)\left(+\frac{1}{\epsilon}+2-\frac{1}{2}(\zeta(2)-6)\epsilon+\left(-\zeta(2)-\frac{7\zeta(3)}{3}+5\right)\epsilon^2\right)
$$

$$
i\,C_A\,n_{+\rho}\,\frac{1}{n_+p}\left(\gamma_{\perp\nu}-\frac{\not{k}_\perp n_{-\nu}}{(n_-k)}\right)\left(-\frac{1}{\epsilon}-2+\frac{1}{2}(\zeta(2)-6)\epsilon+\left(\zeta(2)+\frac{7\zeta(3)}{3}-5\right)\epsilon^2\right)
$$

Results for power suppressed amplitude: Soft

$$
\begin{aligned}
i\, C_A \gamma_{\perp\rho} \frac{1}{(n_+p)(n_-k)} \Bigg(& (n_+k)\, n_{-\nu} \left(+ \frac{1}{\epsilon^2} + \frac{\zeta(2)}{2} - \frac{7}{3}\zeta(3)\epsilon - \frac{1}{16} 39\zeta(4)\epsilon^2 \right) \\
& + (n_-k)\, n_{+\nu} \Big(0 \Big) \\
& + k_{\perp\nu} \left(\frac{1}{\epsilon^2} + \frac{1}{2}\zeta(2) - \frac{7}{3}\zeta(3)\epsilon - \frac{39}{16}\zeta(4)\epsilon^2 \right) \\
& + [\not{k}_\perp, \gamma_{\perp\nu}] \left(+ \frac{1}{2\,\epsilon^2} + \frac{1}{4}\zeta(2) - \frac{7}{6}\zeta(3)\epsilon - \frac{1}{32} 39\zeta(4)\epsilon^2 \right) \Bigg)
\end{aligned}
$$

$$
g\, \frac{C_A}{2} t^b \frac{1}{(n_+p)} n_+^\rho \frac{i\alpha\, e^{\epsilon\gamma_E}}{(4\pi)} \frac{1}{\epsilon^2} \frac{\Gamma[1-\epsilon]^3}{\Gamma[2-2\epsilon]} \Gamma[1+\epsilon]^2
$$

$$
\times \left[-(1-2\epsilon) \frac{\not{k}_\perp n_{-\nu}}{(n_-k)} + 2\gamma_{\perp\nu} + \frac{2\not{k}_\perp k_{\perp\nu}}{(n_-k)(n_+k)} + \frac{\not{k}_\perp n_{+\nu}}{(n_+k)} (1-2\epsilon) \right]
$$

Results for power suppressed amplitude: soft $\times$ hard

$$
\begin{aligned}
iC_F\,\gamma_\perp^\rho\,\frac{1}{(n_+p)(n_-k)}\Bigg(&(n_+k)n_{-\nu}\left(\frac{2}{\epsilon^2}+\frac{1}{\epsilon}+5-\frac{1}{6}\pi^2+\mathcal{O}(\epsilon)\right)\\
&+(n_-k)n_{+\nu}\left(+\frac{2}{\epsilon}+3+\mathcal{O}(\epsilon)\right)\\
&+k_{\perp\nu}\left(\frac{2}{\epsilon^2}+\frac{3}{\epsilon}-\frac{\pi^2}{6}+8+\mathcal{O}(\epsilon)\right)\\
&+[\not{k}_\perp,\gamma_{\perp\nu}]\left(\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}-\frac{\pi^2}{12}+4+\mathcal{O}(\epsilon)\right)\Bigg)
\end{aligned}
$$

$$
i\,gt^b\;n_+^\rho C_F\left(-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}-8+\frac{\pi^2}{6}+\mathcal{O}(\epsilon)\right)
\frac{1}{(n_+p)(n_-k)}\left(\not{k}_\perp n_{-\nu}-(n_-k)\gamma_{\perp\nu}\right)
$$

NLP factorization formula

$$\frac{d\sigma_{\mathrm{DY}}}{dQ^2} = \frac{4\pi\alpha_{\mathrm{em}}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b\, f_{a/A}(x_a) f_{b/B}(x_b)\, \hat{\sigma}_{ab}(z)$$

The $\hat{\sigma}_{ab}(z)$ is now

$$\begin{aligned}
\hat{\sigma}(z) \quad &= \quad \sum_{\text{terms}} \int d\omega_i d\bar{\omega}_i d\omega_i' d\bar{\omega}_i'\, D(-\hat{s};\omega_i,\bar{\omega}_i) D^*(-\hat{s};\omega_i',\bar{\omega}_i') \\
&\quad \times Q^2 \int \frac{d^3\vec{q}}{(2\pi)^3\, 2\sqrt{Q^2+\vec{q}^{\,2}}}\, \frac{1}{2\pi} \int d^4x\, e^{i(x_a p_A + x_b p_B - q)\cdot x} \\
&\quad \times \widetilde{S}(x;\omega_i,\bar{\omega}_i,\omega_i',\bar{\omega}_i')
\end{aligned}$$

and

$$\begin{aligned}
D(-\hat{s};\omega_i,\bar{\omega}_i) \quad &= \quad \int d(n_+p_i) d(n_-\bar{p}_i)\, C(n_+p_i, n_-\bar{p}_i) \\
&\quad \times J(n_+p_i, x_a n_+ p_A;\omega_i)\, \bar{J}(n_-\bar{p}_i, -x_b n_- p_B;\bar{\omega}_i)
\end{aligned}$$

$J_{1,2}$ Collinear function

$$J_{1;\gamma\beta}\,(n_+p, x_a\,n_+p_A;\omega) \quad = \quad \delta_{\gamma\beta}\,\Big[J_{1,1}\,(x_a n_+p_A;\omega)\,\frac{\partial}{\partial(n_+p)}\delta(n_+p - x_a\,n_+p_A) + J_{1,2}\,(x_a n_+p_A;\omega)\,\delta(n_+p - x_a\,n_+p_A)\Big].$$

$$\Delta^{dyn\,(2)}_{\mathrm{NLP-hard}}(z) = 4\left(-H^{(1)}(Q^2) + \epsilon\,H^{(1)}(Q^2)\right)\int d\omega\,S_1^{(1)}\,(\Omega;\,\omega\,)$$

Factorization formula at NLP

Following such simplifications, and defining $\Delta = \hat{\sigma}/z$, we arrive at a final result:

$$\Delta^{dyn}_{\text{NLP}}(z) = -2\, Q \left[\left(\frac{\not{n}_-}{4} \right) \gamma_{\perp\rho} \left(\frac{\not{n}_+}{4} \right) \gamma^{\rho}_{\perp} \right]_{\beta\gamma} \int d(n_+p)\ C^{\,A0}\,(n_+p, x_b n_- p_B)$$

$$\times\, C^{*A0}\,(\,x_a\, n_+p_A,\, x_b n_- p_B) \sum_{i=1}^{5} \int \{d\omega_j\}\ J_i\,(n_+p, x_a\, n_+p_A; \{\omega_j\})\, S_i(\Omega; \{\omega_j\}) + \text{h.c.}$$

where the *generalised* soft functions have the structure:

$$\widetilde{S}_{\,i}\,(x; \{\omega_j\}) = \int \{dz_{j-}\}\ e^{-i\omega_j z_{j-}} \times \frac{1}{N_c} \text{Tr}\langle 0|\bar{\mathbf{T}} \left(\left[Y_+^\dagger\, Y_- \right](x) \right) \mathbf{T} \left(\left[Y_-^\dagger\, Y_+ \right](0)\, \mathfrak{s}_i\,(\,\{z_{j-}\}) \right) |0\rangle$$

with

$$\mathfrak{s}_i(\{z_{j-}\}) \quad \in \quad \left\{ \frac{i\partial_\perp^\mu}{in_-\partial} \mathcal{B}^+_{\mu\perp}(z_{1-}),\, \frac{1}{(in_-\partial)^2} \left[\mathcal{B}^{+\,\mu}{}_\perp(z_{1-}), \left[in_-\partial \mathcal{B}^+_{\mu\perp}(z_{1-}) \right] \right],\right.$$

$$\left. \frac{1}{(in_-\partial)} \left[\mathcal{B}^+_{\mu\perp}(z_{1-}), \mathcal{B}^+_{\nu\perp}(z_{1-}) \right],\, \frac{1}{(in_-\partial)} \mathcal{B}^+_{\mu\perp}(z_{1-}) \mathcal{B}^+_{\nu\perp}(z_{2-}),\, \frac{1}{(in_-\partial)^2} q_{+\sigma}(z_{1-}) \bar{q}_{+\lambda}(z_{2-}) \right\}$$

At NNLO accuracy there are three contributions:

- Collinear: 1-loop collinear and NLO soft functions
- Hard: 1-loop hard and NLO soft functions
- Soft: NNLO soft functions

Only one of the soft building blocks starts with a single gluon emission.

Factorization formula at NLP

Following such simplifications, and defining $\Delta = \hat{\sigma}/z$, we arrive at a final result:

$$\Delta^{dyn}_{\mathrm{NLP}}(z) = -2\, Q \left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\gamma_\perp^\rho\right]_{\beta\gamma}\int d(n_+p)\; C^{\,A0}\left(n_+p, x_b n_-p_B\right)$$

$$\times\, C^{*A0}\left(x_a\, n_+p_A,\, x_b n_-p_B\right)\sum_{i=1}^{5}\int \{d\omega_j\}\; J_i\left(n_+p, x_a\, n_+p_A; \{\omega_j\}\right) S_i(\Omega; \{\omega_j\}) + \text{h.c.}$$

$$\Delta^{dyn\,(2)}_{\mathrm{NLP-coll}}(z) \quad = \quad 4\,Q\int d\omega\; J^{(1)}_{1,2}\left(x_a\, n_+p_A; \omega\right) S^{(1)}_1(\Omega; \omega)$$

At NNLO accuracy there are three contributions:

- Collinear: 1-loop collinear and NLO soft functions
- Hard: 1-loop hard and NLO soft functions
- Soft: NNLO soft functions

Only one of the soft building blocks starts with a single gluon emission.

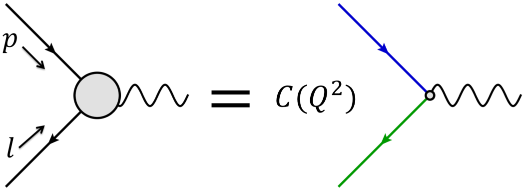
Leading power resummation

Resummation in the Drell-Yan proccess

Each of the objects in the factorization formula depends only on one physical scale.

$$\hat{\sigma}^{\,\mathrm{LP}}(z) = |C(Q^2)|^2 \; Q \; S_{\mathrm{DY}}(\Omega)$$

Hard function is the modulus square of the hard matching coefficient. Soft scale $\Omega = Q(1-z)$

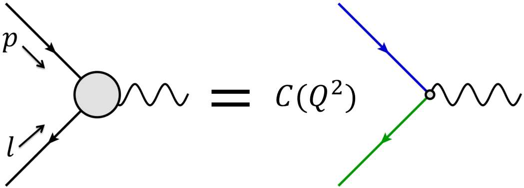


Resummation in the Drell-Yan proccess

Each of the objects in the factorization formula depends only on one physical scale.

$$\hat{\sigma}^{\,\mathrm{LP}}(z) = |C(Q^2)|^2 \; Q \; S_{\mathrm{DY}}(\Omega)$$

Hard function is the modulus square of the hard matching coefficient. Soft scale $\Omega = Q(1-z)$



$$C_V^{\mathrm{bare}}(\epsilon, Q^2) = 1 + \frac{\alpha_s(\mu)}{4\pi}\, C_F \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{\pi^2}{6} + \mathcal{O}(\epsilon)\right)\left(-\frac{\mu^2}{Q^2}\right)^{\epsilon} + \mathcal{O}(\alpha_s^2)$$

with $\overline{\mathrm{MS}}$ renormalized coupling. Absorb divergences into multiplicative Z factor

$$C_V(\epsilon, Q^2) = \lim_{\epsilon\to 0} Z^{-1}(\epsilon, Q^2, \mu)\, C_V^{\mathrm{bare}}(\epsilon, Q^2)$$

$$C_V(Q^2, \mu) = 1 + \frac{\alpha_s(\mu)}{4\pi}\, C_F \left(-\ln^2\left(-\frac{Q^2}{\mu^2}\right) + 3\ln\left(-\frac{Q^2}{\mu^2}\right) - 8 + \frac{\pi^2}{6}\right) + \mathcal{O}(\alpha_s^2)$$

Resummation in the Drell-Yan proccess

Each of the objects in the factorization formula depends only on one physical scale.

$$\hat{\sigma}^{\,\mathrm{LP}}(z) = |C(Q^2)|^2 \; Q \; S_{\mathrm{DY}}(\Omega)$$

Hard function is the modulus square of the hard matching coefficient. Soft scale $\Omega = Q(1-z)$

Hard matching coefficient satisfied renormalization group equation:

$$\frac{d}{d\ln\mu} C_V(Q^2,\mu) = \left[C_F \underbrace{\frac{\alpha_s(\mu)}{\pi}}_{\gamma_{\mathrm{cusp}}(\alpha_s)} \ln\left(-\frac{Q^2}{\mu^2}\right) + \underbrace{\frac{-6C_F\alpha_s(\mu)}{4\pi}}_{\gamma_V(\alpha_s)} \right] C_V(Q^2,\mu)$$

Solution of which is written as

$$C_V(Q^2,\mu) = U(\mu_h,\mu)\, C_V(Q^2,\mu_h)$$

$$C_V^{\mathrm{bare}}(\epsilon,Q^2) = 1 + \frac{\alpha_s(\mu)}{4\pi} C_F \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{\pi^2}{6} + \mathcal{O}(\epsilon)\right)\left(-\frac{\mu^2}{Q^2}\right)^{\epsilon} + \mathcal{O}(\alpha_s^2)$$

with $\overline{\mathrm{MS}}$ renormalized coupling. Absorb divergences into multiplicative Z factor

$$C_V(\epsilon,Q^2) = \lim_{\epsilon\to 0} Z^{-1}(\epsilon,Q^2,\mu)\, C_V^{\mathrm{bare}}(\epsilon,Q^2)$$

$$C_V(Q^2,\mu) = 1 + \frac{\alpha_s(\mu)}{4\pi} C_F \left(-\ln^2\left(-\frac{Q^2}{\mu^2}\right) + 3\ln\left(-\frac{Q^2}{\mu^2}\right) - 8 + \frac{\pi^2}{6}\right) + \mathcal{O}(\alpha_s^2)$$

Resummation in the Drell-Yan proccess

Each of the objects in the factorization formula depends only on one physical scale.

$$\hat{\sigma}^{\,\mathrm{LP}}(z) = |C(Q^2)|^2 \; Q \; S_{\mathrm{DY}}(\Omega)$$

Hard function is the modulus square of the hard matching coefficient. Soft scale $\Omega = Q(1-z)$

Hard matching coefficient satisfied renormalization group equation:

$$\frac{d}{d\ln\mu} C_V(Q^2,\mu) = \left[C_F \; \underbrace{\frac{\alpha_s(\mu)}{\pi}}_{\gamma_{\mathrm{cusp}}(\alpha_s)} \; \ln\left(-\frac{Q^2}{\mu^2}\right) + \underbrace{\frac{-6C_F\alpha_s(\mu)}{4\pi}}_{\gamma_V(\alpha_s)} \right] C_V(Q^2,\mu)$$

Solution of which is written as

$$C_V(Q^2,\mu) = U(\mu_h,\mu)\, C_V(Q^2,\mu_h)$$

$$U(\mu_h,\mu) = \exp\left[2\, C_F S(\mu_h,\mu) - A_{\gamma_V}(\mu_h,\mu)\right] \left(-\frac{Q^2}{\mu_h^2}\right)^{-C_F\, A_{\gamma_{\mathrm{cusp}}}(\mu_h,\mu)}$$

where

$$S(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\gamma_{\mathrm{cusp}}}{\beta(\alpha)} \int_{\alpha_s(\nu)}^{\alpha} \frac{d\alpha'}{\beta(\alpha')} \qquad\qquad A_{\gamma_i}(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\gamma_i(\alpha)}{\beta(\alpha)}$$

$$\frac{d\alpha_s}{\beta} = d\ln\mu$$

Resummation in the Drell-Yan proccess

Each of the objects in the factorization formula depends only on one scale.

$$\hat{\sigma}^{\,\mathrm{LP}}(z) = |C(Q^2)|^2 \; Q \; S_{\mathrm{DY}}(Q(1-z))$$

Hard function is the modulus square of the hard matching coefficient.

This is an example of resummation performed using renormalization group techniques.

Resummation in the Drell-Yan proccess

Each of the objects in the factorization formula depends only on one scale.

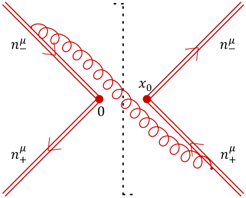
$$\hat{\sigma}^{\,\mathrm{LP}}(z) = |C(Q^2)|^2 \; Q \; S_{\mathrm{DY}}(Q(1-z))$$

Hard function is the modulus square of the hard matching coefficient.

This is an example of resummation performed using renormalization group techniques.

Soft function would be calculated in the same way. In momentum space renormalization is a convolution with the Z factor. → we will discuss this in the context of NLP in detail.

$$S_{\mathrm{DY}}(\Omega) = \int \frac{dx^0}{4\pi}\, e^{ix^0\Omega/2}\, \frac{1}{N_c}\, \mathrm{Tr}\, \langle 0|\bar{\mathbf{T}}(Y_+^\dagger(x^0)Y_-(x^0))\, \mathbf{T}(Y_-^\dagger(0)Y_+(0))|0\rangle$$



$$S_{\mathrm{DY}}(\Omega) \quad = \quad \delta(\Omega) + \frac{\alpha_s\, C_F}{\pi}\, \frac{1}{\Omega} \left(\frac{\mu}{\Omega}\right)^{2\epsilon} \frac{\Gamma[1-\epsilon]}{\epsilon^2\, \Gamma[-2\epsilon]} e^{\epsilon\gamma_E}$$

Resummation in the Drell-Yan proccess

Each of the objects in the factorization formula depends only on one scale.

$$\hat{\sigma}^{\,\mathrm{LP}}(z) = |C(Q^2)|^2 \; Q \; S_{\mathrm{DY}}(Q(1-z))$$

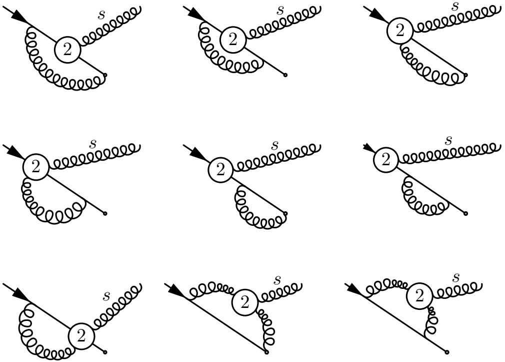
Hard function is the modulus square of the hard matching coefficient.

This is an example of resummation performed using renormalization group techniques.

Soft function would be calculated in the same way. In momentum space renormalization is a convolution with the Z factor. $\rightarrow$ we will discuss this in the context of NLP in detail.

Originally in [G. P. Korchemsky, G. Marchesini, 1993] and for details in SCET can see [T. Becher, M. Neubert, G.Xu, 0710.0680].

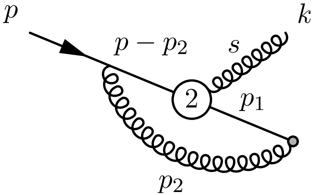
One-loop collinear function calculation



$$\langle g(k)_K|\mathcal{J}^{1g}_{\gamma f}(0)|q(p_A)_q\rangle \quad = \int dt\, dn_+p_1\, e^{itn_+p_1} \int \frac{dn_+p_a}{2\pi} du\, e^{i\,n_+p_a\,u} \int \frac{d\omega}{2\pi}\, dz_-\, e^{-i\,\omega\,z_-}$$

$$\times \int \frac{dn_+p}{2\pi}\, e^{-i\,n_+p\,t}\, J^A_{1;\gamma\beta,fb}\,(n_+p, n_+p_a;\omega)\, \langle 0|\chi^{\rm PDF}_{c,\beta b}\,(un_+)|q(p_A)_q\rangle\, \langle g(k)_K|\mathfrak{s}_{1;A}(z_-)\,|0\rangle$$

One-loop collinear function calculation



$$
\begin{aligned}
\langle g(k)^K | \mathcal{T}^{1g}_{\gamma f}(n_+q) | q(p)_e \rangle_{\text{fig}} \quad &= \quad 2\pi \, \frac{g_s \alpha_s}{4\pi} \left(C_F - \frac{1}{2} C_A \right) \frac{\mathbf{T}^K_{fe}}{(n_+p)} \left[\frac{(n_+p)(n_-k)}{\mu^2} \right]^{-\epsilon} \\
& \times \Bigg\{ \delta(n_+q - n_+p) \Bigg[2\,\delta_{\gamma\beta} \left(\frac{(n_+k)}{(n_-k)} n_-^\nu - n_+^\nu \right) \\
& + \delta_{\gamma\beta} \left(\frac{k_\perp^2 \, n_-^\nu}{(n_-k)^2} - \frac{k_\perp^\nu}{(n_-k)} \right) \left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon} + 2 + \frac{\pi^2}{6} \right) + \frac{[\gamma_\perp^\nu , \not{k}_\perp]_{\gamma\beta}}{(n_-k)} \left(-\frac{1}{\epsilon^2} + \frac{\pi^2}{12} \right) \Bigg] \\
& + (n_+p) \frac{\partial}{\partial n_+q} \, \delta(n_+q - n_+p) \, \delta_{\gamma\beta} \left(\frac{(n_+k)}{(n_-k)} n_-^\nu - n_+^\nu \right) \\
& \times \left(-\frac{2}{\epsilon^2} - \frac{2}{\epsilon} - 4 + \frac{\pi^2}{6} \right) \Bigg\} u_{c,\beta}(p) \epsilon^*_\nu(k) \; + \mathcal{O}(\epsilon)
\end{aligned}
$$

Endpoint divergent convolutions

Resummations at next-to-leading power in SCET

Leading logarithms:

Subleading power resummed thrust spectrum for $H \to gg$

[I. Moult, I. Stewart, G. Vita, H. Zhu, 1804.04665]

Drell-Yan production at threshold

[M. Beneke, A.Broggio, M. Garny, **SJ**, R. Szafron, L. Vernazza, J.Wang, 1809.10631]

Higgs production via gluon fusion at threshold

[M. Beneke, M. Garny, **SJ**, R. Szafron, L. Vernazza, J.Wang, 1910.12685]

Subleading power resummation of rapidity logarithms: the energy-energy correlator in N=4 SYM

[I. Moult, G. Vita, K. Yan, 1912.02188]

Next-to-leading logarithms :

Factorization at Subleading Power and Endpoint Divergences in $h \to \gamma\gamma$ Decay: II. Renormalization and Scale Evolution

[Z. L. Liu, B. Mecaj, M.Neubert, X.Wang, 2009.06779]

Consistency relations

- We know that an observable must be a finite quantity.
- Imposing the constraint allows us to infer structure of partonic objects.

Consistency relations

- We know that an observable must be a finite quantity.
- Imposing the constraint allows us to infer structure of partonic objects.

The hadronic tensor is given by

$$W = \sum_i W_{\phi,i} \, f_i \, ,$$

related to their finite counterparts through

$$\tilde{f}_k = Z_{ki} f_i , \qquad W_{\phi,i} = \tilde{C}_{\phi,k} Z_{ki} \, ,$$

such that

$$W_{\phi,i} f_i = \tilde{C}_{\phi,k} \tilde{f}_k \, .$$

The splitting kernels are given by

$$P_{ij} = -\gamma_{ij} = \frac{dZ_{ik}}{d \ln \mu} (Z^{-1})_{kj} \, .$$

Consistency relations

- We know that an observable must be a finite quantity.
- Imposing the constraint allows us to infer structure of partonic objects.

Focusing on the quark initiated NLP contribution

$$\sum_i (W_{\phi,i} f_i)^{NLP} = \left(W_{\phi,q}^{NLP} U_{qq}^{LP} + W_{\phi,g}^{LP} U_{gq}^{NLP} \right) f_q(\Lambda)$$

where U_{ij} are the evolution factors

$$f_i(\mu) = U_{ij}(\mu) f_j(\Lambda)$$

The general expansion for the cross section is

$$\sum_i (W_{\phi,i} f_i)^{NLP} = f_q(\Lambda) \times \frac{1}{N} \sum_{n=1} \left(\frac{\alpha_s}{4\pi} \right)^n \frac{1}{\epsilon^{2n-1}} \sum_{k=0}^{n} \sum_{j=0}^{n} c_{kj}^{(n)}(\epsilon) \left(\frac{\mu^{2n} N^j}{Q^{2k} \Lambda^{2(n-k)}} \right)^{\epsilon}$$

The scaling of the regions: hard (Q^2), anti-hardcollinear (Q^2/N), collinear (Λ^2), softcollinear (Λ^2/N)

Consistency relations

- We know that an observable must be a finite quantity.
- Imposing the constraint allows us to infer structure of partonic objects.

Invoking pole cancellation, the consistency relations allow us to determine all $(n+1)^2$ coefficients $c_{kj}^{(n)}$ in terms of three unknowns at every order n.
We then need "initial conditions":

$$c_{n0}^{(n)} = 0 \qquad , c_{00}^{(n)} = 0 \qquad \text{for all } n\,.$$

and the third initial condition is taken from the conjectured exponentiation of the momentum distribution function which gives the series of terms $c_{n1}^{(n)}$.

$$\sum_i (W_{\phi,i} f_i)^{NLP} = f_q(\Lambda) \times \frac{1}{N} \sum_{n=1} \left(\frac{\alpha_s}{4\pi}\right)^n \frac{1}{\epsilon^{2n-1}} \sum_{k=0}^{n} \sum_{j=0}^{n} c_{kj}^{(n)}(\epsilon) \left(\frac{\mu^{2n} N^j}{Q^{2k}\Lambda^{2(n-k)}}\right)^{\epsilon}$$

The scaling of the regions: hard (Q^2), anti-hardcollinear (Q^2/N), collinear (Λ^2), softcollinear (Λ^2/N)

Consistency relations

- We know that an observable must be a finite quantity.
- Imposing the constraint allows us to infer structure of partonic objects.

Invoking pole cancellation, the consistency relations allow us to determine all $(n+1)^2$ coefficients $c_{kj}^{(n)}$ in terms of three unknowns at every order n.
We then need "initial conditions":

$$c_{n0}^{(n)} = 0 \qquad , c_{00}^{(n)} = 0 \qquad \text{for all } n\,.$$

and the third initial condition is taken from the conjectured exponentiation of the momentum distribution function which gives the series of terms $c_{n1}^{(n)}$.

These considerations lead us to a solution for $W_{\phi,q}^{NLP,LL}$ which is in agreement with [A. Vogt, 1005.1606] and we obtain the same splitting kernels.

Thank you