$B \to X_u \ell \nu$ and V_{ub} determinations: challenges and theoretical framework(s)

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Outline

Motivation: Standard Model parameters

Standard Model (*input*) parameters

The **Standard Model** (SM) Lagrangian

$$\begin{aligned}\mathcal{L}_{\rm SM} = i &\left(\overline{\ell_L}\not{D}\ell_L + \overline{e_R}\not{D}e_R + \overline{Q_L}\not{D}Q_L + \overline{u_R}\not{D}u_R + \overline{d_R}\not{D}d_R\right) \\ &- \frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &+ (D_\mu\phi)^\dagger(D^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda\left(\phi^\dagger\phi\right)^2 \\ &- \left(\overline{\ell_L}Y^\ell\phi e_R + \overline{Q_L}Y^u\epsilon\phi^* u_R + \overline{Q_L}Y^d\phi d_R + \text{h.c.}\right),\end{aligned}$$

with

$$(D_\mu\psi)^{\alpha j} = \left[\delta_{\alpha\beta}\delta_{jk}\left(\partial_\mu + ig' Y_q B_\mu\right) + ig\delta_{\alpha\beta}S^I_{jk}W^I_\mu + ig_s\delta_{jk}T^A_{\alpha\beta}G^A_\mu\right]\psi^{\beta k},$$

depends on a (reduced) set of fundamental (*input*) parameters:

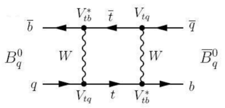
- ⇒ Three *gauge couplings*: g', g and g_s.
- ⇒ *Higgs vev* and *Higgs mass*: $v = \sqrt{-\mu^2/\lambda}$ and $m_H = \sqrt{2\lambda}\, v$.
- ⇒ Three *lepton masses*: $m_i^\ell = \lambda_i^\ell(v/\sqrt{2})$ with $Y^\ell_{ij} = \lambda^\ell_i\,\delta_{ij}$.
- ⇒ Six *quark masses*: $m_i^q = V_L^q Y^q V_R^{q\dagger}(v/\sqrt{2})$ $(q = u, d)$ with $V^{u,d}_{L,R}$ unitary matrices.
- ⇒ Four *CKM matrix elements*: $V_{\rm CKM} = V_L^u V_L^{d\dagger}$.
- ⇒ + one *strong CP angle*: $\Delta\mathcal{L} \sim \bar\theta\, G^A_{\mu\nu}\tilde{G}^{A\mu\nu}$.
- ⇒ + three *neutrino masses* + four (or five) *PMNS matrix elements*.

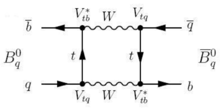
The importance of the CKM matrix

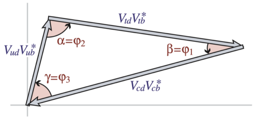
- CKM matrix parametrisation $\Rightarrow$ $3\Re + 1\Im$ parameters:

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

- The CKM matrix sets the strength of quark-level transitions $J_L \sim V_{ij} W^+_\mu \bar{q}_i \gamma^\mu (1-\gamma_5) q_i$.
  - $\Rightarrow$ Hadronic lifetimes.
  - $\Rightarrow$ Branching ratios.
- Imaginary phase of the CKM matrix $\Rightarrow$ only CP-violation (CPV) source in the SM.
  - $\Rightarrow$ CKM triangles.
  - $\Rightarrow$ CP-violation observables (including possible CPV New Physics): $K \to \pi\pi$, $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$, $B^0_s - \bar{B}^0_s$, etc.







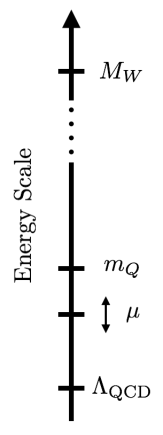
The structure of the CKM matrix: the *Flavour Puzzle*

[Fuentes-Martín]

- 13 parameters characterise the Yukawa sector of the $\mathcal{L}_{\mathrm{SM}}$: 3 lepton masses, 6 quark masses, 4 CKM elements.
- Strong hierarchical structure between quark masses and CKM elements.
  - ⇒ Why this hierarchy? SM does **not** provide an answer.
  - ⇒ *Flavour Puzzle*.
- Satisfactory NP must provide a mechanism:
  - ⇒ Hierarchy dynamically generated through local interactions (w/ symmetry breaking).
  - ⇒ Quark generations as excitations of more fundamental constituents.
- First, we need **precise determinations** of V_{CKM}.

Brief review of HQET

Effective Field Theories



- Heavy modes $E_H \sim M$ cannot be resolved at low energies $E \sim \Lambda$ $(M > \Lambda)$. We can *integrate them out* of the theory:

$$\mathcal{Z} = \int D\phi_L D\phi_H \mathrm{e}^{iS(\phi_L,\phi_H)+i\int d^Dx\, J_L(x)\phi_L(x)}$$

$$\Rightarrow \mathcal{Z}_\Lambda = \int D\phi_L \mathrm{e}^{iS_\Lambda(\phi_L)+i\int d^Dx\, J_L(x)\phi_L(x)}.$$

- The *effective action* S_Λ is *nonlocal.*
- *Local effective action* from an operator product expansion (OPE) of S_Λ with $1/M$ expansion parameter:

$$\mathcal{L}_{\mathrm{eff}} \sim \frac{1}{M}\sum_i \mathcal{C}_i(\mu)\,\mathcal{O}_i + O\left(\frac{1}{M^2}\right).$$

- Short-distance dynamics incorporated by *matching* the full theory onto the effective theory at a high scale μ_0

$$\mathcal{A}(M_1 \to M_2) = \langle M_2|\,\mathcal{L}_{\mathrm{full}}\,|M_1\rangle$$

$$= \frac{1}{M}\sum_i \mathcal{C}_i(\mu_0)\,\langle M_2|\,\mathcal{O}_i\,|M_1\rangle\,(\mu_0).$$

- Use *renormalisation group techniques* to run down the value of the *effective coefficients* from μ_0 to Λ.

Heavy Quark Effective Theory

"The heavy quark effective theory (HQET) is constructed to provide a simplified description of processes where a heavy quark interacts with light degrees of freedom by the exchange of soft gluons." [Neubert]

$\Rightarrow$ m_Q sets the high-scale.

$\Rightarrow$ Λ_{QCD} scale of hadronic physics we want to describe.

- *Soft dominance*: a heavy quark inside a hadron moves with nearly the hadron's velocity v and is almost on-shell.
  - $\Rightarrow$ $p_Q = m_Q v + k$, with $|k| \lll |m_Q v|$.
  - $\Rightarrow$ Heavy quark interactions with light degrees of freedom change its momentum by $\Delta k \sim \Lambda_{\text{QCD}} \Rightarrow \Delta v \sim \Lambda_{\text{QCD}}/m_Q \to 0$.
- HQET describes properties of heavy hadrons.
  - $\Rightarrow$ Heavy quark fields cannot be fully integrated out.
  - $\Rightarrow$ Only the "small components" of the heavy quark fields are removed.

The HQET Lagrangian

- Let $Q(x)$ be a heavy quark field. We project out its "large component" and "small component" fields:

$$h_v(x) = \mathrm{e}^{im_Q v\cdot x} P_+ Q(x),$$
$$H_v(x) = \mathrm{e}^{im_Q v\cdot x} P_- Q(x),$$

with the projectors $P_\pm = (1 \pm \not{v})/2$.

- The HQET Lagrangian takes the form of an OPE in $1/m_Q$:

$$\mathcal{L}_{\mathrm{HQET}} = \bar{h}_v\, iv\cdot D_s\, h_v + \frac{\mathcal{C}_{\mathrm{kin}}}{2m_Q}\bar{h}_v\,(iD_{s\perp})^2\, h_v$$
$$+ \frac{\mathcal{C}_{\mathrm{mag}}\, g_s}{4m_Q}\bar{h}_v \sigma_{\mu\nu}\, G_s^{\mu\nu}\, h_v + O(1/m_Q^2),$$

with $D_\perp^\mu = D^\mu - v^\mu(v\cdot D)$.

⇒ Leading term $\mathcal{L}_{\mathrm{HQET}}^{m_Q\to\infty} = \bar{h}_v\, iv\cdot D_s\, h_v$ invariant under $SU(2N_H)$ (N_H number of heavy flavours).

⇒ *Heavy quark symmetry*: form factor constraints, Isgur-Wise functions, relations between hadron masses, etc.

⇒ Wilson coefficients $\mathcal{C}_{\mathrm{kin}} = \mathcal{C}_{\mathrm{mag}} = 1 + O(\alpha_s)$.

Power corrections

- *Power corrections* of $O(1/m_b)$ parametrised in terms of the operators:

$$\mathcal{O}_{\text{kin}} = -\bar{h}_v\,(iD_{s\perp})^2\,h_v, \quad \mathcal{O}_{\text{mag}} = \frac{g_s}{2}\bar{h}_v\sigma_{\mu\nu}G_s^{\mu\nu}\,h_v.$$

  ⇒ $\mathcal{O}_{\text{kin}}$ kinetic energy of the heavy quark inside the hadron (*Fermi motion*).

  ⇒ $\mathcal{O}_{\text{mag}}$ chromomagnetic interaction of the heavy quark spin with the gluon field.

- Forward B-meson matrix elements:

$$\lambda_1 = \frac{1}{2m_B}\langle\bar{B}(v)|\mathcal{O}_{\text{kin}}|\bar{B}(v)\rangle, \quad \lambda_2 = -\frac{1}{6m_B}\langle\bar{B}(v)|\mathcal{O}_{\text{mag}}|\bar{B}(v)\rangle,$$

  ⇒ QCD corrections: $\lambda_2 = \lambda_2(\mu)$, such that $\mathcal{C}_{\text{mag}}(\mu)\lambda_2(\mu)$ is scale independent.

  ⇒ Instead, λ_1 is protected by *reparametrisation invariance* and, hence, scale-independent: $\mathcal{C}_{\text{kin}}(\mu) = 1$ to all orders.

  ⇒ $|\bar{B}(v)\rangle$ in $\lambda_{1,2}$ eigenstates of $\mathcal{L}_{\text{HQET}}^{m_Q\to\infty}$. In terms of QCD states:

$$\mu_\pi^2 = -\lambda_1 + O(1/m_b), \quad \mu_G^2 = 3\lambda_2 + O(1/m_b).$$

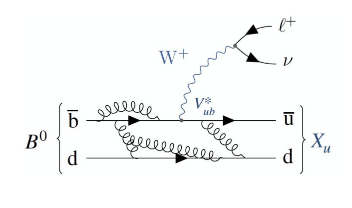
  ⇒ $\lambda_{1,2}$ (or $\mu^2_{\pi,G}$) used in hadron spectroscopy and to parameterise inclusive decays.

- Connection between QCD and HQET fields at $O(1/m_b)$ but LO in α_s:

$$Q(x) = \mathrm{e}^{-im_Q v\cdot x}\left(1 + i\frac{\not{D}_\perp}{2m_Q} + \dots\right)h_v(x).$$

Theoretical framework for inclusive semileptonic B-decays

Differential Decay Distribution and the Hadronic Tensor



- Decay distribution ($\bar{B} \to X_c \ell \nu$):

$$\frac{d\Gamma}{d\hat{q}^2\, d\hat{E}_\ell\, d\hat{E}_\nu} \sim \sum_{X_c} \sum_{\text{pols.}} \frac{|\langle X_c \ell \nu | \mathcal{H}_{\text{eff}} | B \rangle|^2}{2 m_B} \delta^4(\hat{p}_B - \hat{p}_{X_c} - \hat{q})$$

$$= \frac{G_F^2\, |V_{ub}|^2}{8\pi^3} L_{\mu\nu} W^{\mu\nu}.$$

$\Rightarrow$ Effective Hamiltonian $\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} J_L^{\mu\dagger} J_{L\mu}$

with $J_L^{\mu\dagger} = \bar{c}\gamma^\mu P_L b$.

$\Rightarrow$ *quark-hadron duality*: sum over all X_c hadronic states eliminates individual hadronic bound-state effects.

- Hadronic Tensor:

$$W^{\mu\nu}(\hat{p}_B, \hat{q}) \sim \sum_{X_c} (2\pi)^3 \delta^4(\hat{p}_B - \hat{p}_{X_c} - \hat{q}) \frac{1}{2m_B} \langle \bar{B}(p_B)|\, J_L^{\dagger\mu}\, |X_c(p_{X_c})\rangle \langle X_c(p_{X_c})|\, J_L^\nu\, |\bar{B}(p_B)\rangle$$

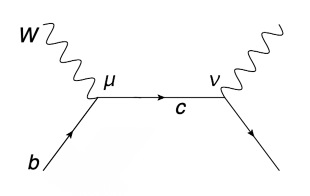
$\Rightarrow$ Decomposition in terms of "*form factors*" $W_i(\hat{q}_0, \hat{q}^2)$ (with $q = p_\ell + p_\nu$):

$$m_b\, W^{\mu\nu}(\hat{p}_B, \hat{q}) = -W_1\, g^{\mu\nu} + W_2\, v^\mu v^\nu + i W_3\, \epsilon^{\mu\nu\rho\sigma} v_\rho \hat{q}_\sigma + \ldots$$

$\Rightarrow$ *Optical Theorem*: $W^{\mu\nu}$ from B-meson forward scattering amplitude:

$$W^{\mu\nu} = 2\,\text{Im}\, T^{\mu\nu}, \qquad T^{\mu\nu} = \frac{i}{\pi m_B} \int d^4x\, \mathrm{e}^{-iq\cdot x} \langle \bar{B}(p_B)|\, T\left\{ J_L^{\mu\dagger}(x) J_L^\nu(0) \right\} |\bar{B}(p_B)\rangle\,.$$

The Local OPE



- Partonic dynamics: $b(p) \to c(p')\ell(p_\ell)\nu(p_\nu)$ with $p = m_b v + k$ and $k \sim \Lambda_{\text{QCD}}$.

  $\Rightarrow$ Decay variables:

$$\rho = \frac{m_c^2}{m_b^2}, \quad \hat{u} = \frac{(p-q)^2 - m_c^2}{m_b^2}, \quad \hat{q}^2 = \frac{q^2}{m_b^2}.$$

  $\Rightarrow$ $\hat{u}$ replaces $\hat{E}_\nu$ in the decay distribution.

  $\Rightarrow$ At fixed $\hat{q}^2$ and $\hat{E}_\ell$, integration over $\hat{u}$ averages over hadronic masses M_{X_c}.

- $T^{\mu\nu}$ can be expanded in an OPE:

$$\int d^4x\, \mathrm{e}^{-iq\cdot x}\, T\left\{J_L^{\mu\dagger}(x) J_L^\nu(0)\right\} \sim \frac{1}{(m_b v + k - q)^2 - m_c^2 + i\varepsilon} \bar{b}_v \gamma^\mu P_L (m_b \not{v} + \not{k} - \not{q}) \gamma^\nu P_L b_v$$

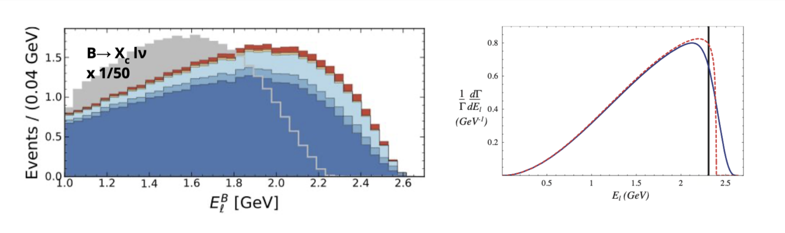
$$\overset{k\sim 0}{\sim} \frac{1}{\Delta_0} \sum_i a_i \mathcal{O}_i^{\text{HQET}}, \text{ with } \Delta_0 = (m_b v + k - q)^2 - m_c^2 + i\varepsilon \sim \hat{u} m_b^2.$$

  $\Rightarrow$ Form factors as a double series in α_s and $1/m_b$.

$$W_i = W_i^{(0)} + \frac{\mu_\pi^2}{2m_b^2} W_i^{(\pi,0)} + \frac{\mu_G^2}{2m_b^2} W_i^{(G,0)}$$
$$+ \frac{\alpha_s}{\pi}\left[C_F W_i^{(1)} + C_F \frac{\mu_\pi^2}{2m_b^2} W_i^{(\pi,1)} + \frac{\mu_G^2}{2m_b^2} W_i^{(G,1)}\right] + \ldots$$

Inclusive V_{ub} determinations: challenges and the GGOU/NNVub approach

Experimental backgrounds and phase space cuts



[Lu Cao at ICHEP 2020; Luke]

- $B \to X_c \ell \bar{\nu}$ very CKM favoured w.r.t. $B \to X_u \ell \bar{\nu}$ ($|V_{cb}/V_{ub}| \sim 10$).
  - $\Rightarrow$ Large charm backgrounds.
  - $\Rightarrow$ $B \to X_u \ell \bar{\nu}$ signal difficult to measure.
  - $\Rightarrow$ Need to impose kinematic cuts:

$$E_\ell \sim \frac{m_B^2 - m_D^2}{2m_B} \sim E_\ell^{\max} \sim \frac{m_b}{2} \quad \text{and} \quad m_X^2 \sim \hat{u} m_b^2 + m_c^2 \sim m_D^2.$$

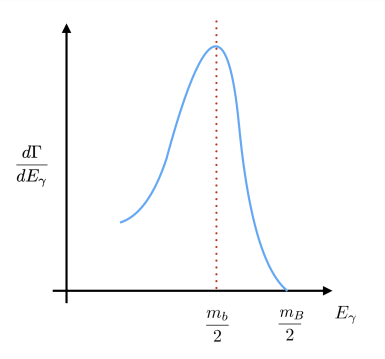
- Convergence of the local OPE is destroyed within the region allowed by the kinematic cuts.
  - $\Rightarrow$ $(m_b v + k - q)^2 \nsim (m_b v - q)^2 + O(\Lambda_{\mathrm{QCD}}/m_b) + O(\Lambda_{\mathrm{QCD}}^2)$ since $(m_b v - q)^2 \sim 2k \cdot (m_b v - q)$.
  - $\Rightarrow$ OPE coefficient $\Delta_0 \sim 0$.

[Neubert; Luke]

Shape function(s): $\bar{B} \to X_s\gamma$

- The residual $\sim \Lambda_{\rm QCD}$ momentum of the b-quark in the B-meson cannot be encoded into the non-perturbative matrix elements of the OPE. Needs to be resumed into a non-perturbative *Shape Function*.

[Neubert; Bigi, Shifman, Uraltsev, Vainshtein]



- Partonic decay (tree level): $b(p) \to s(p')\gamma(q)$ with $p = m_b v$.
  - $\Rightarrow$ Infinitely narrow photon line at $E_\gamma^{(0)} = \frac{m_b}{2}$.
- Hadronic level: $\bar{B}(p_B) \to X_s(p_{X_s})\gamma(q)$.
  - $\Rightarrow$ Hadronic kinematic boundary at $E_\gamma^{\rm max} = \frac{m_B}{2}$.
  - $\Rightarrow$ Partonic vs hadronic dynamics: $E_\gamma^{\rm max} - E_\gamma^{(0)} = \frac{m_B - m_b}{2} \sim \frac{\Lambda_{\rm QCD}}{2}$.
  - $\Rightarrow$ Partonic dynamics: $b(p) \to s(p')\gamma(q)$ with $p = m_b v + k$ and $k \sim \Lambda_{\rm QCD}$.

- Decay distribution $d\Gamma/dE_\gamma$ is smeared due to purely non-perturbative effects:

$$\frac{d\Gamma}{dE_\gamma} = \int dk_+ \, F(k_+) \frac{d\Gamma^{\rm pert}}{dE_\gamma}\left(E_\gamma - \frac{k_+}{2}\right).$$

[Bigi, Shifman, Uraltsev, Vainshtein]

Shape function(s) in $\bar{B} \to X_u \ell \nu$

- *Factorisation formula* for the W_i structure functions:

$$W_i(q_0, q^2) = \int dk_+ F(k_+)\, W_i^{pert}\left[q_0 - \frac{k_+}{2}\left(1 - \frac{q^2}{m_b M_B}\right), q^2\right] + O(1/m_b).$$

  ⇒ At LO the Shape Function (SF) $F(k_+)$ is *universal*, i.e. shared by inclusive radiative and semileptonic decays.

  ⇒ The SF is the parton distribution function for the b quark in the B meson.

  ⇒ At NLO, non-universal subleading SFs emerge.

  ⇒ SFs modelling is one of the dominant uncertainties in $|V_{ub}|$ determinations.

- Different approaches for the estimation of the shape functions.

  ⇒ OPE constrains on the SF moments + parametrisation with / without resummation (**GGOU**, **BLNP**).

  ⇒ Theory prediction based on resummed pQCD (**DGE**, **ADFR**).

  ⇒ Global fit radiative + $b \to u \ell \bar{\nu}$ (**SIMBA**).

GGOU Framework

- Subleading $O(1/m_b)$ corrections absorbed into non-universal q^2-dependent SFs:

$$W_i(q_0, q^2) = \int dk_+ F_i(k_+, q^2)\, W_i^{pert}\left[q_0 - \frac{k_+}{2}\left(1 - \frac{q^2}{m_b M_B}\right), q^2\right].$$

$\Rightarrow$ $W_i(q_0, q^2)$ up to $O(\beta_0 \alpha_s^2)$ and $O(1/m_b^3)$ ($\mu_{\pi,G}^2$ and $\mu_{D,LS}^3$ from $B \to X_c \ell \nu$).

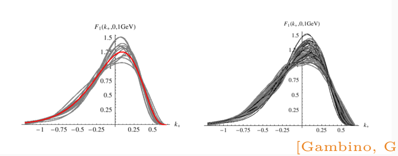
$\Rightarrow$ Matching consistency implies W_i up to $O(1/m_b^3)$ and W_i^{pert} at tree-level in the convolution formula.

- SFs can be constrained by matching with the $\hat{q}_0$-moments of the OPE for the structure functions:

$$\int dk_+ k_+^n F_i(k_+, q^2) = \left(\frac{2}{\Delta}\right)^n \left[\delta_{n0} + \frac{J_i^{(n,0)}}{I_i^{(0,0)}}\right]$$

$\Rightarrow$ $I_i^{(n,0)}$ and $J_i^{(n,0)}$ are the nth central $\hat{q}_0$-moments of W_i^{tree} and W_i^{tree} (up to $O(1/m_b^3)$), respectively.

$\Rightarrow$ Different parametric families for $F_i(k_+, q^2)$ are used to estimate the theoretical errors.



[Gambino, Giordano, Ossola, Uraltsev]

Tensions in V_{ub}: exclusive vs inclusive

- Inclusive ($\bar{B} \to X_u \ell \nu$):

$$\Rightarrow |V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(\bar{B} \to X_u \ell \nu)_{\text{cuts}}}{\tau_B \Delta\Gamma(\bar{B} \to X_u \ell \nu)_{\text{cuts}}}}.$$

$$|V_{ub}^{\text{incl.}}| = \left(4.32 \pm 0.12^{+0.12}_{-0.13}\right) \times 10^{-3}.$$

[HFLAV 2019]

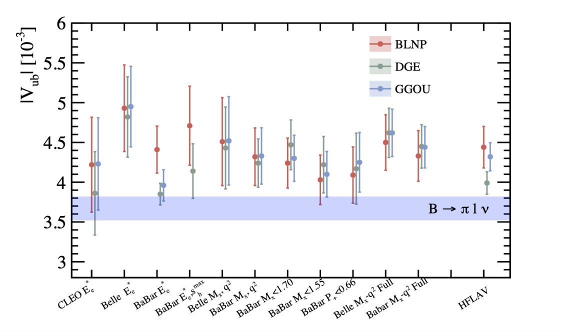
- Exclusive ($B \to \pi \ell^+ \nu$):

$$\Rightarrow \frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |p_\pi|^3 |f_+(q^2)|^2.$$

$\Rightarrow$ Form factor $f_+(q^2)$ from Lattice and / or LCSRs.

$$|V_{ub}^{\text{excl.}}| = (3.67 \pm 0.09 \pm 0.12) \times 10^{-3}.$$

[HFLAV 2019]



[Gambino, Kronfeld, Rotondo, Schwanda, Bernlochner, et al]

$\Rightarrow$ Tensions (inclusive vs exclusive) $\sim 1 - 3\sigma$ (depending on the theo. and exp. approaches).

NNVub Framework: GGOU + Neural Networks (update)

- In GGOU $W_i(q_0, q^2)$ known through $O(\beta_0 \alpha_s^2)$ and $O(1/m_b^3)$.

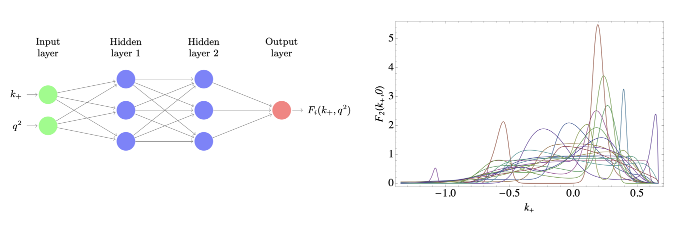
  $\Rightarrow$ + $O(\alpha_s \Lambda^2_{\rm QCD}/m_b^2)$ corrections.

  $\Rightarrow$ + $O(\alpha_s^2)$ corrections.

- Knowing the $O(\alpha_s \Lambda^2_{\rm QCD}/m_b^2)$ corrections to $W_i(q_0, q^2)$ allows to constrain the SFs moments up to $O(\alpha_s)$.

$$\int dk_+ k_+^n F_i(k_+, q^2) = \left(\frac{2}{\Delta}\right)^n \left[\delta_{n0} + \frac{J_i^{(n,0)}}{I_i^{(0,0)}} + O(\alpha_s)\right]$$

- Employ artificial *Neural Networks* as unbiased interpolants for the SFs, instead of relying on different particular parametrisations.



[Gambino, Healey, Mondino]

Perturbative power suppressed effects in $\bar{B} \to X_u \ell \nu$

$W_i^{(1)}$, $W_i^{(\pi,1)}$ and $W_i^{(G,1)}$ with massive charm

- Explicit calculation at $O(\alpha_s)$ available for $\bar{B} \to X_c \ell \bar{\nu}$.

  $\Rightarrow$ Schematic structure:

$$W_i^{(1)} = w_i^{(0)} \left\{ S_i\, \delta(\hat{u}) - 2\,(1 - E_0 I_1) \left[\frac{1}{\hat{u}}\right]_+ + \frac{\theta(\hat{u})}{(\rho + \hat{u})} \right\} + R_i\, \theta(\hat{u}),$$

$$R_i = \frac{r_i^{(1)}\, \hat{u} + r_i^{(2)} \rho}{(\hat{u} + \rho)^2} + \frac{s_i}{\hat{u} + \rho} + t_i.$$

[Aquila, Gambino, Ridolfi, Uraltsev]

- Explicit calculation at $O(\alpha_s \Lambda^2_{\mathrm{QCD}}/m_b^2)$ available for $\bar{B} \to X_c \ell \bar{\nu}$.

  $\Rightarrow$ Schematic structure:

$$W_i^{(\pi,1)} = w_i^{(0)} \frac{\lambda_0}{3} \Big(S_i + 3(1 - E_0 I_{1,0}) \Big) \delta''(\hat{u}) + b_i\, \delta'(\hat{u}) + c_i\, \delta(\hat{u}) \\ + \; d_i \left[\frac{1}{\hat{u}^3}\right]_+ + e_i \left[\frac{1}{\hat{u}^2}\right]_+ + f_i \left[\frac{1}{\hat{u}}\right]_+ + R_i^{(\pi)}\, \theta(\hat{u}),$$

$$R_i^{(\pi)} = \frac{p_i^{(1)}\, \hat{u} + p_i^{(2)} \rho}{(\hat{u} + \rho)^4} + \frac{q_i}{(\hat{u} + \rho)^3} + \frac{r_i}{(\hat{u} + \rho)^2} + \frac{s_i}{\hat{u} + \rho} + t_i.$$

[Alberti, Ewerth, Gambino, Nandi; Alberti, Gambino, Nandi]

- The generalized plus distributions are defined by

$$\int d\hat{u} \left[\frac{\ln^n \hat{u}}{\hat{u}^m}\right]_+ f(\hat{u}) = \int_0^1 d\hat{u}\, \frac{\ln^n \hat{u}}{\hat{u}^m} \left[f(\hat{u}) - \sum_{p=0}^{m-1} \frac{\hat{u}^p}{p!} f^{(p)}(0) \right]$$

[Alberti, Ewerth, Gambino, Nandi; Alberti, Gambino, Nandi]

Cancellation of collinear divergences in the massless limit

- Compute the limit $\rho \to 0$ of the structures $W_i^{(1)}$, $W_i^{(\pi,1)}$ and $W_i^{(G,1)}$.
- *Collinear divergences* emerge under phase-space integration, together with $\rho \to 0$.

  $\Rightarrow$ $\hat{u}$ allowed range: $0 \leq \hat{u} \leq \hat{u}_+ = (1-\sqrt{\hat{q}^2})^2 - \rho$.

  $\Rightarrow$ $\lim_{\rho\to 0} \int_0^1 d\hat{u}\, \dfrac{1}{(\hat{u}+\rho)^n} \to \infty$.
- Make collinear divergences apparent ($\sim \ln\rho$, $\sim \ln^2\rho$, $\sim 1/\rho$, $\sim 1/\rho^2$, etc.) by introducing the appropriate distributions.

$$\frac{1}{(\hat{u}+\rho)^3} \to \left[\frac{1}{\hat{u}^3}\right]_+ + \frac{1}{2}\left(\frac{1}{\rho^2}-1\right)\delta(\hat{u}) - \left(\frac{1}{2\rho}-1\right)\delta'(\hat{u}) - \frac{1}{4}(3+2\ln\rho)\,\delta''(\hat{u})$$

$\Rightarrow$ Collinear divergences in more complicated structures of the type $I_1\left[\dfrac{1}{\hat{u}^n}\right]$ are more involved but can be extracted.

$$I_1\left[\frac{1}{\hat{u}}\right]_+ \to -\frac{1}{6\hat{w}}\left(\pi^2+3\ln^2\rho\right)\delta(\hat{u}) + \frac{2\ln\hat{w}}{\hat{w}}\left[\frac{1}{\hat{u}}\right]_+ \frac{1}{\hat{w}}\left[\frac{\ln\hat{u}}{\hat{u}}\right]_+ + \frac{1}{\hat{u}\hat{w}}\left(\ln\frac{\hat{u}}{\hat{w}^2} + \hat{w}\,\mathcal{I}_1\right)$$

$\Rightarrow$ $\hat{q}^2$ distr. and $\hat{E}_\ell$ distr. are observables: divergences cancel between virtual and real radiation structures.

[BC, Gambino, Nandi]

Applications: total width and $\hat{q}^2$ distribtion

- $O(\alpha_s \Lambda^2_{\mathrm{QCD}}/m_b^2)$ corrections to the total rate (pole mass scheme):

$$\Gamma_{B\to X_u \ell\nu} = \Gamma_0 \left[\left(1 - 2.41\,\frac{\alpha_s}{\pi}\right)\left(1 - \frac{\mu_\pi^2}{2m_b^2}\right) - \left(1.5 + 4.98\,\frac{\alpha_s}{\pi}\right)\frac{\mu_G^2(m_b)}{m_b^2}\right]$$

[BC, Gambino, Nandi]

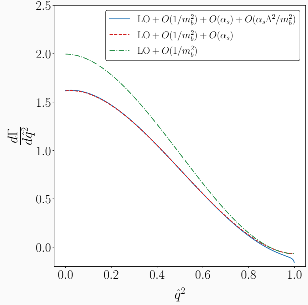
$\Rightarrow$ $\Gamma_0 = G_F^2 |V_{ub}|^2 m_b^5/192\pi^3$ is the lowest order result.

$\Rightarrow$ $O(\alpha_s \mu_\pi^2/m_b^2)$ comply with *Reparametrisation Invariance*.

$\Rightarrow$ $O(\alpha_s \mu_G^2/m_b^2)$ agrees with previous result in the literature.

[Mannel, Pivovarov, Rosenthal]

- $O(\alpha_s \Lambda^2_{\mathrm{QCD}}/m_b^2)$ corrections to the $\hat{q}^2$ distribution.



$\Rightarrow$ Total correction very small over the whole $\hat{q}^2$ range, except close to the endpoint (soft dynamics dominated region). [BC, Gambino, Nandi]

Applications: $\hat{q}_0$ moments

- $\hat{q}_0$ moments of the $O(\alpha_s \mu_\pi^2/m_b^2)$ and $O(\alpha_s \mu_G^2/m_b^2)$ structures will place further constraints on the SFs moments.

- Central moments of the perturbative and power suppressed contributions

$$J_{i,X}^{(n,j)}(\hat{q}^2) = \int_0^\infty (\hat{q}_0 - \hat{q}_0{}^{max})^n \, W_i^{(X,j)}(\hat{q}_0, \hat{q}^2) \, d\hat{q}_0,$$

with $j = 0, 1$ and $X = \pi, G$ and $\hat{q}_0{}^{max} = \frac{1+\hat{q}^2}{2}$.

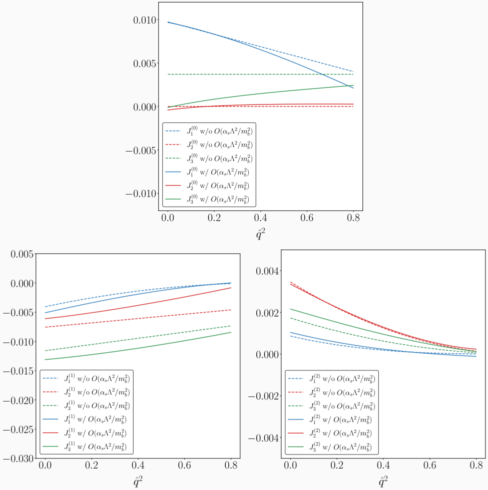
- We also define,

$$J_i^{(n)}(\hat{q}^2) = \frac{\mu_\pi^2}{2m_b^2} J_{i,\pi}^{(n,0)}(\hat{q}^2) + \frac{\mu_G^2}{2m_b^2} J_{i,G}^{(n,0)}(\hat{q}^2) + \frac{\alpha_s}{\pi} \left[\frac{\mu_\pi^2}{2m_b^2} C_F J_{i,\pi}^{(n,1)}(\hat{q}^2) + \frac{\mu_G^2}{2m_b^2} J_{i,G}^{(n,1)}(\hat{q}^2) \right].$$

[BC, Gambino, Nandi]

Applications: $\hat{q}_0$ moments

- Compare the moments $J_i^{(n)}$ with and without the $O(\alpha_s\Lambda^2/m_b^2)$ corrections.



⇒ $O(\alpha_s\Lambda^2/m_b^2)$ corrections to the zeroth moments are relatively small in most of the $\hat{q}^2$ range for $J_{1,2}^{(0)}$, and significant for $J_3^{(0)}$.

⇒ $O(\alpha_s\Lambda^2/m_b^2)$ corrections to the higher moments are generally moderate.

[BC, Gambino, Nandi]

Final remarks

Final remarks

- Long standing discrepancy in the determination of $|V_{ub}|$ between $1-3\sigma$.
  - $\Rightarrow$ V_{ub} is important to understand the structure of CP-violation and to obtain high precision calculations for NP searches.
  - $\Rightarrow$ Work needs to be done to better understand the non-perturbative dynamics in $\bar{B} \to X_u \ell \nu$, in particular the SFs.
- Performed the calculation of $O(\alpha_s \Lambda_{\mathrm{QCD}}^2/m_b^2)$ corrections to $\bar{B} \to X_u \ell \nu$:
  - $\Rightarrow$ This will allow us to include $O(\alpha_s)$ contributions to constrain the moments of the SFs.
  - $\Rightarrow$ Computed corrections to the total width and found agreement with previous calculations and reparametrisation invariance.
  - $\Rightarrow$ Computed central $\hat{q}_0$-moments of these structures.

Thank you!