Positivity bounds and the inverse problem in SMEFT

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Based on 2005.03047 with S.-Y. Zhou (PRL 125, 201601), 2009.02212 with B. Fuks, Y. Liu and S.-Y. Zhou, and other ongoing works.

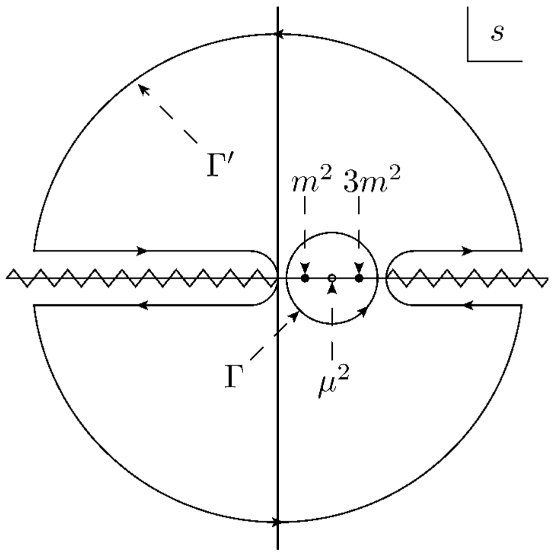
“Positivity bounds”

- Not all EFTs have a UV completion.
- Bounds from axiomatic principles of QFT (causality, unitarity, etc.), on the signs of (combinations of) Wilson coefficients.
- 2-to-2 elastic amplitude $A_{2\to 2}(s,t) = c_0 + c_2 s^2 + c_{2,1} s^2 t + c_4 s^4 + \cdots + c_{n,m} s^n t^m$
- $c_2 > 0$; Often in SMEFT: $C^{(8)} > 0$. [A. Adams et al., JHEP 06]

Positivity from elastic scattering

- Unitarity: $A(s,0) < \mathcal{O}(s\ln^2 s)$

$$f = \frac{1}{2\pi i}\oint_\Gamma \mathrm{d}s \frac{A(s,0)}{(s-\mu^2)^3}$$



[Cheung, Remmen,1601.04068]

- Analyticity:

$$f = \frac{1}{2\pi i}\oint_\Gamma \mathrm{d}s \frac{A(s,0)}{(s-\mu^2)^3} = \frac{1}{2\pi i}\left(\int_{-\infty}^{0} + \int_{4m^2}^{\infty}\right)\mathrm{d}s \frac{\mathrm{Disc}A(s,0)}{(s-\mu^2)^3}$$

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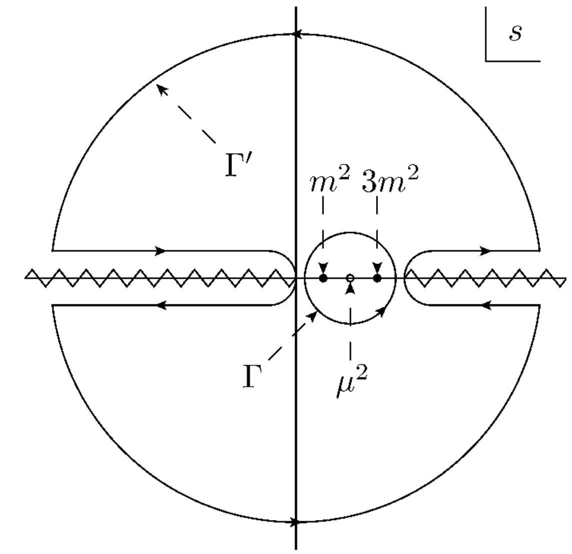

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IR

Calculable in EFT

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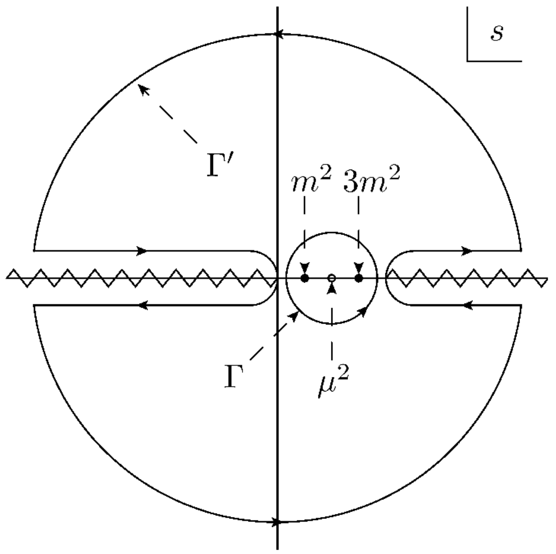

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IR

Calculable in EFT

UV

Disc > 0 by optical theorem + (s-u) crossing

$$A_{2\to 2}(s,t) = c_0 + c_2 s^2 + c_{2,1}s^2 t + c_4 s^4 + \cdots + c_{n,m}s^n t^m$$

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- 2-to-2 elastic amplitude $A_{2\to 2}(s,t) = c_0 + c_2 s^2 + c_{2,1} s^2 t + c_4 s^4 + \cdots + c_{n,m} s^n t^m$
- $c_2 > 0$; Often in SMEFT: $C^{(8)} > 0$. [A. Adams et al., JHEP 06]
- More bounds on higher-s (and t) dependence.
  See recent developments [B. Bellazzini et al., 2011.00037] [A. Tolley et al., 2011.02400] [Caron-Huot and Van Duong, 2011.02957] [Arkani-Hamed et al., 2012.15849]

Positivity in SMEFT at dim-8

- ✦ From a “phenomenological” point of view: SMEFT beyond dim-8 seems hard.
    - ➡ Focus on dim-8 in SMEFT (with E^4)
- ✦ Complication: many fields in SM, and very large-dimensional parameter space.
    - ✦ Elastic scattering is not enough
- ✦ Convex geometry helps solving the full bounds
    - ✦ Study the “generators” of the parameter space
      [CZ, S.-Y. Zhou, 2005.03047]
    - ✦ Connection with the so called “inverse problem”
    - ✦ Alternative approach, using the dual space and semidefinite programming
      [X. Li et al., 2101.01191]

B^4 operators	
$\mathcal{O}_1^{B^4}$	$(BB)(BB)$
$\mathcal{O}_2^{B^4}$	$(B\widetilde{B})(B\widetilde{B})$
$\widetilde{\mathcal{O}}_1^{B^4}$	$(BB)(B\widetilde{B})$
W^4 operators	
$\mathcal{O}_1^{W^4}$	$(W^IW^I)(W^JW^J)$
$\mathcal{O}_2^{W^4}$	$(W^I\widetilde{W}^I)(W^J\widetilde{W}^J)$
$\mathcal{O}_3^{W^4}$	$(W^IW^J)(W^IW^J)$
$\mathcal{O}_4^{W^4}$	$(W^I\widetilde{W}^J)(W^I\widetilde{W}^J)$
$\widetilde{\mathcal{O}}_1^{W^4}$	$(W^IW^I)(W^J\widetilde{W}^J)$
$\widetilde{\mathcal{O}}_2^{W^4}$	$(W^IW^J)(W^I\widetilde{W}^J)$
G^4 operators	
$\mathcal{O}_1^{G^4}$	$(G^aG^a)(G^bG^b)$
$\mathcal{O}_2^{G^4}$	$(G^a\widetilde{G}^a)(G^b\widetilde{G}^b)$
$\mathcal{O}_3^{G^4}$	$(G^aG^b)(G^aG^b)$
$\mathcal{O}_4^{G^4}$	$(G^a\widetilde{G}^b)(G^a\widetilde{G}^b)$
$\mathcal{O}_5^{G^4}$	$d^{abe}d^{cde}(G^aG^b)(G^cG^d)$
$\mathcal{O}_6^{G^4}$	$d^{abe}d^{cde}(G^a\widetilde{G}^b)(G^c\widetilde{G}^d)$
$\widetilde{\mathcal{O}}_1^{G^4}$	$(G^aG^a)(G^b\widetilde{G}^b)$
$\widetilde{\mathcal{O}}_2^{G^4}$	$(G^aG^b)(G^a\widetilde{G}^b)$
$\widetilde{\mathcal{O}}_3^{G^4}$	$d^{abe}d^{cde}(G^aG^b)(G^c\widetilde{G}^d)$
$F_1^2F_2^2/F_1F_2^3$ cross-quartics	
$\mathcal{O}_1^{B^2W^2}$	$(BB)(W^IW^I)$
$\mathcal{O}_2^{B^2W^2}$	$(B\widetilde{B})(W^I\widetilde{W}^I)$
$\mathcal{O}_3^{B^2W^2}$	$(BW^I)(BW^I)$
$\mathcal{O}_4^{B^2W^2}$	$(B\widetilde{W}^I)(B\widetilde{W}^I)$
$\widetilde{\mathcal{O}}_1^{B^2W^2}$	$(B\widetilde{B})(W^IW^I)$
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$\widetilde{\mathcal{O}}_3^{B^2G^2}$	$(BG^a)(B\widetilde{G}^a)$
$\mathcal{O}_1^{W^2G^2}$	$(W^IW^I)(G^aG^a)$
$\mathcal{O}_2^{W^2G^2}$	$(W^I\widetilde{W}^I)(G^a\widetilde{G}^a)$
$\mathcal{O}_3^{W^2G^2}$	$(W^IG^a)(W^IG^a)$
$\mathcal{O}_4^{W^2G^2}$	$(W^I\widetilde{G}^a)(W^I\widetilde{G}^a)$
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$\mathcal{O}_1^{BG^3}$	$d^{abc}(BG^a)(G^bG^c)$
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$(DH)^4$ operators	
$\mathcal{O}_1^{H^4}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$\mathcal{O}_2^{H^4}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$\mathcal{O}_3^{H^4}$	$(D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$
$(DH)^2F^2$ cross-quartics	
$\mathcal{O}_1^{H^2B^2}$	$(D^\mu H^\dagger D^\nu H)B_{\mu\rho}B_\nu{}^\rho$
$\mathcal{O}_2^{H^2B^2}$	$(D^\mu H^\dagger D_\mu H)B_{\rho\sigma}B^{\rho\sigma}$
$\widetilde{\mathcal{O}}_1^{H^2B^2}$	$(D^\mu H^\dagger D_\mu H)B_{\rho\sigma}\widetilde{B}^{\rho\sigma}$
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$\mathcal{O}_2^{H^2W^2}$	$(D^\mu H^\dagger D_\mu H)W^I_{\rho\sigma}W^{I\rho\sigma}$
$\mathcal{O}_3^{H^2W^2}$	$i\,\epsilon^{IJK}(D^\mu H^\dagger\tau^I D^\nu H)W^J_{\mu\rho}W^{K\rho}_\nu$
$\widetilde{\mathcal{O}}_1^{H^2W^2}$	$(D^\mu H^\dagger D_\mu H)W^I_{\rho\sigma}\widetilde{W}^{I\rho\sigma}$
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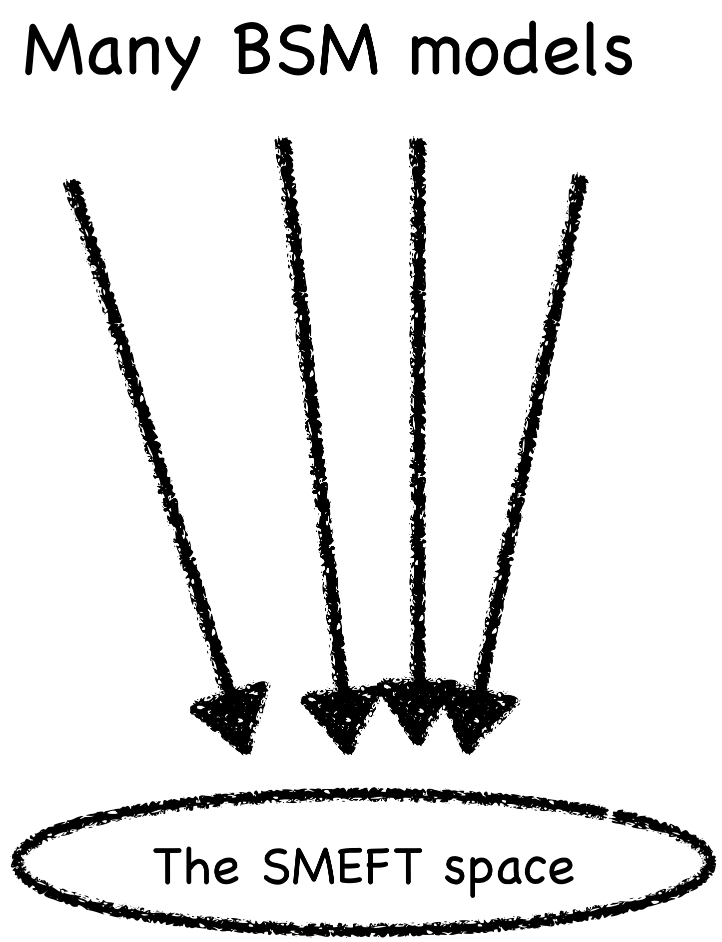
[Remmen, Rodd,1908.09845]

The inverse problem



- SMEFT is useful because it describes (infinitely) many models by finitely many Wilson coefficients.
- They are being determined by global fit to LHC data.

See e.g.
[1901.05965 N. P. Hartland et al. [SMEFiT]],
[1910.03606 J. Ellis et al.]
and more

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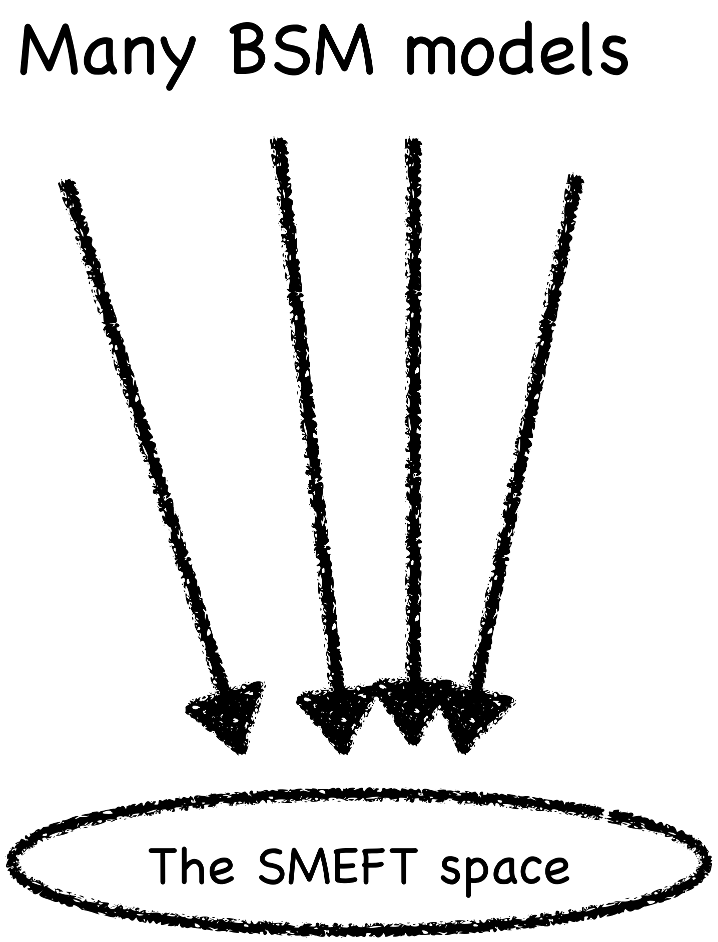

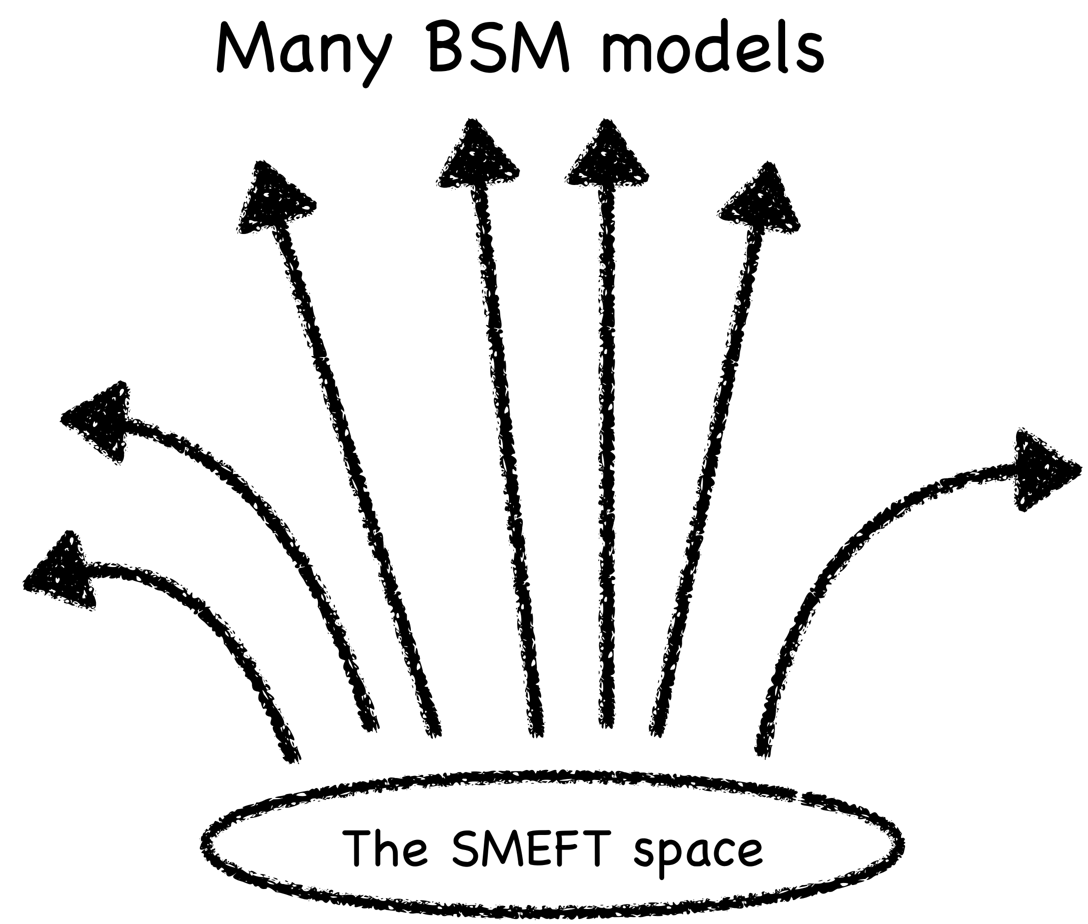

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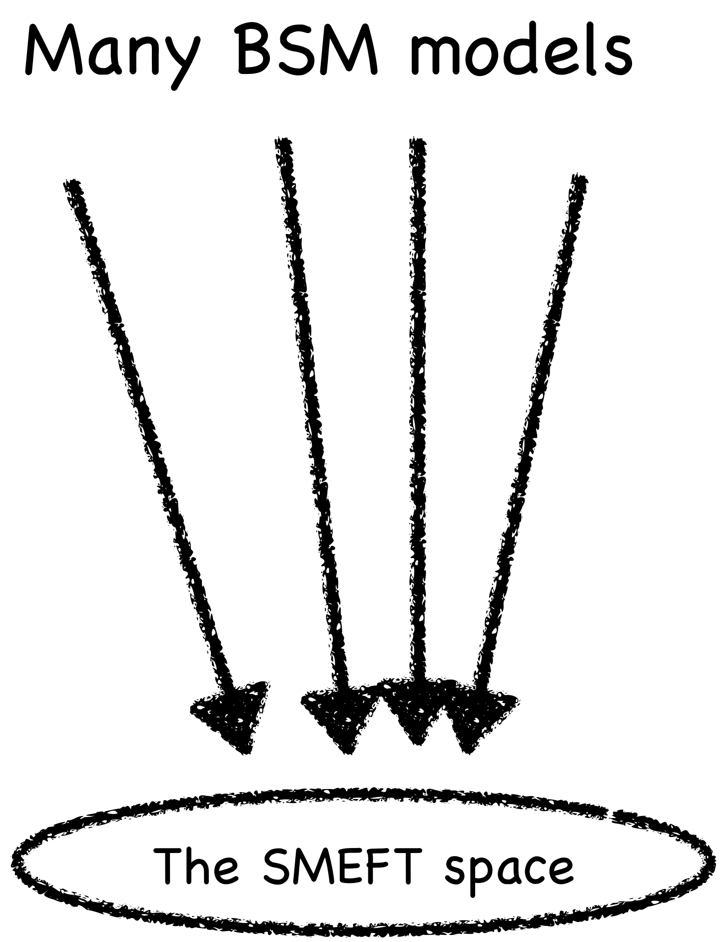

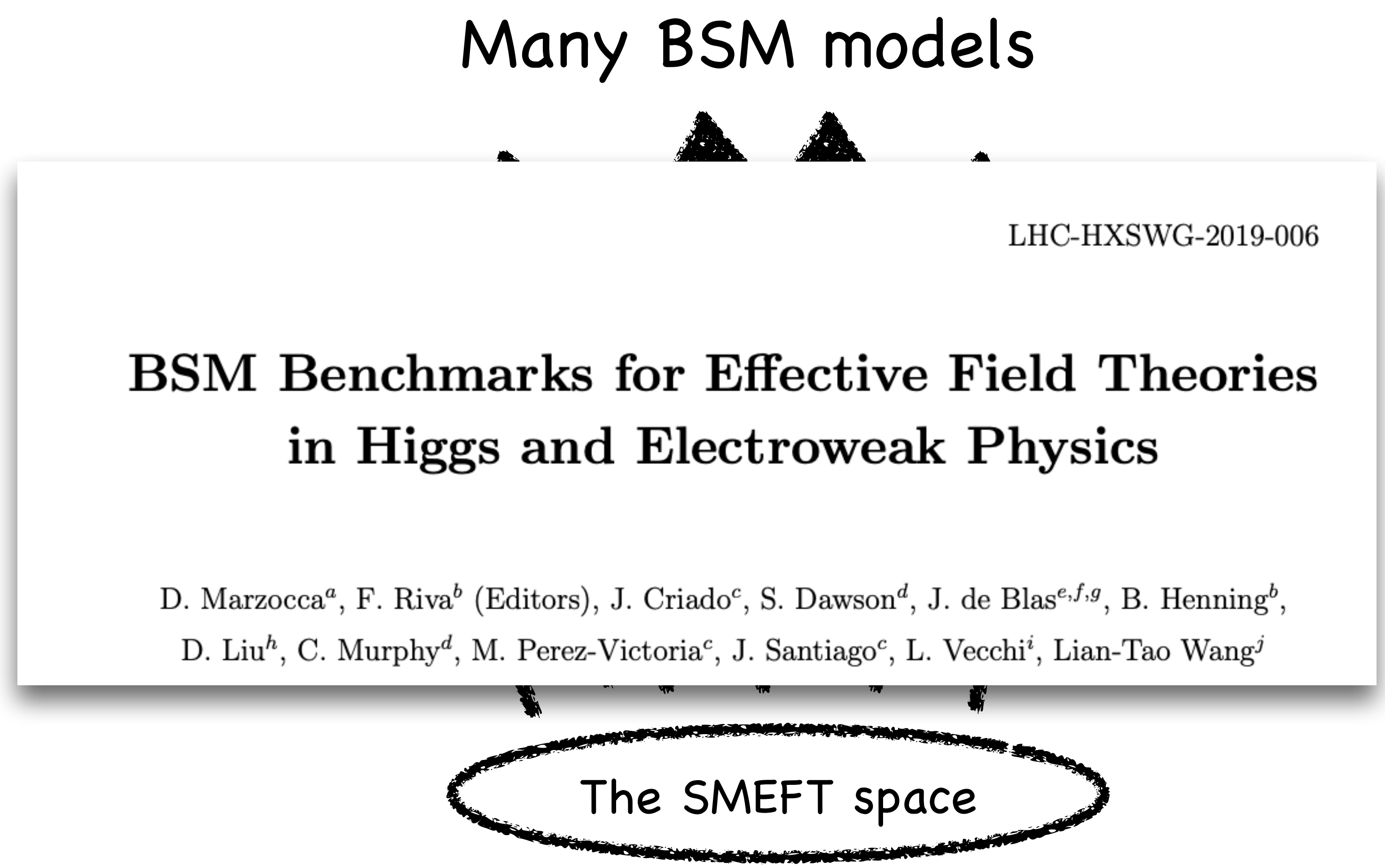

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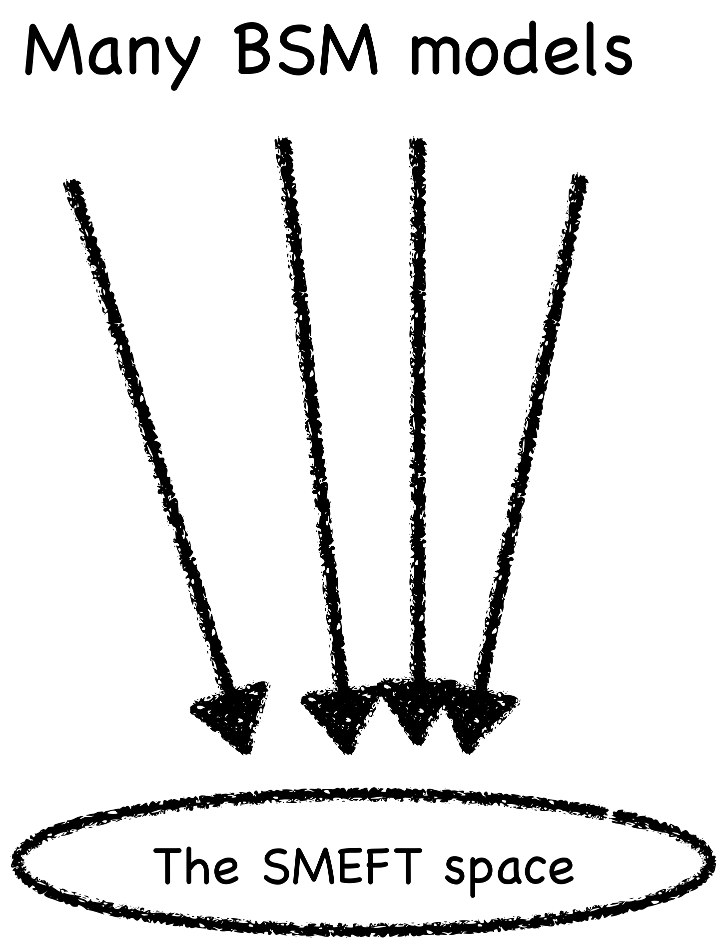

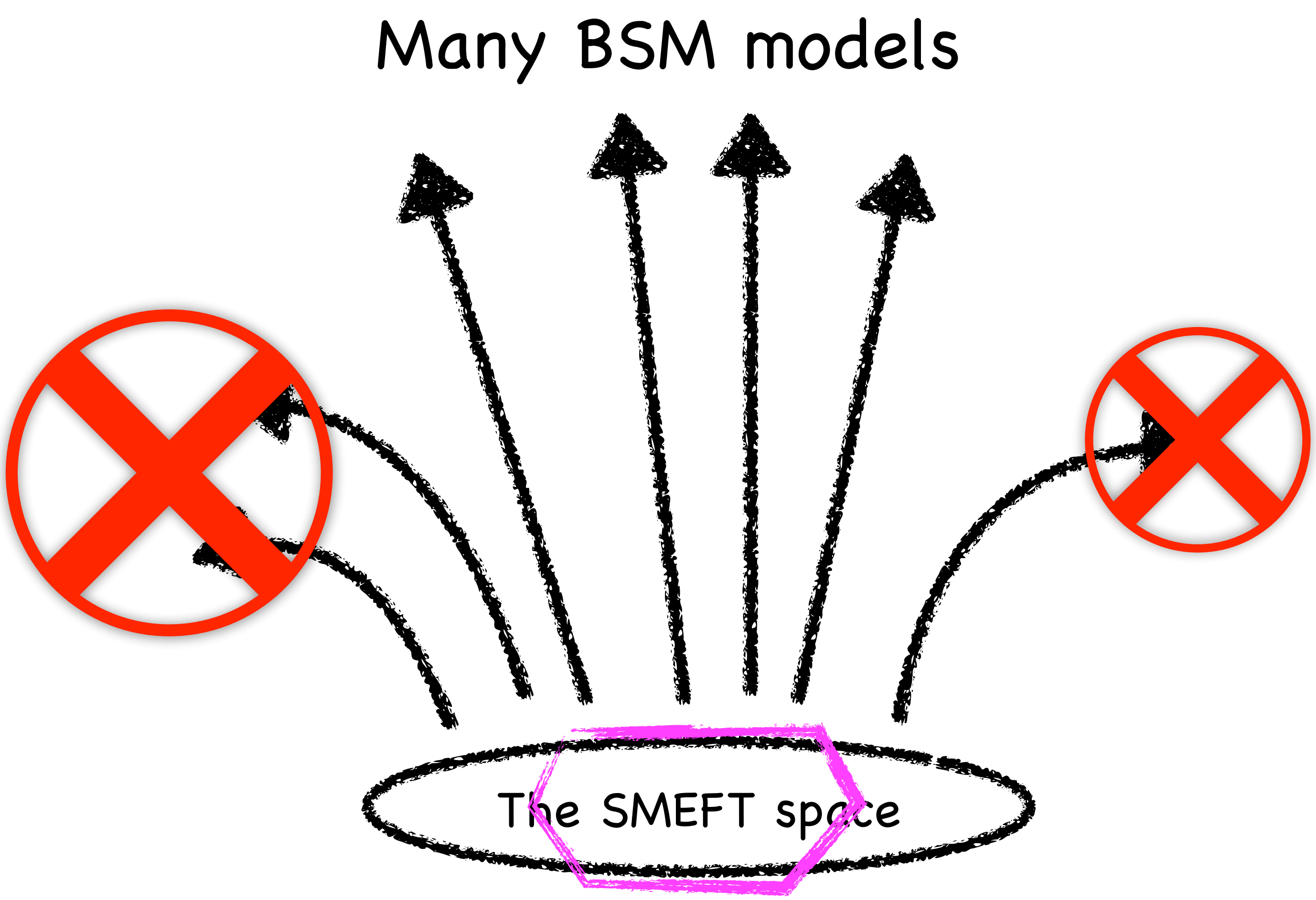

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- Positivity tells us when this is impossible, and much more.

A toy example

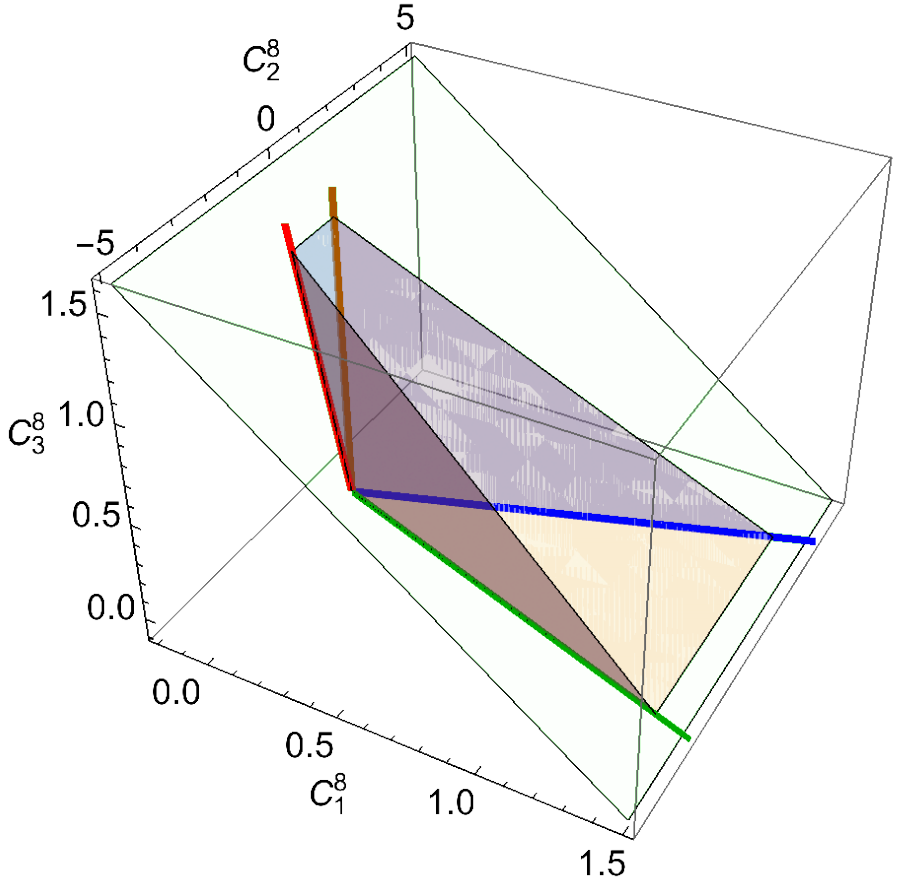
- Consider a two scalar EFT, with two discrete symmetries:
  - $\phi_1 \to -\phi_1$
  - $\phi_1 \leftrightarrow \phi_2$
- 3 dim-8 operators (E4):

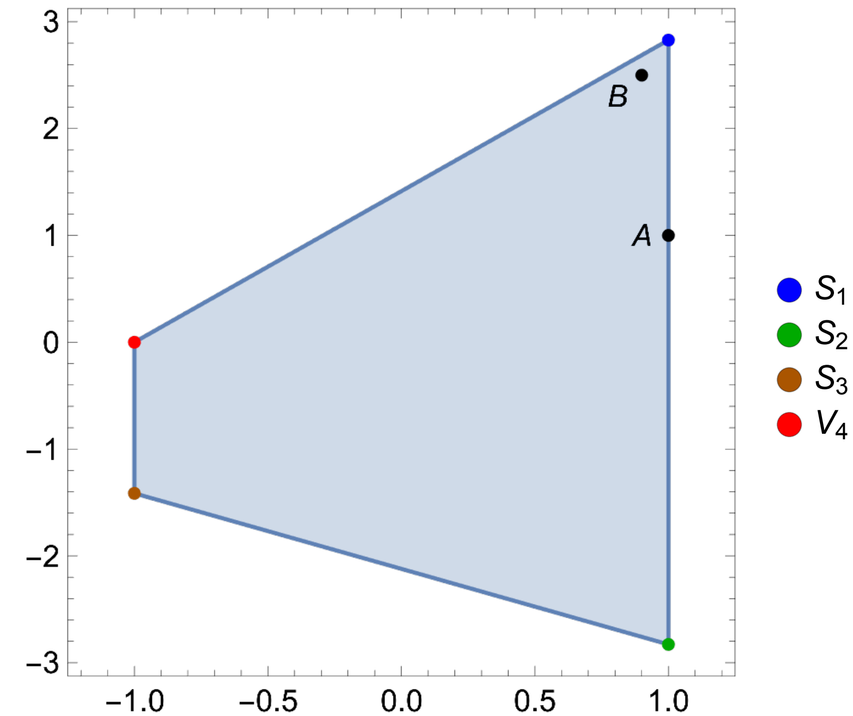
$$O_1^8 = \partial_\mu\phi_1\partial^\mu\phi_1\partial_\nu\phi_1\partial^\nu\phi_1 + \partial_\mu\phi_2\partial^\mu\phi_2\partial_\nu\phi_2\partial^\nu\phi_2$$
$$O_2^8 = \partial_\mu\phi_1\partial^\mu\phi_1\partial_\nu\phi_2\partial^\nu\phi_2$$
$$O_3^8 = \partial_\mu\phi_1\partial^\mu\phi_2\partial_\nu\phi_1\partial^\nu\phi_2$$

- Each elastic channel gives one bound:
  - Superposition: $|\phi_\pm\rangle \equiv \frac{1}{\sqrt{2}}|\phi_1\rangle \pm \frac{1}{\sqrt{2}}|\phi_2\rangle$

$$\frac{1}{2}\frac{d^2}{ds^2}M_{\phi_1\phi_1\to\phi_1\phi_1}(s) = 4C_1^8 \geq 0,$$
$$\frac{1}{2}\frac{d^2}{ds^2}M_{\phi_+\phi_+\to\phi_+\phi_+}(s) = 4(2C_1^8 + C_2^8 + C_3^8) \geq 0,$$
$$\frac{1}{2}\frac{d^2}{ds^2}M_{\phi_+\phi_-\to\phi_+\phi_-}(s) = 4(2C_1^8 - C_2^8) \geq 0,$$
$$\frac{1}{2}\frac{d^2}{ds^2}M_{\phi_1\phi_2\to\phi_1\phi_2}(s) = C_3^8 \geq 0.$$

- Four bounds in total
  - Any other superposition gives redundant bounds





Q: where do the UV models live? Tree level UV completion:

particle	spin	parities $(\phi_1 \to -\phi_1, \phi_1 \leftrightarrow \phi_2)$	Interaction	ER	$\vec{c} = (C_1, C_2, C_3)$
S_1	0	$++$	$g_1 M_1(\phi_1^2 + \phi_2^2) S_1$	✓	$2 \times (1, 2, 0)$
S_2	0	$+-$	$g_2 M_2(\phi_1^2 - \phi_2^2) S_2$	✓	$2 \times (1, -2, 0)$
S_3	0	$-+$	$g_3 M_3 \phi_1 \phi_2 S_3$	✓	$2 \times (0, 0, 1)$
V_4	1	$--$	$g_4(\phi_1 \overleftrightarrow{D}_\mu \phi_2) V_4^\mu$	✓	$2 \times (0, -1, 1)$

$\times \dfrac{g^2}{M^4}$

Exactly on the edges!

Edge vector ↔ "one particle extension"

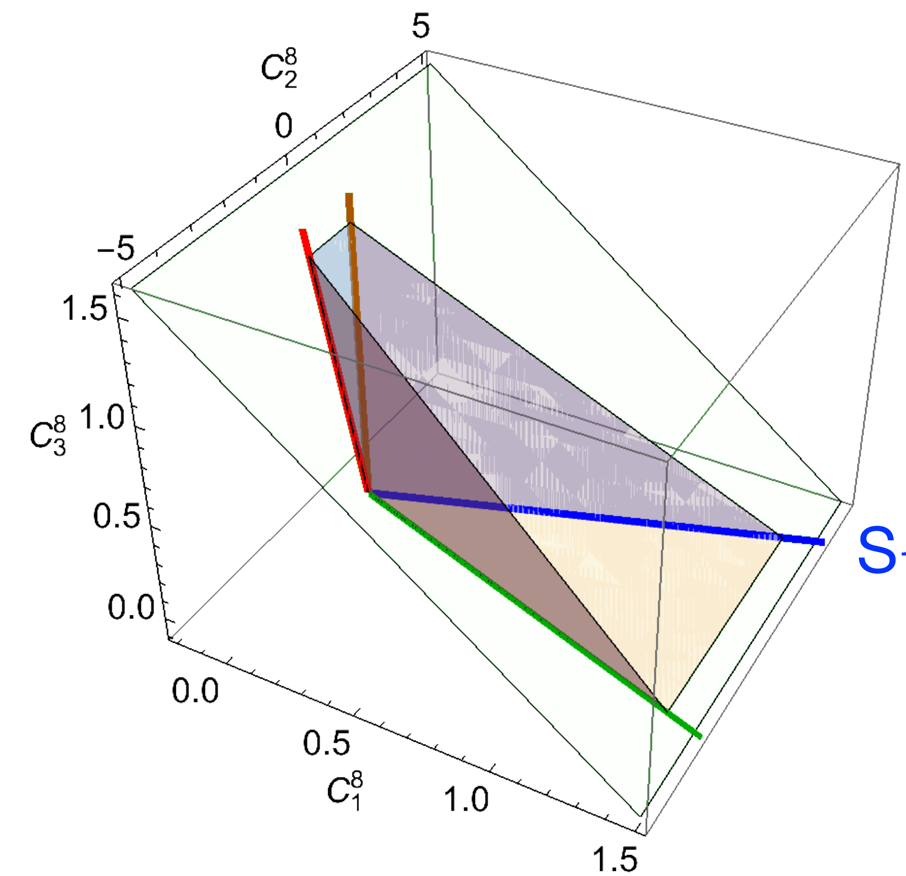
- Positivity bounds describe the 4 faces of the pyramid.

- Alternatively, the pyramid is generated its edge vectors. They are the **generators**.

- On the physics side, integrating out each heavy particle gives:

$$\vec{C} = (C_1, C_2, C_3) = \frac{g^2}{M^4}\vec{c}$$

- In total: $\vec{C} = \sum_{X=S_{1,2,3},V} w_X \vec{c}_X\,, \quad w_X \equiv \frac{g_X^2}{M_X^4} \geq 0$



- One particle extension are the **generators**.

- <u>It is also (often) the unique UV completion of EFTs on edge vectors.</u>

Extremal rays

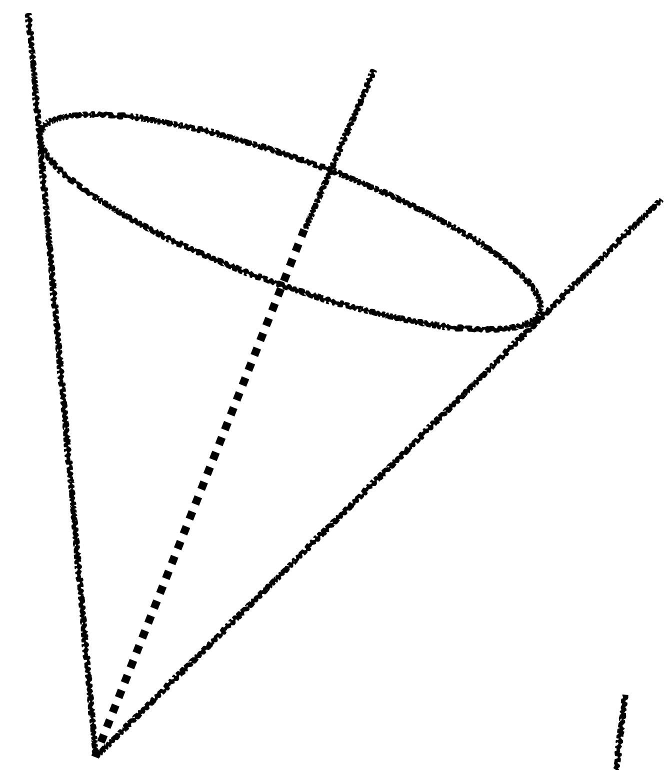
- **Extremal Ray (ER)**: A ray is an extremal ray of cone C, if it cannot be split into two other vectors in C, which are linearly independent.

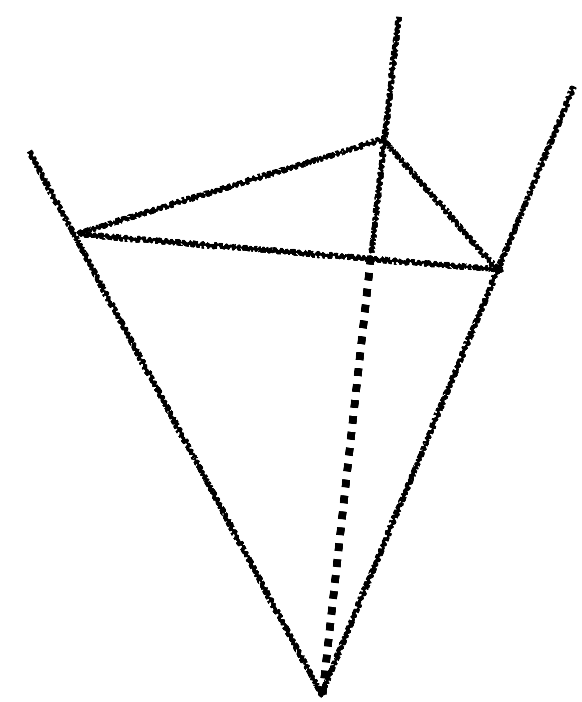
- In polyhedral cones, ERs are the edge vectors.

- Being **not splittable**, the corresponding UV completion cannot have more than one (type of) particles.

  - If the measured coefficients are on an ER, the UV particle content is **uniquely determined**.

  - Not true at dim-6.





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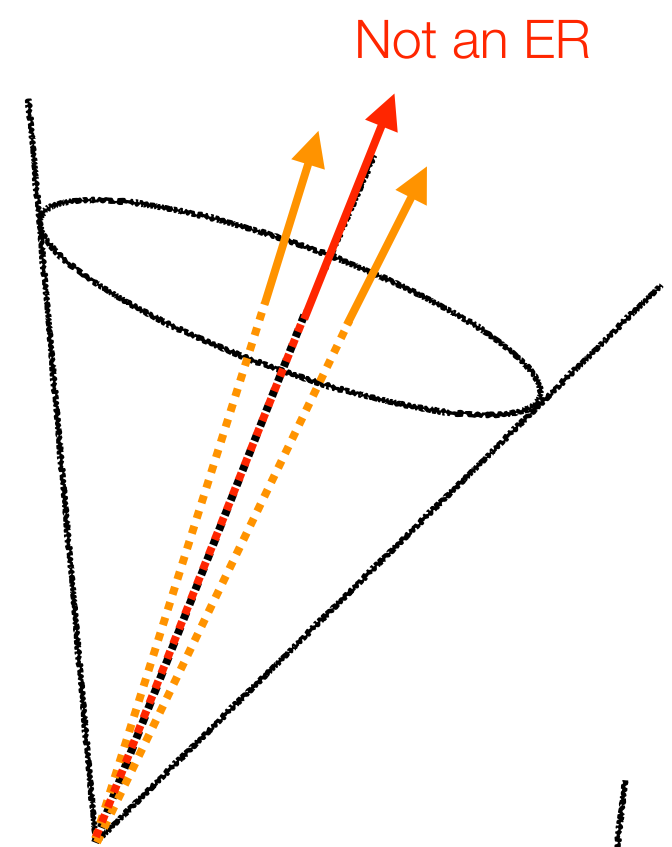

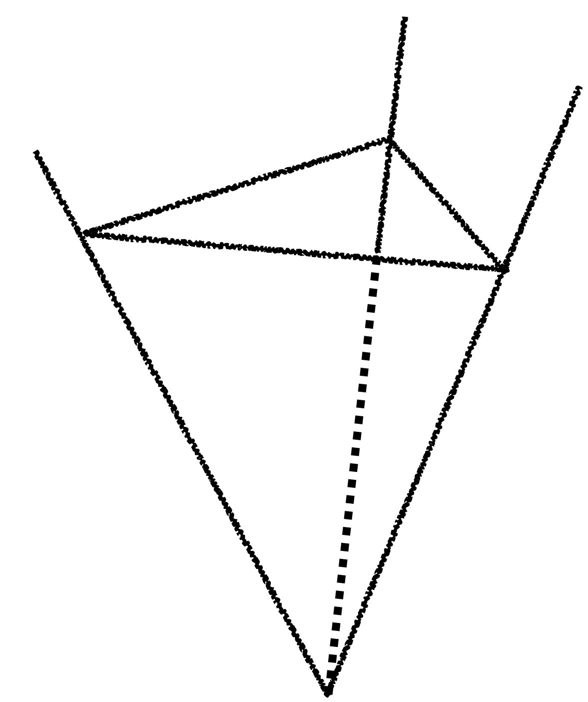

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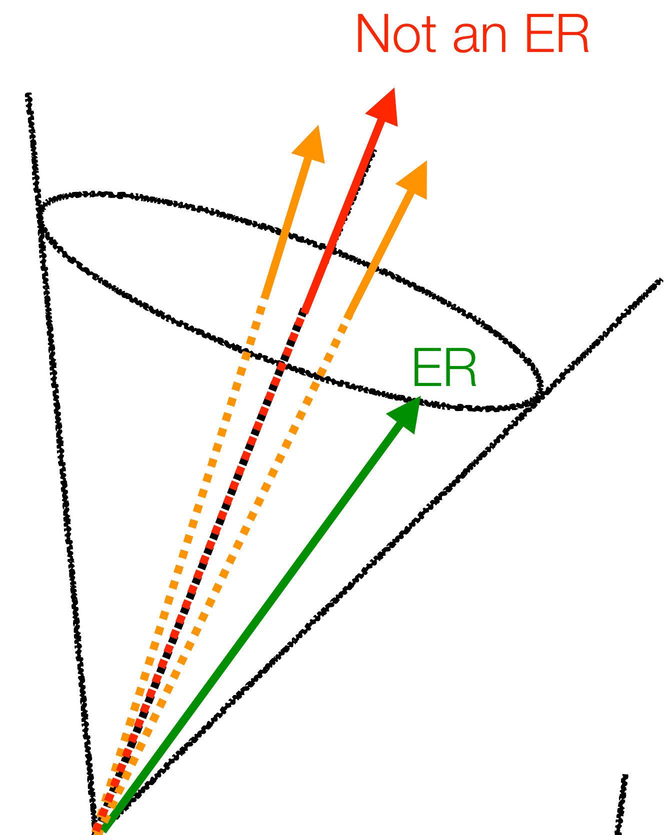

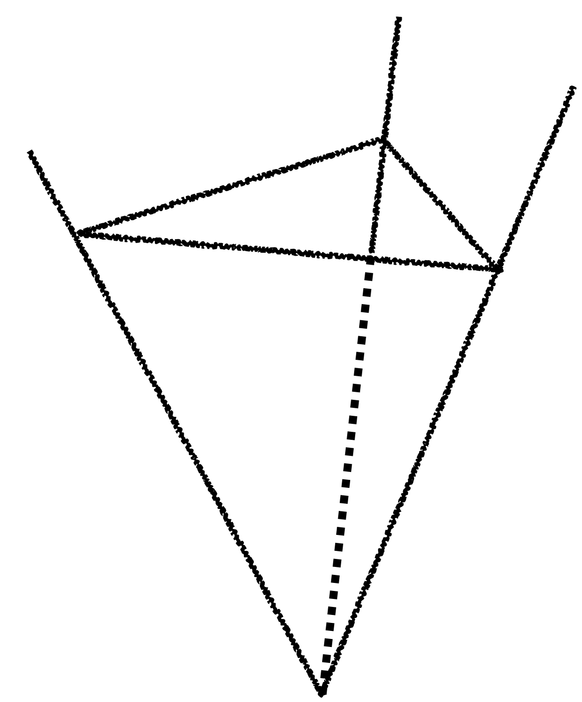

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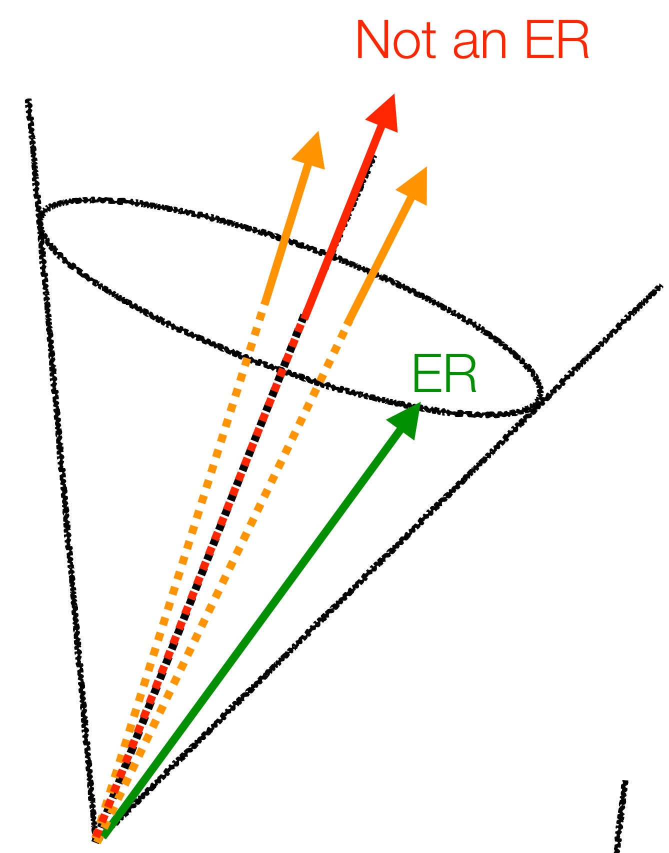
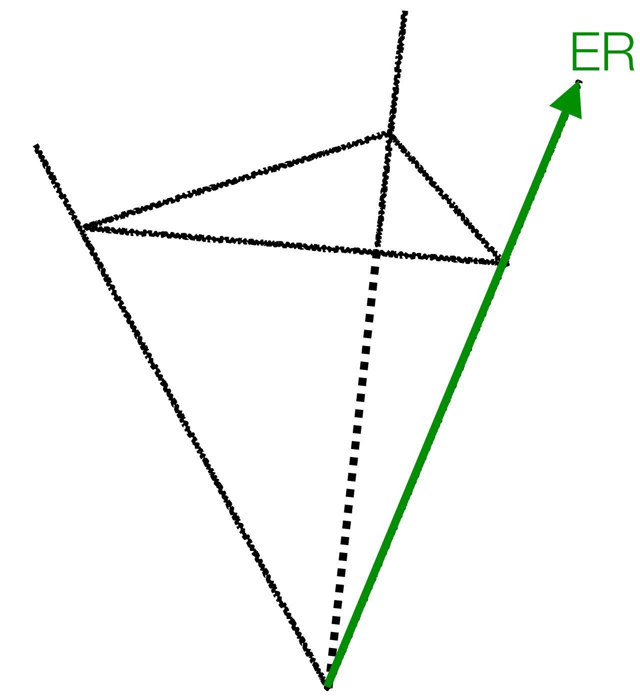
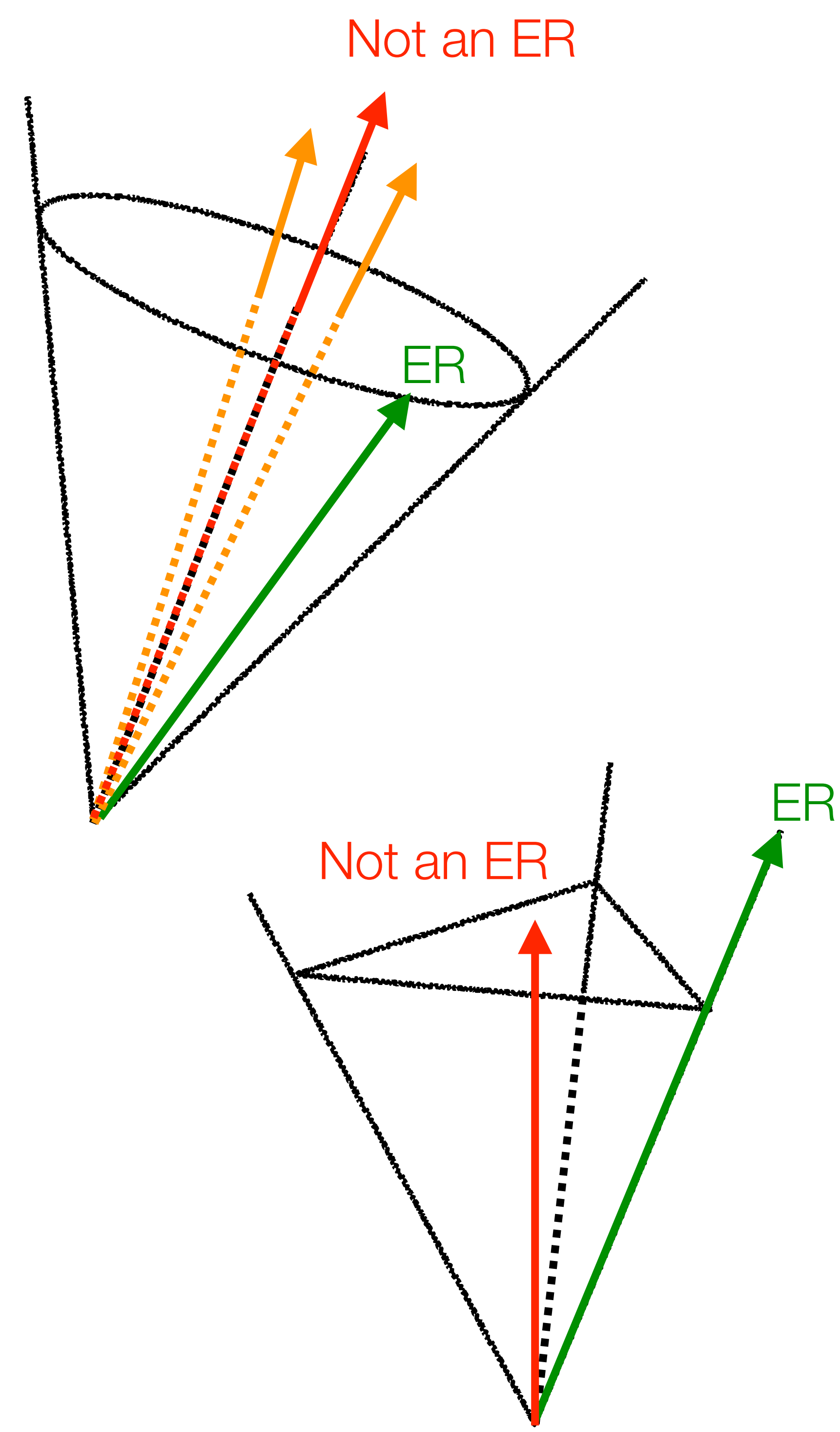




Extremal rays

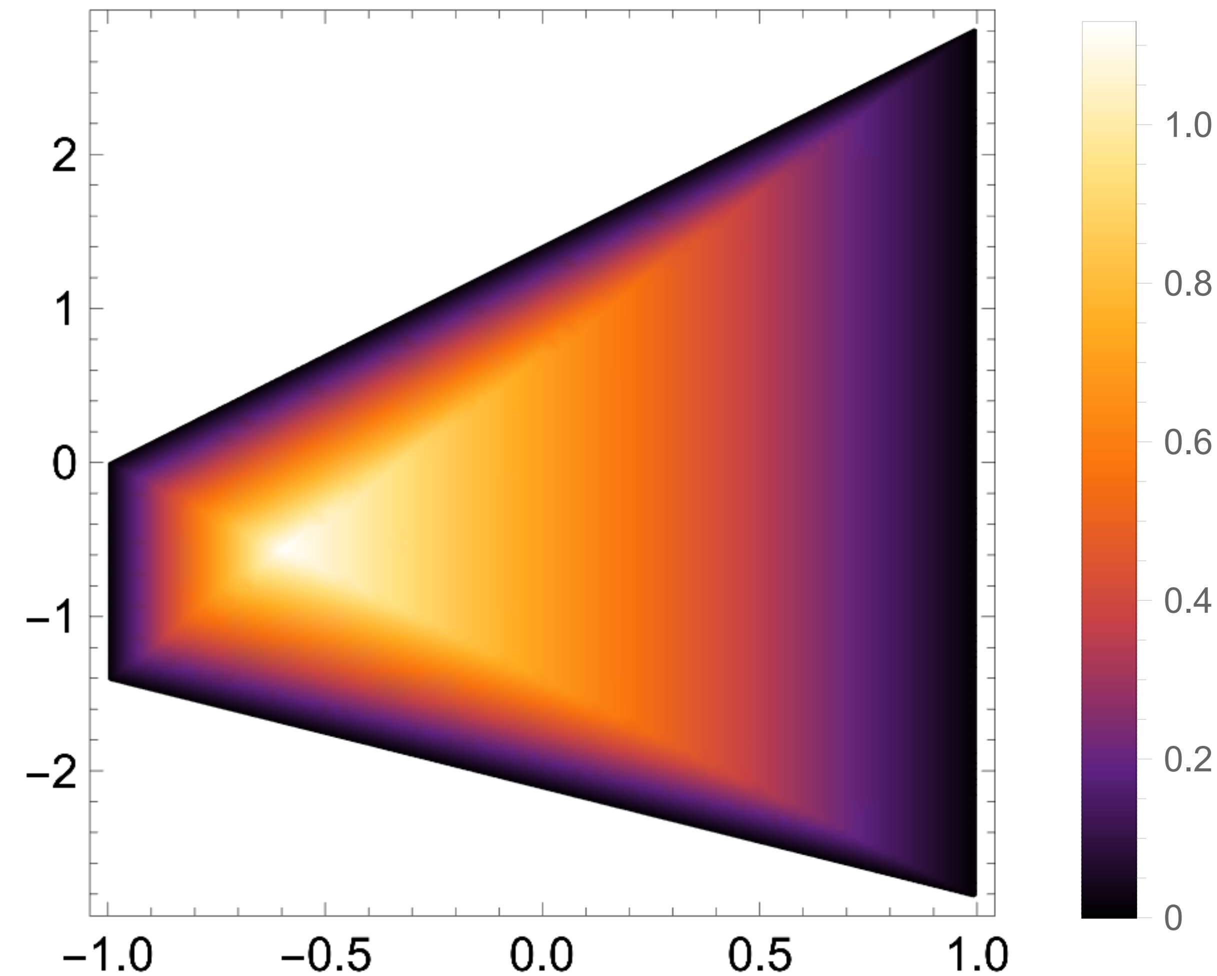
- **Extremal Ray (ER)**: A ray is an extremal ray of cone C, if it cannot be split into two other vectors in C, which are linearly independent.
- In polyhedral cones, ERs are the edge vectors.
- Being **not splittable**, the corresponding UV completion cannot have more than one (type of) particles.
  - If the measured coefficients are on an ER, the UV particle content is **uniquely determined**.
  - Not true at dim-6.



$$\vec{C} = \sum_{X=S_{1,2,3},V} w_X \vec{c}_X, \quad w_X \equiv \frac{g_X^2}{M_X^4} \geq 0$$

- To what extend can we determine the weights w, given the measured coefficients C? (With **less** coefficients than particle types X)
- ER: unique solution.
- Face: (in this example) unique solution.
- Inside -> more arbitrariness

Uncertainty on w's



Outline

- Positivity bound from the generator point of view
- The inverse problem
- A realistic example: e+e- scattering at ILC

Positivity bound from generators

Positively constructing the SMEFT space

- Identify the generators and edge vectors (or ERs) of the dim-8 SMEFT coefficient space [CZ, S.-Y. Zhou, 2005.03047] [T. Trott, 2011.10058]

- Tool: dispersion relation

    - We are going to work under the assumption of SM masses -> 0, focus on the 2nd s derivative of the forward amplitude, but **not necessarily elastic**

$$\mathcal{M}^{ijkl} \equiv \frac{\mathrm{d}^2}{\mathrm{d}s^2} A_{ij\to kl}(s)\Big|_{s\to 0}$$

$$
\begin{aligned}
\mathcal{M}^{ijkl} &= \frac{1}{2\pi i}\left(\int_{-\infty}^{0} + \int_{4m^2}^{\infty}\right) \mathrm{d}s \frac{\mathrm{Disc}A_{ij\to kl}(s,0)}{(s-2m^2)^3} \\
&\Rightarrow \frac{1}{2\pi i}\left(\int_{-\infty}^{-(\epsilon\Lambda)^2} + \int_{(\epsilon\Lambda)^2}^{\infty}\right) \mathrm{d}s \frac{\mathrm{Disc}A_{ij\to kl}(s,0)}{(s-2m^2)^3} \\
&= \frac{1}{2\pi i}\int_{(\epsilon\Lambda)^2}^{\infty} \mathrm{d}s \frac{\mathrm{Disc}A_{ij\to kl}(s,0) + \mathrm{Disc}A_{i\bar{l}\to k\bar{j}}(s,0)}{(s-2m^2)^3}
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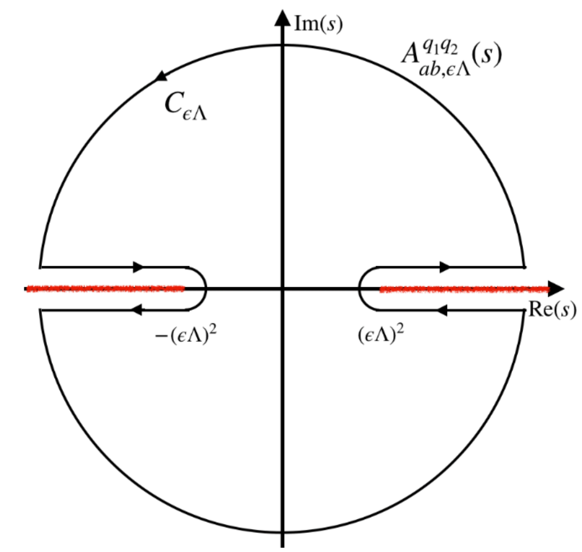
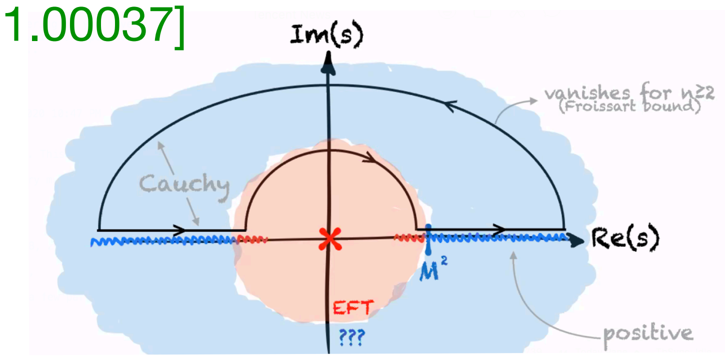
$$



$\epsilon\Lambda$ is some scale comparable but below cutoff so the EFT is still valid; see “improved positivity” of [C. de Rham et al., 1710.09611]; and the “arc”s in [B. Bellazzini et al., 2011.00037]



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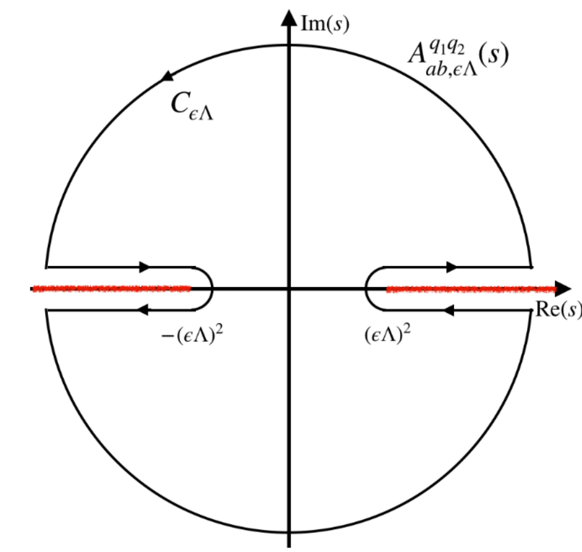

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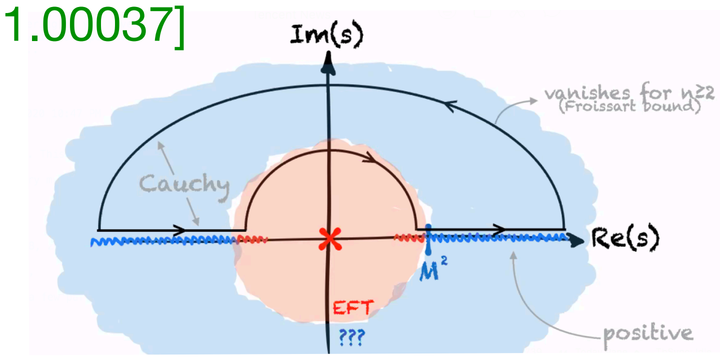
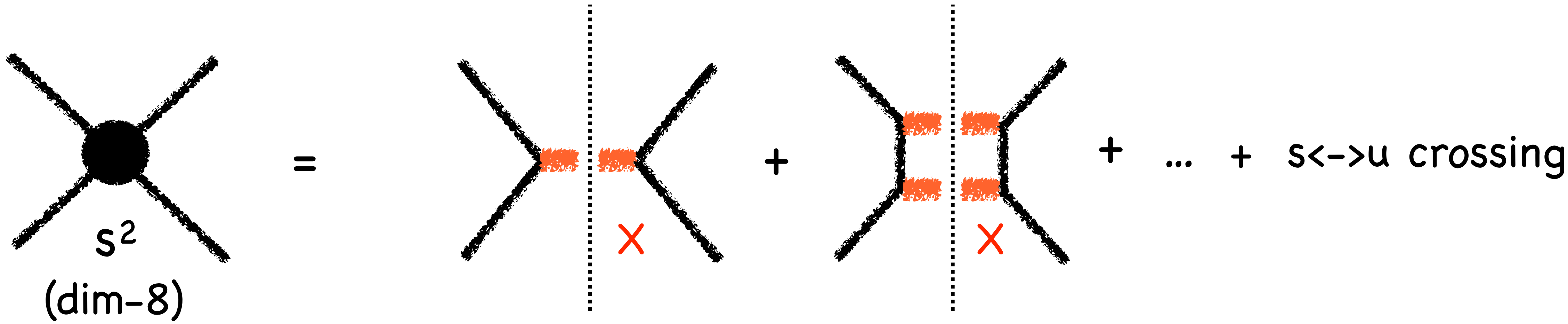


Generalized optical theorem:

$$\mathrm{Disc}A_{ij\to kl}(s) = A_{ij\to kl}(s) - A_{kl\to ij}(s)^* = i\sum_X M_{ij\to X}(s)M_{kl\to X}(s)^*$$

$$\boxed{\mathcal{M}^{ijkl} = \frac{1}{2\pi}\int_{(\epsilon\Lambda)^2}^{\infty} \frac{\mathrm{d}s}{s^3}\sum_X\left(\mathcal{M}_{ij\to X}\mathcal{M}^*_{kl\to X} + \mathcal{M}_{i\bar{l}\to X}\mathcal{M}^*_{k\bar{j}\to X}\right)}$$



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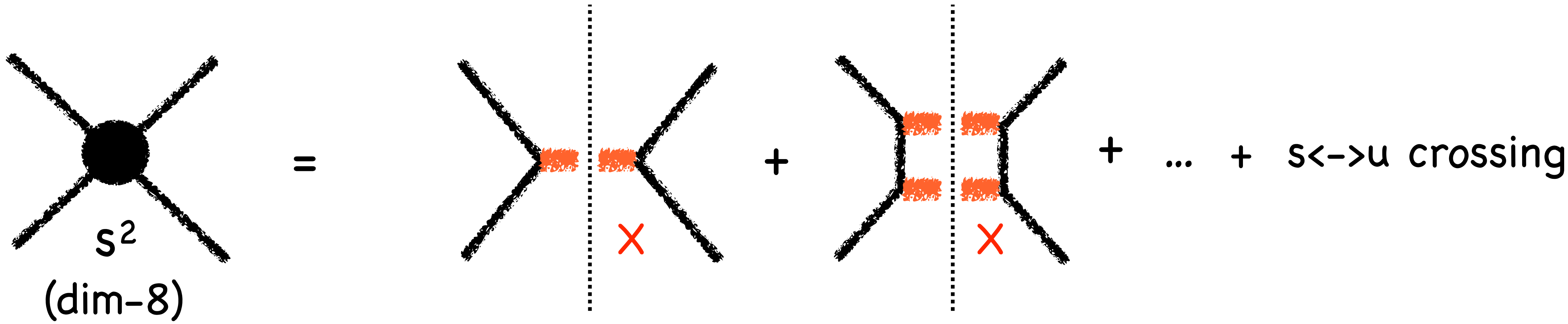

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M^{ijkl} can be mapped to coefficients
e.g. 2-scalar theory

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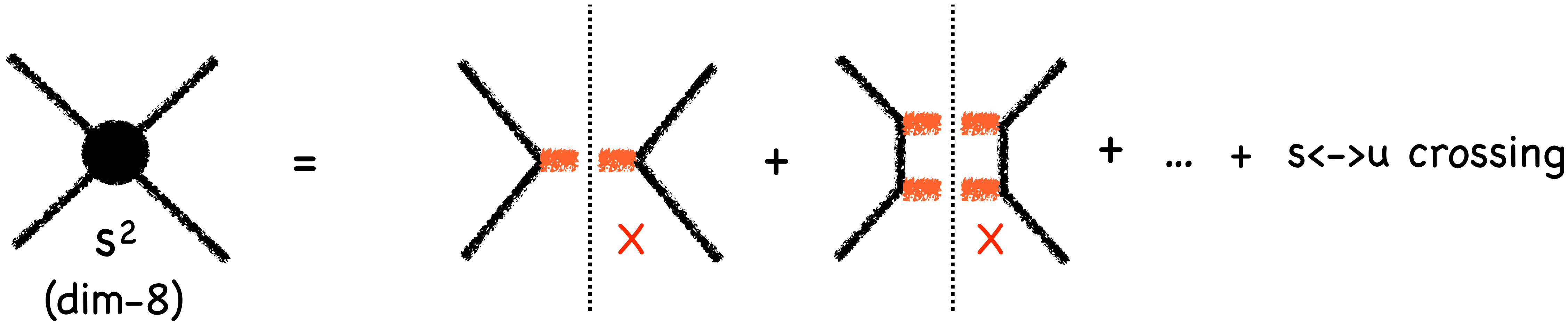

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$M_{ij\to X}$ describes unknown UV physics.
Restricted by only symmetries.

- Magnitude does not matter (remove positive factors)
- Deal with tensors and matrices

$$\mathcal{M}^{ijkl} = \sum_X \lambda_X \left(m_X^{ij} m_X^{*kl} + m_X^{i\bar{l}} m_X^{*k\bar{j}}\right), \;\; \lambda_X \geq 0$$

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Define the "directional" information of $m_X^{ij} m_X^{*kl} + m_X^{i\bar{l}} m_X^{*k\bar{j}}$ as the "generator"

$$\mathcal{G}_X^{ijkl} \equiv m_X^{ij} m_X^{*kl} + m_X^{i\bar{l}} m_X^{*k\bar{j}}$$

Define all allowed values of *M* by "**C**". If we enumerate all possible ***m*** matrices (up to normalization), then

$$\mathbf{C} \equiv \left\{ \mathcal{M}^{ijkl} \right\} = \mathrm{cone}\left(\left\{ \mathcal{G}^{ijkl} \right\} \right)$$

The physical parameter space is the "conical hull" of all generators.

Convex cones, hulls, representations of cones

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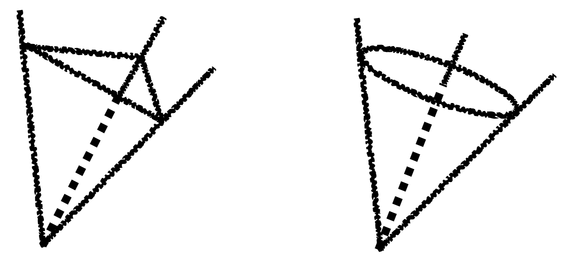

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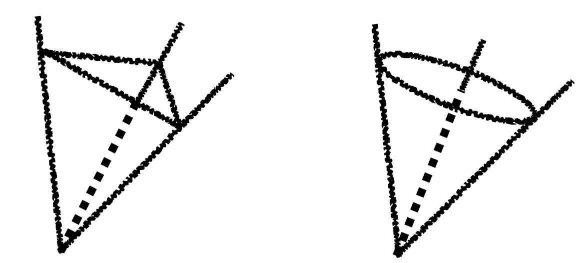

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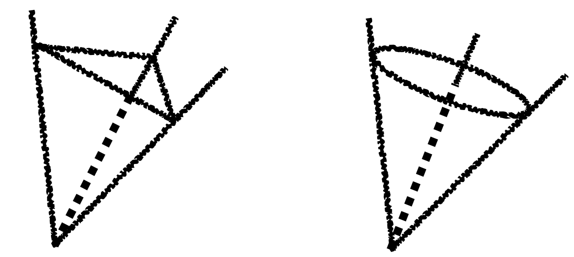

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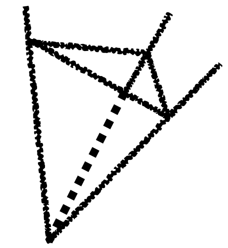

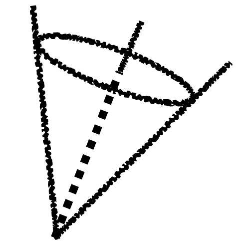

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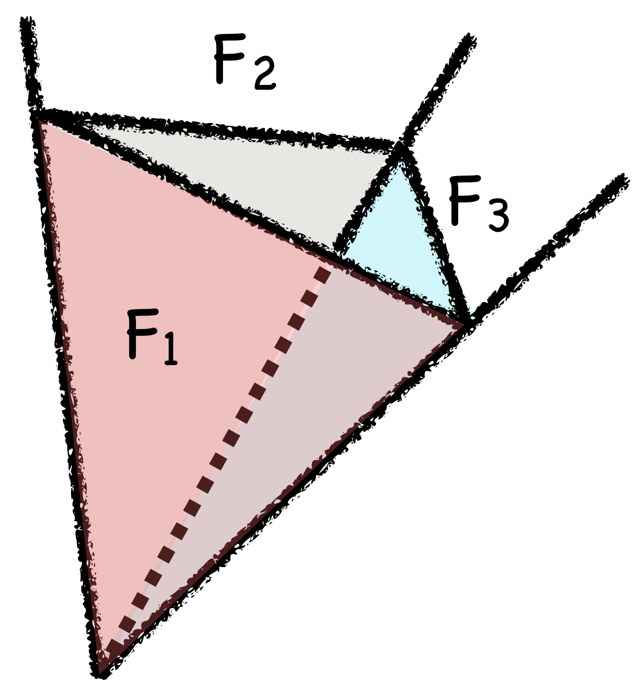

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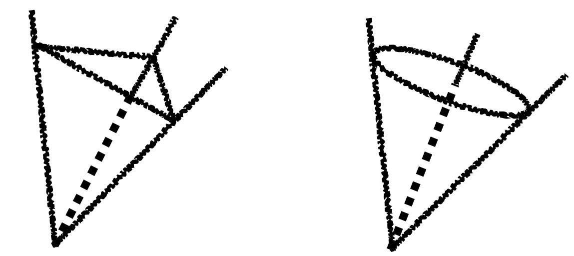

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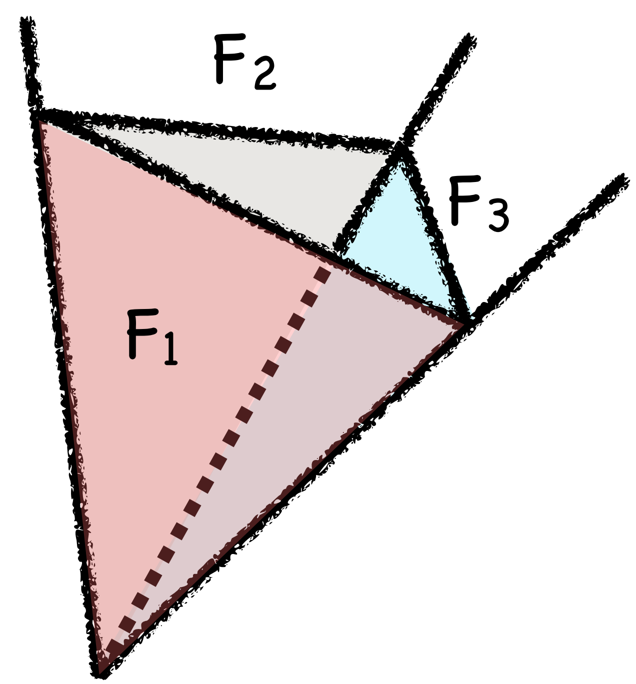
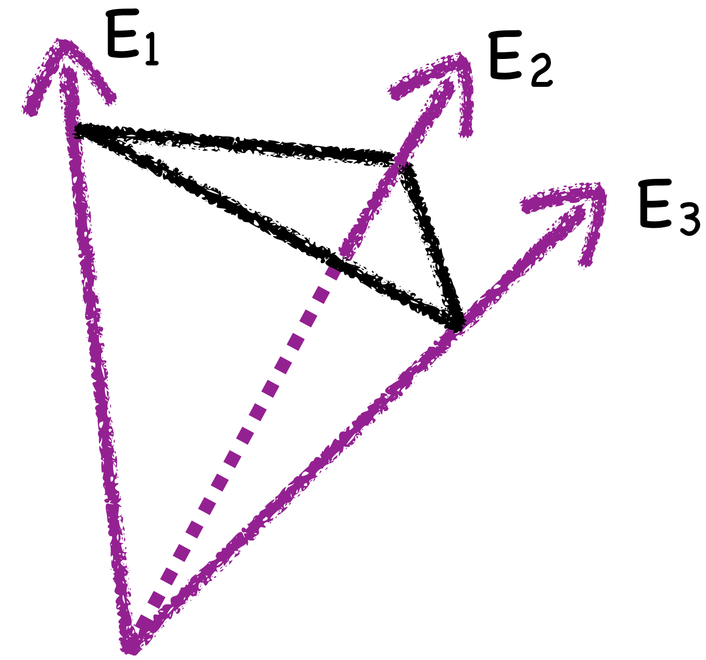
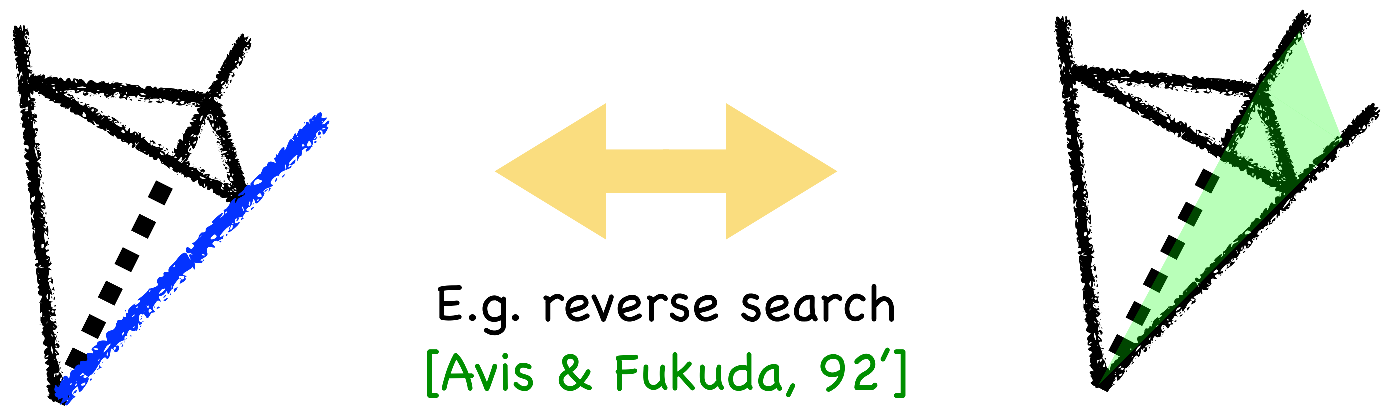
  - Edge representation: the cone is generated by its edge vectors





Convex cones, hulls, representations of cones

- **Vertex enumeration**: computes one representation from the other.
    - Allows to derive bound from “generators”



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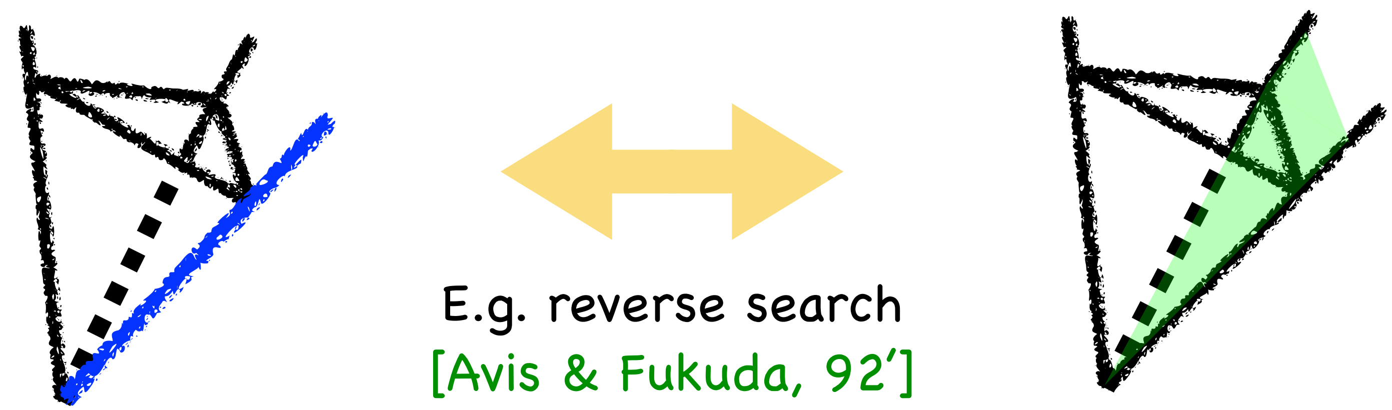

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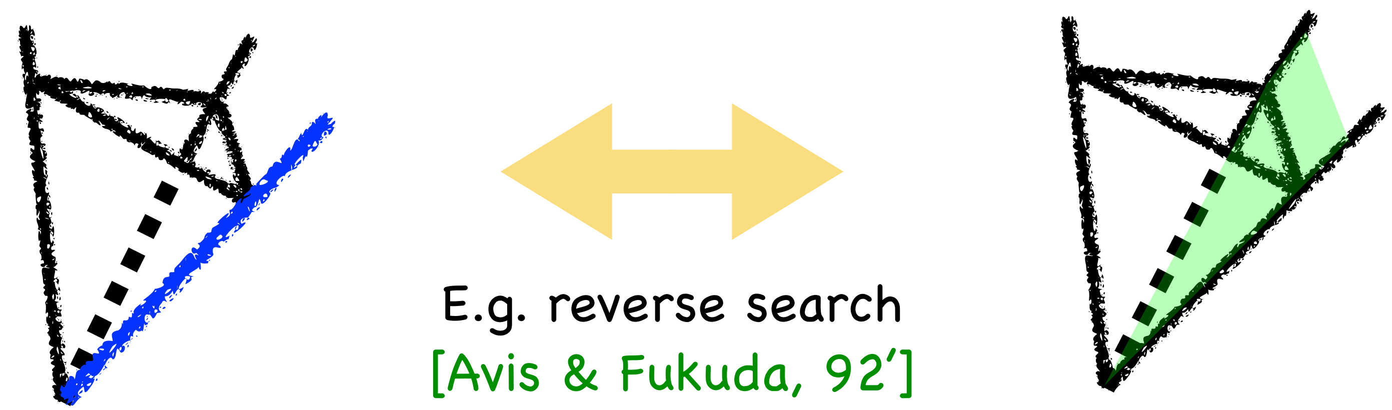

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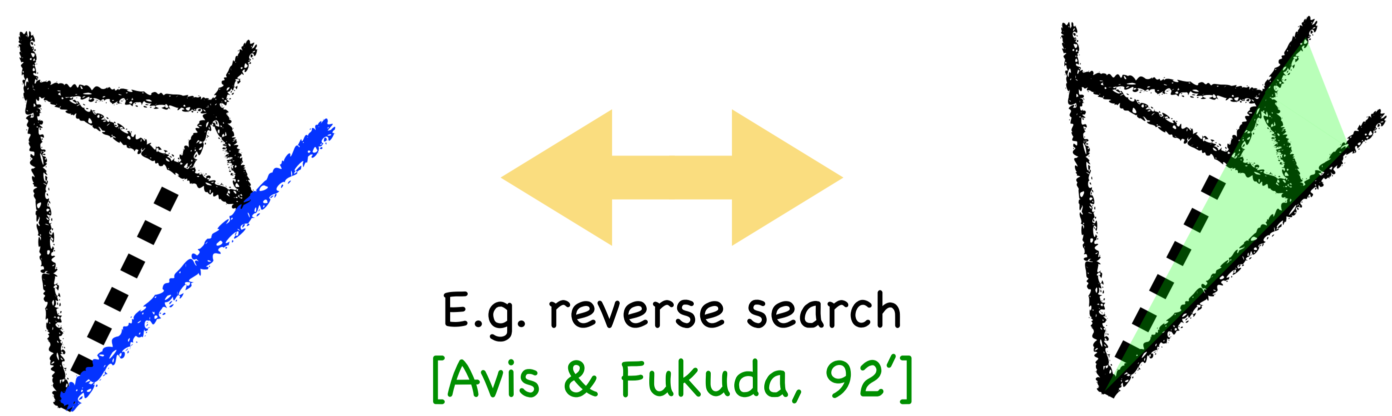

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  - **Krein-Milman theorem**: a salient cone **C** is a conical hull of its ERs. C = cone(“ERs”). ERs always exist, they are a subset of $\{\mathcal{G}^{ijkl}\}$

Example 1: two scalars with SO(2)

✦ Operators:

$$O_{ijkl} = (\partial_\mu \phi_i \partial^\mu \phi_j)(\partial_\nu \phi_k \partial^\nu \phi_l)$$
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✦ Assuming a SO(2) symmetry. Write in terms of complex field

$$\phi = \phi_1 + i\phi_2$$

$$O'_1 = |\partial_\mu \phi \partial^\mu \phi|^2$$

$$O'_2 = \left|\partial_\mu \phi^\dagger \partial^\mu \phi\right|^2$$

	$\phi_1\phi_1$	$\phi_2\phi_2$	$\phi_1\phi_2$	$\phi_2\phi_1$
$\phi_1\phi_1$	$4(C'_1+C'_2)$	$2C'_2$	0	0
$\phi_2\phi_2$	$2C'_2$	$4(C'_1+C'_2)$	0	0
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✦ To enumerate the generators: SO(2) fixes the ***m*** matrices in

$$\mathcal{G}_X^{ijkl} \equiv m_X^{ij} m_X^{*kl} + m_X^{i\bar{l}} m_X^{*k\bar{j}}$$

$$\mathbf{2} \otimes \mathbf{2} = \mathbf{1}_S \oplus \mathbf{1}_A \oplus \mathbf{2}$$

$\mathbf{1}_S$: xx+yy $\quad$ $\mathbf{1}_A$: xy-yx

2 : (xx-yy, xy+yx)

Example 1: two scalars with SO(2)

✦ ***m*** matrices: the CG coefficients

$$m_{\mathbf{1_S}} = \begin{array}{c|c|c|} & \phi_1 & \phi_2 \\ \hline \phi_1 & 1 & 0 \\ \hline \phi_2 & 0 & 1 \\ \hline \end{array}, \qquad m_{\mathbf{1_A}} = \begin{array}{c|c|c|} & \phi_1 & \phi_2 \\ \hline \phi_1 & 0 & 1 \\ \hline \phi_2 & -1 & 0 \\ \hline \end{array}$$

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✦ Generators: the projective operators, with (j,l) symmetrized

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$$P_{\mathbf{1}_A}^{i(j|k|l)} = \frac{1}{2}\left(\delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk}\right)$$

$$P_{\mathbf{2}}^{i(j|k|l)} = \frac{1}{2}\left(\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk} - \delta^{ij}\delta^{kl}\right)$$

$$G_{\mathbf{1}_S}^{ijkl} = \begin{array}{|c|c|c|c|} \hline 2 & 1 & 0 & 0 \\ \hline 1 & 2 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 1 & 0 \\ \hline \end{array} \propto P_{\mathbf{1}_S}^{i(j|k|l)} \qquad G_{\mathbf{2}}^{ijkl} = \begin{array}{|c|c|c|c|} \hline 1 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \\ \hline \end{array} \propto P_{\mathbf{2}}^{i(j|k|l)}$$

$$G_{\mathbf{1}_A}^{ijkl} = \begin{array}{|c|c|c|c|} \hline 0 & -1 & 0 & 0 \\ \hline -1 & 0 & 0 & 0 \\ \hline 0 & 0 & 2 & -1 \\ \hline 0 & 0 & -1 & 2 \\ \hline \end{array} \propto P_{\mathbf{1}_A}^{i(j|k|l)}$$

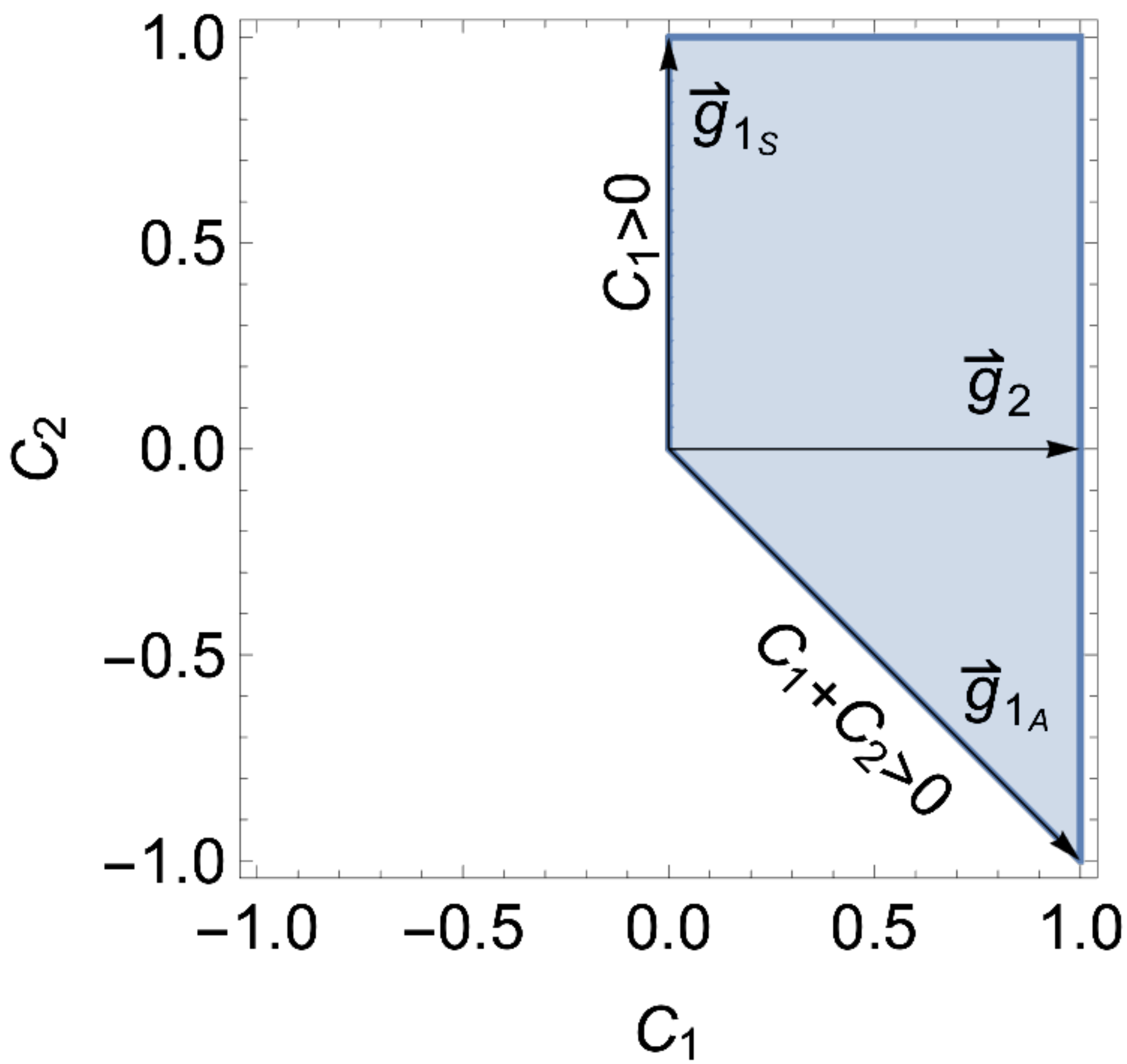
✦ Compare with the amplitude:

	$\phi_1\phi_1$	$\phi_2\phi_2$	$\phi_1\phi_2$	$\phi_2\phi_1$
$\phi_1\phi_1$	$4(C_1'+C_2')$	$2C_2'$	0	0
$\phi_2\phi_2$	$2C_2'$	$4(C_1'+C_2')$	0	0
$\phi_2\phi_1$	0	0	$4C_1'$	$2C_2'$
$\phi_1\phi_2$	0	0	$2C_2'$	$4C_1'$

$$\vec{g} = (C_1', C_2')$$

$$\vec{g}_{\mathbf{1}S} = (0,1), \quad \vec{g}_{\mathbf{1}A} = (1,-1), \quad \vec{g}_{\mathbf{2}} = (1,0).$$

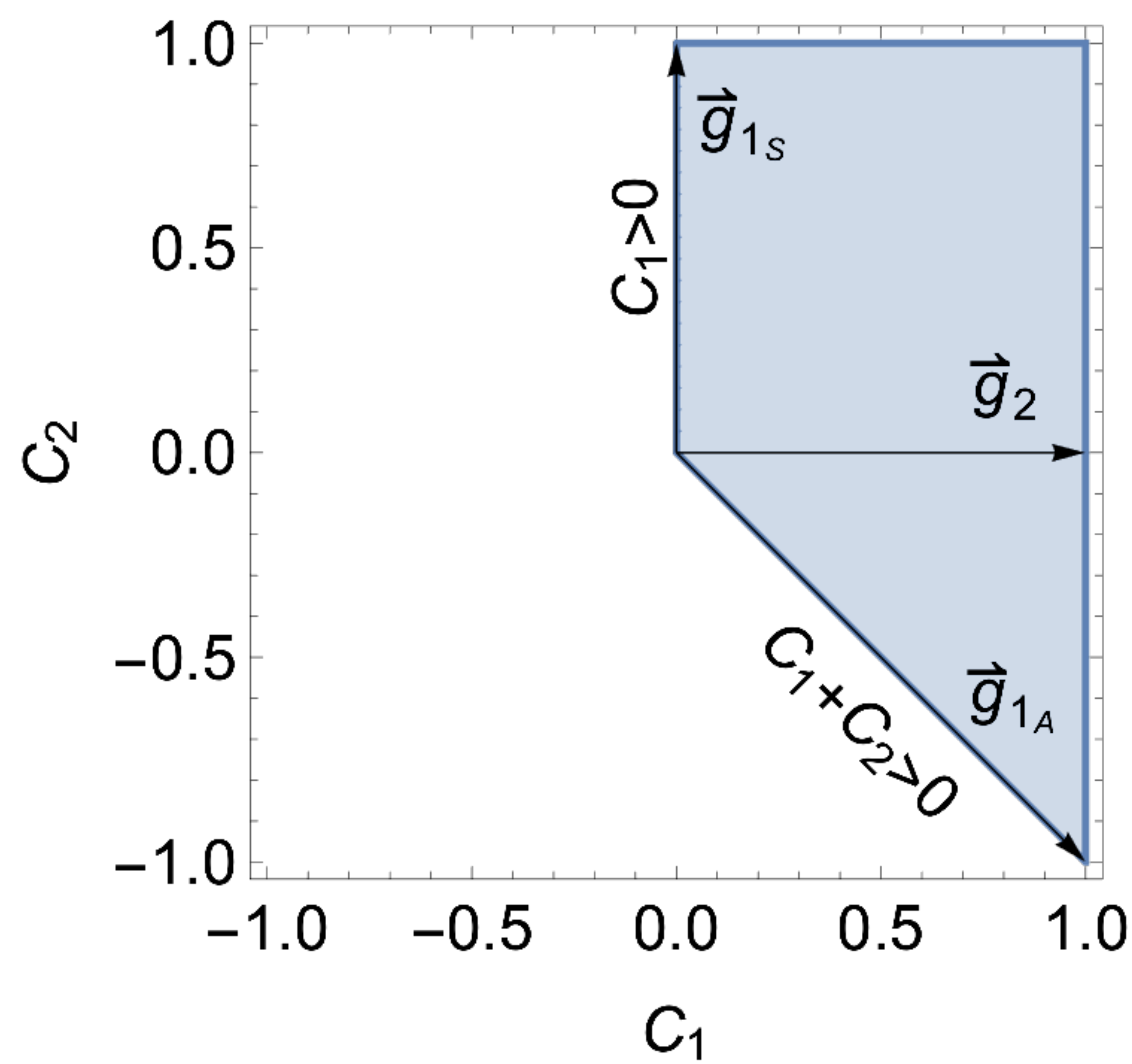
Example 1: two scalars with SO(2)



$$O_1' = |\partial_\mu \phi \partial^\mu \phi|^2$$

$$O_2' = \left|\partial_\mu \phi^\dagger \partial^\mu \phi\right|^2$$

Example 1: two scalars with SO(2)



$$O'_1 = |\partial_\mu \phi \partial^\mu \phi|^2$$

$$O'_2 = \left|\partial_\mu \phi^\dagger \partial^\mu \phi\right|^2$$

- Bounds are

$$C'_1 \geq 0, \quad C'_1 + C'_2 \geq 0$$

- Same bound from conventional approach based on elastic scattering

	$\phi_1\phi_1$	$\phi_2\phi_2$	$\phi_1\phi_2$	$\phi_2\phi_1$
$\phi_1\phi_1$	$4(C'_1+C'_2)$	$2C'_2$	0	0
$\phi_2\phi_2$	$2C'_2$	$4(C'_1+C'_2)$	0	0
$\phi_2\phi_1$	0	0	$4C'_1$	$2C'_2$
$\phi_1\phi_2$	0	0	$2C'_2$	$4C'_1$

- Generators correspond to 1-particle UVs

State	Spin	Charge	$\mathcal{L}_{\text{int}}$	ER	$\vec{c}/\frac{g^2}{M^4}$
S_1	0	0	$S_1\phi^\dagger\phi$	✓	$(0,2)$
V	1	0	$V^\mu\phi^\dagger i\overleftrightarrow{D}_\mu\phi$	✓	$(-2,-2)$
S_2	0	2	$S_2^\dagger\phi^2$	✗	$(4,0)$

Example 2: two scalars with discrete symmetries

✦ Two scalars with discrete symmetries

$$\phi_1 \to -\phi_1 \text{ and } \phi_1 \leftrightarrow \phi_2$$

$$O_{ijkl} = (\partial_\mu \phi_i \partial^\mu \phi_j)(\partial_\mu \phi_k \partial^\mu \phi_l)$$

$$\mathcal{L} = \frac{1}{\Lambda^4}\left[C_1(O_{1111} + O_{2222}) + C_2 O_{1122} + C_4 O_{1212}\right]$$

✦ ***m*** matrices are fixed by the parities under $\phi_1 \to -\phi_1$ and $\phi_1 \leftrightarrow \phi_2$

$$m_{++} = \begin{array}{|c|c|}\hline 1 & 0 \\ \hline 0 & 1 \\ \hline\end{array}, \quad m_{+-} = \begin{array}{|c|c|}\hline 1 & 0 \\ \hline 0 & -1 \\ \hline\end{array},$$

$$m_{-+} = \begin{array}{|c|c|}\hline 0 & 1 \\ \hline 1 & 0 \\ \hline\end{array}, \quad m_{--} = \begin{array}{|c|c|}\hline 0 & 1 \\ \hline -1 & 0 \\ \hline\end{array}$$

Previously we had

$$m_{\mathbf{1_S}} = \begin{array}{c|c|c|}\multicolumn{1}{c}{} & \multicolumn{1}{c}{\phi_1} & \multicolumn{1}{c}{\phi_2} \\ \cline{2-3} \phi_1 & 1 & 0 \\ \cline{2-3} \phi_2 & 0 & 1 \\ \cline{2-3}\end{array}, \quad m_{\mathbf{1_A}} = \begin{array}{c|c|c|}\multicolumn{1}{c}{} & \multicolumn{1}{c}{\phi_1} & \multicolumn{1}{c}{\phi_2} \\ \cline{2-3} \phi_1 & 0 & 1 \\ \cline{2-3} \phi_2 & -1 & 0 \\ \cline{2-3}\end{array}$$

$$m_{\mathbf{2}}^{\alpha} = \left(\begin{array}{c|c|c|}\multicolumn{1}{c}{} & \multicolumn{1}{c}{\phi_1} & \multicolumn{1}{c}{\phi_2} \\ \cline{2-3} \phi_1 & 1 & 0 \\ \cline{2-3} \phi_2 & 0 & -1 \\ \cline{2-3}\end{array}, \begin{array}{c|c|c|}\multicolumn{1}{c}{} & \multicolumn{1}{c}{\phi_1} & \multicolumn{1}{c}{\phi_2} \\ \cline{2-3} \phi_1 & 0 & 1 \\ \cline{2-3} \phi_2 & 1 & 0 \\ \cline{2-3}\end{array}\right)$$

✦ Generators

$$G^{ijkl}_{+\pm} = \begin{array}{|c|c|c|c|}\hline 2 & \pm1 & 0 & 0 \\ \hline \pm1 & 2 & 0 & 0 \\ \hline 0 & 0 & 0 & \pm1 \\ \hline 0 & 0 & \pm1 & 0 \\ \hline\end{array}, \qquad G^{ijkl}_{-\pm} = \begin{array}{|c|c|c|c|}\hline 0 & \pm1 & 0 & 0 \\ \hline \pm1 & 0 & 0 & 0 \\ \hline 0 & 0 & 2 & \pm1 \\ \hline 0 & 0 & \pm1 & 2 \\ \hline\end{array}$$

✦ Compare with amplitude:

$$\mathcal{M}^{ijkl} = \begin{array}{c|c|c|c|c|}\multicolumn{1}{c}{} & \multicolumn{1}{c}{\phi_1\phi_1} & \multicolumn{1}{c}{\phi_2\phi_2} & \multicolumn{1}{c}{\phi_1\phi_2} & \multicolumn{1}{c}{\phi_2\phi_1} \\ \cline{2-5} \phi_1\phi_1 & 4C_1 & \bar{C}_2 & C_5 & C_5 \\ \cline{2-5} \phi_2\phi_2 & \bar{C}_2 & 4C_3 & C_6 & C_6 \\ \cline{2-5} \phi_2\phi_1 & C_5 & C_6 & C_4 & \bar{C}_2 \\ \cline{2-5} \phi_1\phi_2 & C_5 & C_6 & \bar{C}_2 & C_4 \\ \cline{2-5}\end{array}$$

$$\bar{C}_2 \equiv C_2 + \tfrac{1}{2}C_4$$
$$C_1 = C_3, \; C_5 = C_6 = 0$$

Example 2: two scalars with discrete symmetries

✦ Two scalars with discrete symmetries

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Previously we had

$$m_{1_S} = \begin{array}{c|c|c|}& \phi_1 & \phi_2 \\ \hline \phi_1 & 1 & 0 \\ \hline \phi_2 & 0 & 1 \\ \hline\end{array}, \quad m_{1_A} = \begin{array}{c|c|c|}& \phi_1 & \phi_2 \\ \hline \phi_1 & 0 & 1 \\ \hline \phi_2 & -1 & 0 \\ \hline\end{array}$$

$$m_2^\alpha = \left(\begin{array}{c|c|c|}& \phi_1 & \phi_2 \\ \hline \phi_1 & 1 & 0 \\ \hline \phi_2 & 0 & -1 \\ \hline\end{array}, \begin{array}{c|c|c|}& \phi_1 & \phi_2 \\ \hline \phi_1 & 0 & 1 \\ \hline \phi_2 & 1 & 0 \\ \hline\end{array} \right)$$

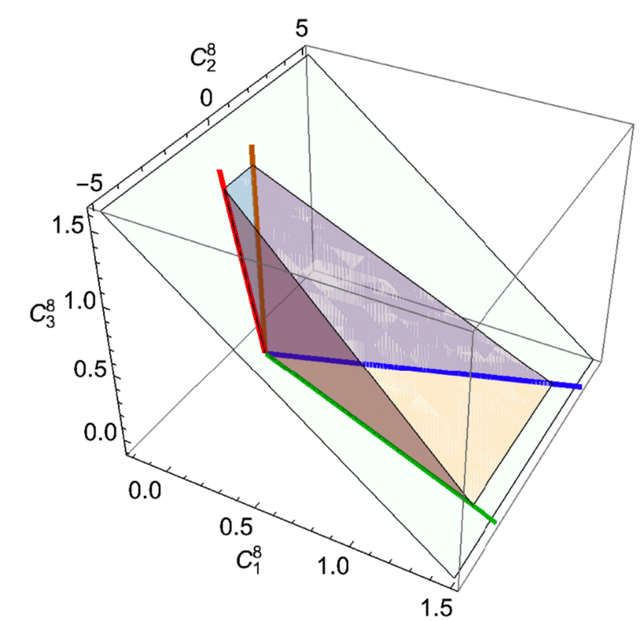
✦ Generators

$$G^{ijkl}_{+\pm} = \begin{array}{|c|c|c|c|}\hline 2 & \pm1 & 0 & 0 \\ \hline \pm1 & 2 & 0 & 0 \\ \hline 0 & 0 & 0 & \pm1 \\ \hline 0 & 0 & \pm1 & 0 \\ \hline\end{array}, \quad G^{ijkl}_{-\pm} = \begin{array}{|c|c|c|c|}\hline 0 & \pm1 & 0 & 0 \\ \hline \pm1 & 0 & 0 & 0 \\ \hline 0 & 0 & 2 & \pm1 \\ \hline 0 & 0 & \pm1 & 2 \\ \hline\end{array}$$

✦ Compare with amplitude:

$$\vec{g} = (C_1, C_2, C_3)$$

$$\vec{g}_{++} = (1, 2, 0)$$
$$\vec{g}_{+-} = (1, -2, 0)$$
$$\vec{g}_{-+} = (0, 0, 4)$$
$$\vec{g}_{--} = (0, -4, 4)$$



Example 3: two scalars with infinitely many ERs

- Remove $\phi_1 \leftrightarrow \phi_2$, keep $\phi_1 \to -\phi_1$

$$O_{ijkl} = (\partial_\mu \phi_i \partial^\mu \phi_j)(\partial_\mu \phi_k \partial^\mu \phi_l)$$

$$\mathcal{L} = \frac{1}{\Lambda^4}\left[C_1 O_{1111} + C_2 O_{1122} + C_3 O_{2222} + C_4 O_{1212}\right]$$

- ***m*** matrices are fixed by the parity and exchange symmetry (i ↔ j)

(Previously) $m_{++} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad m_{+-} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$

↓

$$m_{+} = \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix},$$

$$m_{-S} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad m_{-A} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

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✦ Generators

$$G_{+}^{ijkl} = \begin{pmatrix} 2x^2 & xy & 0 & 0 \\ xy & 2y^2 & 0 & 0 \\ 0 & 0 & 0 & xy \\ 0 & 0 & xy & 0 \end{pmatrix},$$

$$G_{-S}^{ijkl} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}, \quad G_{-A}^{ijkl} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

↓

$$\vec{g}_{+}(x,y) = (x^2, 2xy, y^2, 0),$$

$$\vec{g}_{-S} = (0,0,0,4), \quad \vec{g}_{-A} = (0,-4,0,4).$$

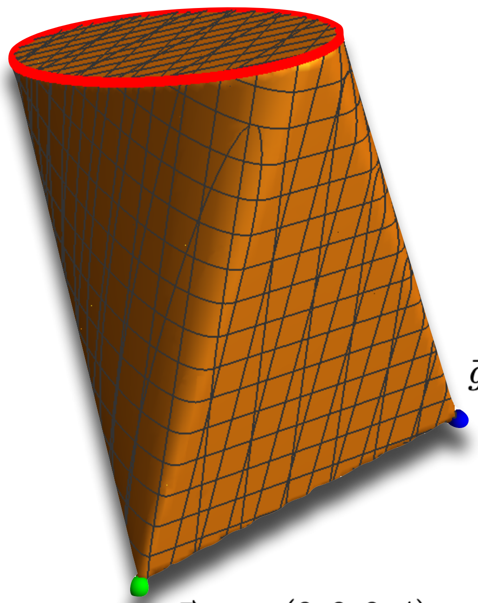
✦ x,y are free real parameters.

Example 3: two scalars with infinitely many ERs

- Bounds ("continuous vertex enumeration")

$$C_1 \geq 0, \quad C_3 \geq 0, \quad C_4 \geq 0$$
$$2\sqrt{C_1 C_3} \geq C_2, \quad 2\sqrt{C_1 C_3} \geq -(C_2 + C_4)$$

3D "cross section" of the 4D cone



- UV states

Particle	spin	Parity($\phi_1 \to -\phi_1$)	Interaction	ER	$\vec{c}$
S_1	0	$+$	$g_1 M_1 (x\phi_1^2 + y\phi_2^2) S_1$	✓	$2 \times (x^2, 2xy, y^2, 0)$
S_3	0	$-$	$g_3 M_3 \phi_1 \phi_2 S_3$	✓	$2 \times (0,0,0,1)$
V_4	1	$-$	$g_4(\phi_1 \overleftrightarrow{D}_\mu \phi_2) V_4^\mu$	✓	$2 \times (0,-1,0,1)$

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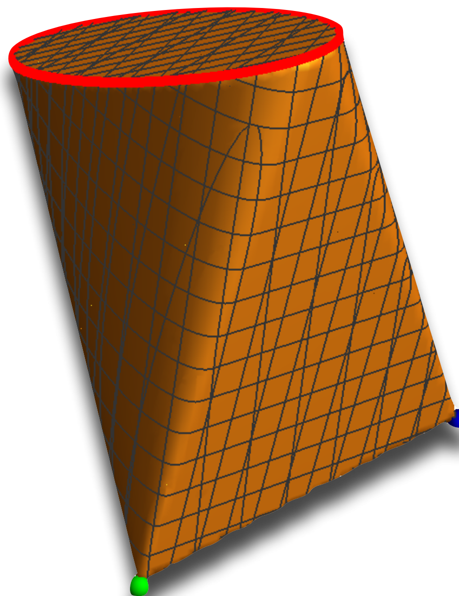


3D "cross section" of the 4D cone

- UV states

Particle	spin	Parity($\phi_1 \to -\phi_1$)	Interaction	ER	$\vec{c}$
S_1	0	+	$g_1 M_1 (x\phi_1^2 + y\phi_2^2) S_1$	✓	$2 \times (x^2, 2xy, y^2, 0)$
S_3	0	−	$g_3 M_3 \phi_1 \phi_2 S_3$	✓	$2 \times (0, 0, 0, 1)$
V_4	1	−	$g_4 (\phi_1 \overleftrightarrow{D}_\mu \phi_2) V_4^\mu$	✓	$2 \times (0, -1, 0, 1)$

- What if we remove all symmetries?

$$\vec{g}_S(x,y,z) = (x^2, 2xy, y^2, 4z^2, 4xz, 4yz), \quad \vec{g}_A = (0, -1, 0, 1, 0, 0)$$

Hard with generator approach. But may resort to the SDP approach. [X. Li et al., 2101.01191]

Example 4: SM Higgs

- ✦ Operators

[C. Murphy, 2005.00059]

$\mathbf{4}:\boldsymbol{H^4 D^4}$	
$Q^{(1)}_{H^4}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$Q^{(2)}_{H^4}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$Q^{(3)}_{H^4}$	$(D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$

- ✦ Coefficients

C_1

C_2

C_3

- ✦ Amplitude in terms complex fields

$$\mathcal{M}^{ijkl} = \begin{array}{c|c|c|c|c|} & h_c h_d & \bar{h}^c \bar{h}^d & h_c \bar{h}^d & \bar{h}^c h_d \\ \hline h^a h^b & \mathcal{M}(hh \to hh)^{ab}{}_{cd} & & & \\ \hline \bar{h}_a \bar{h}_b & & \mathcal{M}(\bar{h}\bar{h} \to \bar{h}\bar{h})_{ab}{}^{cd} & & \\ \hline h^a \bar{h}_b & & & \mathcal{M}(h\bar{h} \to h\bar{h})^{a}{}_{bc}{}^{d} & \mathcal{M}(h\bar{h} \to \bar{h}h)^{a}{}_{b}{}^{c}{}_{d} \\ \hline \bar{h}_a h^b & & & \mathcal{M}(\bar{h}h \to h\bar{h})_{a}{}^{b}{}_{c}{}^{d} & \mathcal{M}(\bar{h}h \to \bar{h}h)_{a}{}^{bc}{}_{d} \\ \hline \end{array}$$

$$\mathcal{M}(hh \to hh)^{ab}{}_{cd} = \frac{1}{2}\left[(C_2 + C_3)\,\delta^a_d \delta^b_c + (C_1 + C_2)\,\delta^a_c \delta^b_d\right]$$

$$\mathcal{M}(\bar{h}\bar{h} \to \bar{h}\bar{h})_{ab}{}^{cd} = \frac{1}{2}\left[(C_2 + C_3)\,\delta^d_a \delta^c_b + (C_1 + C_2)\,\delta^c_a \delta^d_b\right]$$

$$\mathcal{M}(h\bar{h} \to h\bar{h})^{a}{}_{bc}{}^{d} = \frac{1}{2}\left[(C_1 + C_2)\,\delta^a_c \delta^d_b + (C_2 + C_3)\,\delta^a_b \delta^d_c\right]$$

$$\mathcal{M}(\bar{h}h \to \bar{h}h)_{a}{}^{bc}{}_{d} = \frac{1}{2}\left[(C_1 + C_2)\,\delta^c_a \delta^b_d + (C_2 + C_3)\,\delta^b_a \delta^c_d\right]$$

$$\mathcal{M}(h\bar{h} \to \bar{h}h)^{a}{}_{b}{}^{c}{}_{d} = \frac{1}{2}(C_1 + C_3)\,(\delta^a_d \delta^c_b + \delta^a_b \delta^c_d)$$

$$\mathcal{M}(\bar{h}h \to h\bar{h})_{a}{}^{b}{}_{c}{}^{d} = \frac{1}{2}(C_1 + C_3)\left(\delta^d_a \delta^b_c + \delta^b_a \delta^d_c\right)$$

Example 4: SM Higgs

- Intermediate states couple to $hh, \bar{h}\bar{h}, h\bar{h}, \bar{h}h$: $\mathbf{1}_1, \mathbf{3}_1, \mathbf{1}_{0S}, \mathbf{1}_{0A}, \mathbf{3}_{0S}, \mathbf{3}_{0A}$

- Generators

$$m_1 = \begin{array}{c|c|c|} & h^b & \bar{h}_b \\ \hline h^a & \epsilon^{ab} & 0 \\ \hline \bar{h}_a & 0 & 0 \\ \hline \end{array}, \quad m_3^I = \begin{array}{c|c|c|} & h^b & \bar{h}_b \\ \hline h^a & [\epsilon\tau^I]^{ab} & 0 \\ \hline \bar{h}_a & 0 & 0 \\ \hline \end{array} \qquad m_{1[S,A]} = \begin{array}{c|c|c|} & h^b & \bar{h}_b \\ \hline h^a & 0 & \delta^a_b \\ \hline \bar{h}_a & [\pm]\delta^b_a & 0 \\ \hline \end{array}, \quad m^I_{3[S,A]} = \begin{array}{c|c|c|} & h^b & \bar{h}_b \\ \hline h^a & 0 & {\tau^I}^a{}_b \\ \hline \bar{h}_a & [\pm]{\tau^I}^b{}_a & 0 \\ \hline \end{array}$$

$\mathcal{G}^{ijkl}_{1,3}$

${P_{1,3}}^{ab}{}_{cd}$			
	${P_{1,3}}^{cd}{}_{ab}$		
		${P_{1,3}}^{ad}{}_{cb}$	
			${P_{1,3}}^{cb}{}_{ad}$

$\mathcal{G}^{ijkl}_{1,3[S,A]}$

${P_{1,3}}^a{}_{dc}{}^b$			
	${P_{1,3}}^c{}_{ba}{}^d$		
		${P_{1,3}}^a{}_{bc}{}^d$	$\pm {P_{1,3}}^a{}_{bd}{}^c \pm {P_{1,3}}^a{}_{db}{}^c$
		$\pm {P_{1,3}}^b{}_{ac}{}^d \pm {P_{1,3}}^b{}_{ca}{}^d$	${P_{1,3}}^c{}_{da}{}^b$

$$\vec{g} = (C_1, C_2, C_3)$$

$\vec{g}_1 = (1, 0, -1)$ $\quad \vec{g}_{1S} = (0, 0, 2)$ $\quad \vec{g}_{3S} = (4, 0, -2)$

$\vec{g}_3 = (0, 1, 0)$ $\quad \vec{g}_{1A} = (-2, 2, 0)$ $\quad \vec{g}_{3A} = (2, 2, -4)$

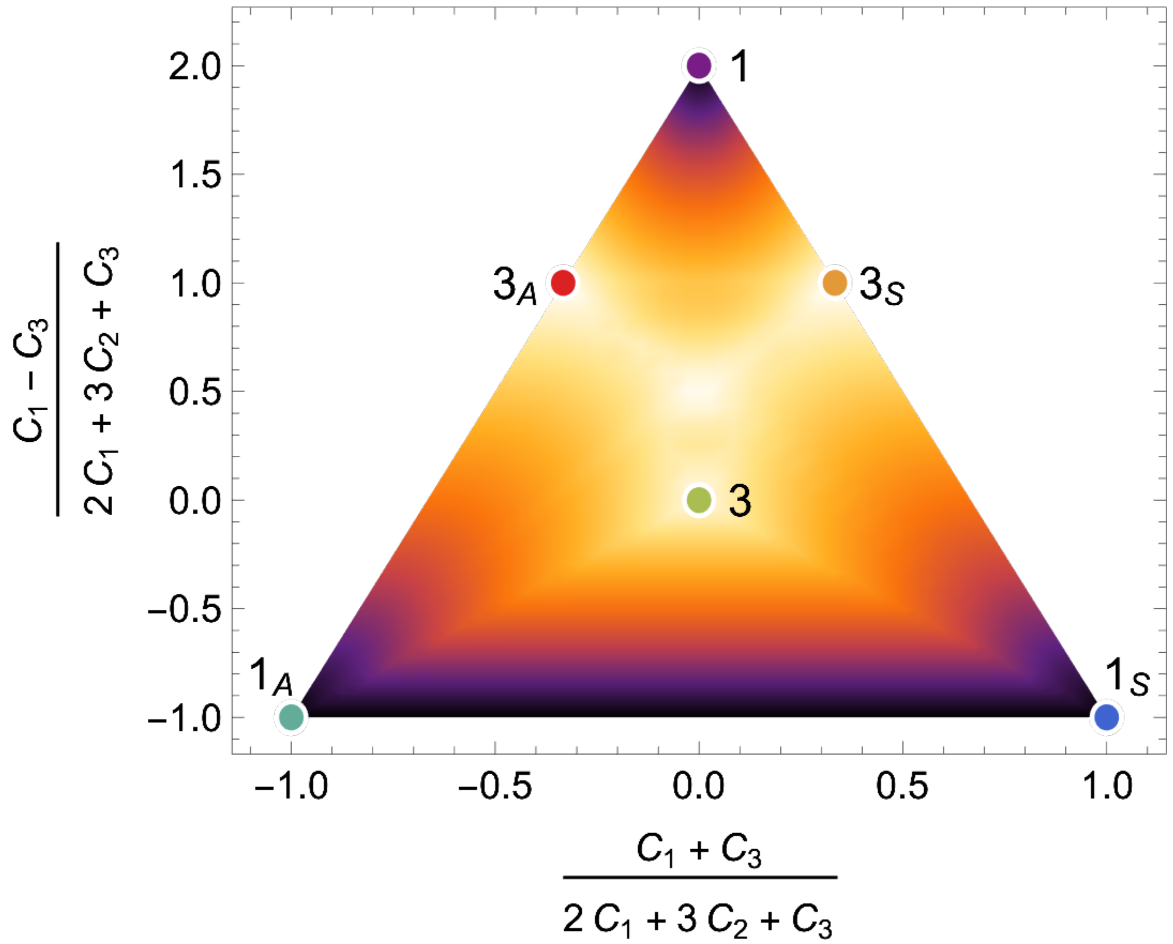
Example 4: SM Higgs

- 1, 1S, 1A are extremal. Triangular cone.

$\vec{g}_1 = (1, 0, -1)$ $\quad \vec{g}_{1S} = (0, 0, 2)$ $\quad \vec{g}_{3S} = (4, 0, -2)$

$\vec{g}_3 = (0, 1, 0)$ $\quad \vec{g}_{1A} = (-2, 2, 0)$ $\quad \vec{g}_{3A} = (2, 2, -4)$

- Take the cross section of triangular cone



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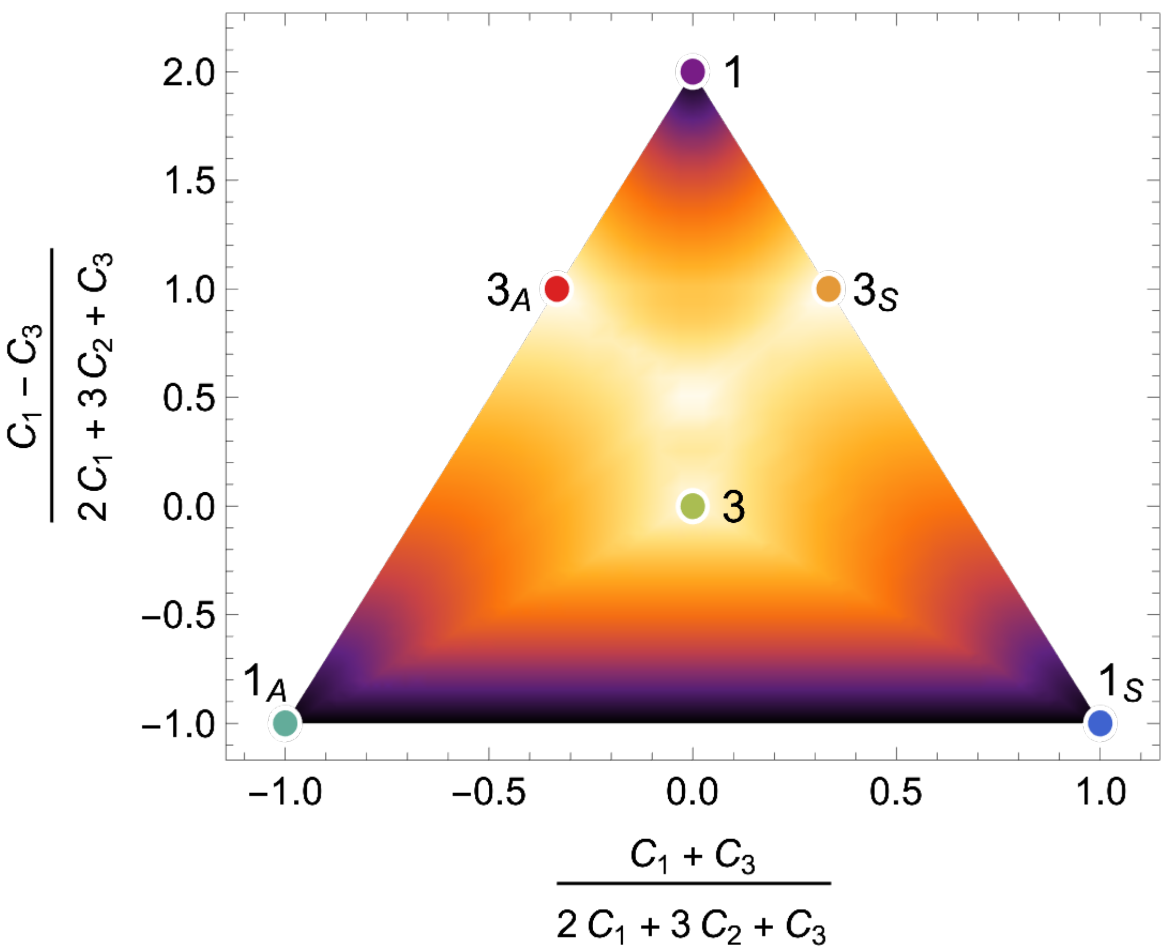
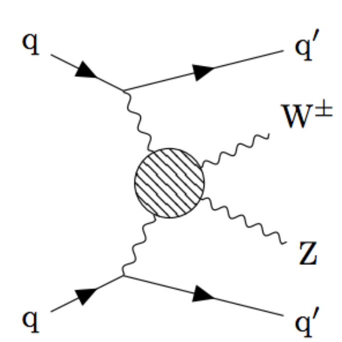
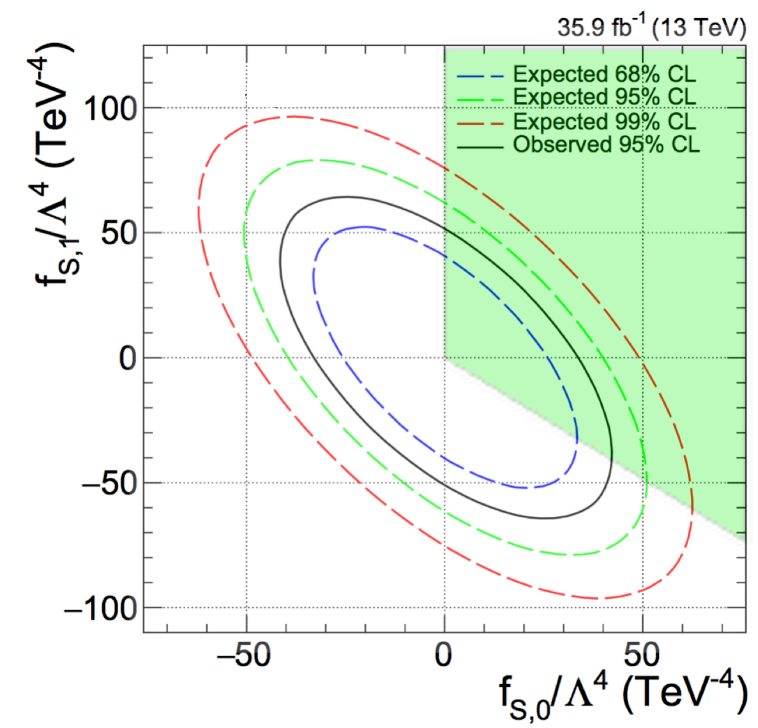


- Bounds on "aQGC"



$$\mathcal{O}_{S,0} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi\right] \times \left[(D^\mu\Phi)^\dagger D^\nu\Phi\right]$$
$$\mathcal{O}_{S,1} = \left[(D_\mu\Phi)^\dagger D^\mu\Phi\right] \times \left[(D_\nu\Phi)^\dagger D^\nu\Phi\right]$$
$$\mathcal{O}_{S,2} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi\right] \times \left[(D^\nu\Phi)^\dagger D^\mu\Phi\right]$$



WZjj (CMS-PAS-SMP-18-001)

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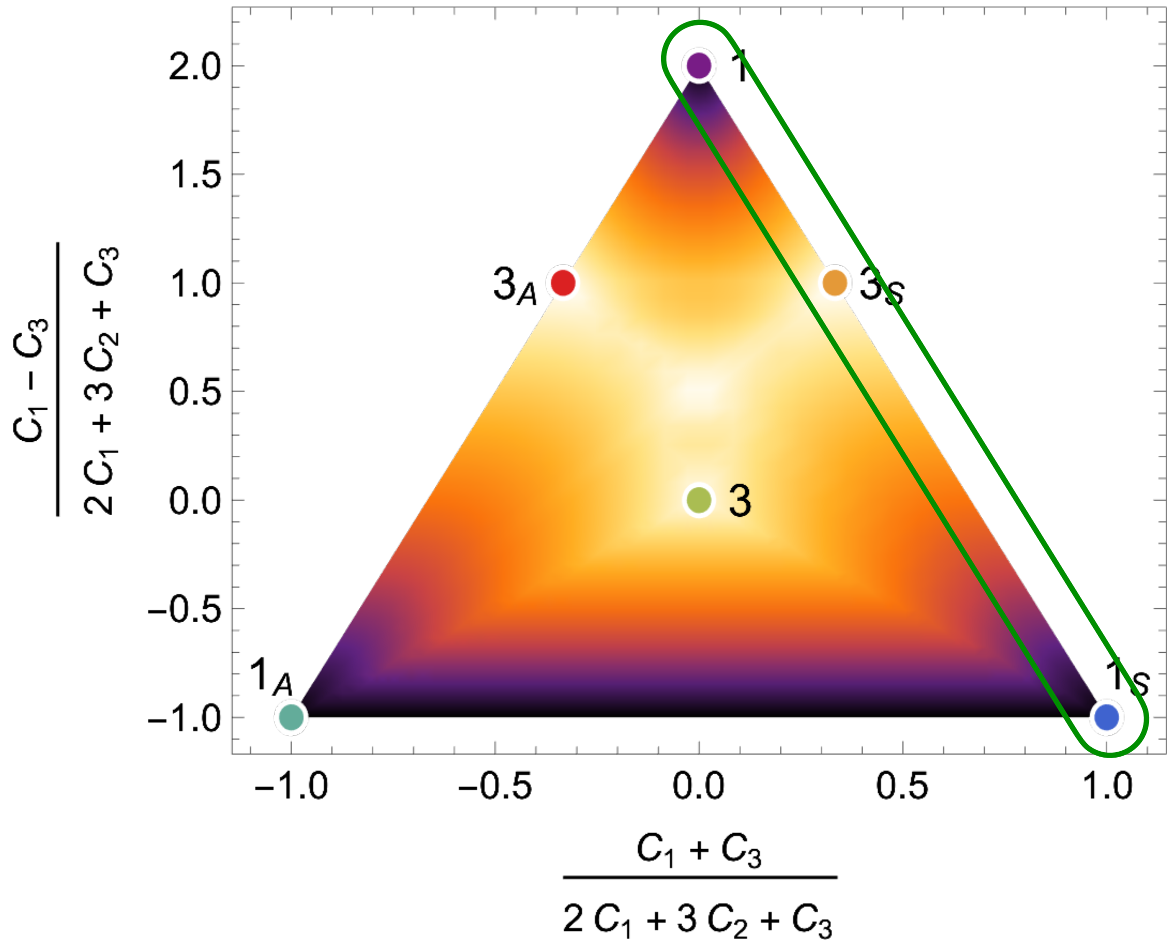
- 1, 1S, 1A are extremal. Triangular cone.

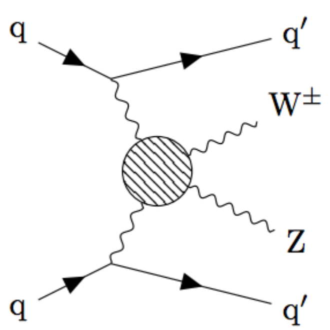
$\vec{g}_1 = (1,0,-1)$ $\vec{g}_{1S} = (0,0,2)$ $\vec{g}_{3S} = (4,0,-2)$

$\vec{g}_3 = (0,1,0)$ $\vec{g}_{1A} = (-2,2,0)$ $\vec{g}_{3A} = (2,2,-4)$

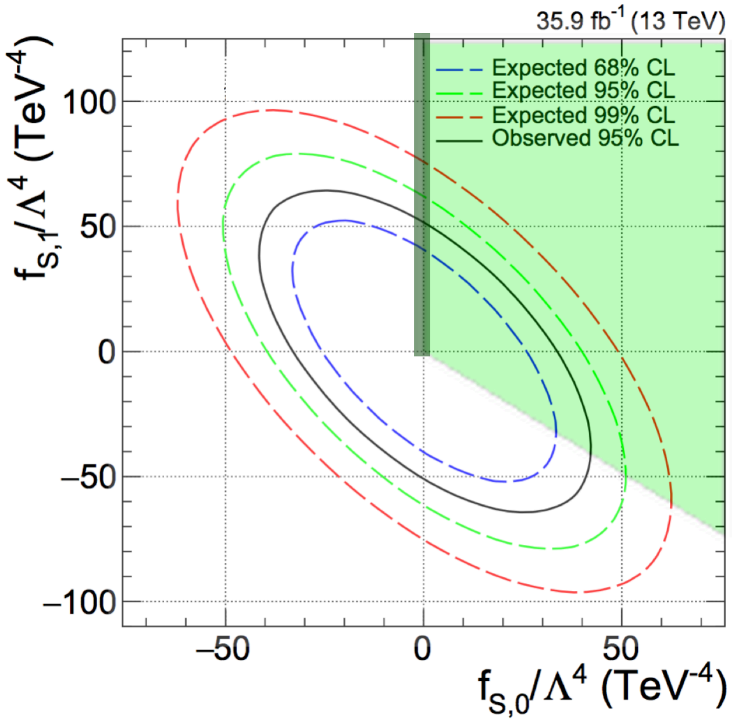
- Take the cross section of triangular cone



- Bounds on "aQGC"



$$\mathcal{O}_{S,0} = [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\mu\Phi)^\dagger D^\nu\Phi]$$
$$\mathcal{O}_{S,1} = [(D_\mu\Phi)^\dagger D^\mu\Phi] \times [(D_\nu\Phi)^\dagger D^\nu\Phi]$$
$$\mathcal{O}_{S,2} = [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\nu\Phi)^\dagger D^\mu\Phi]$$



WZjj (CMS-PAS-SMP-18-001)

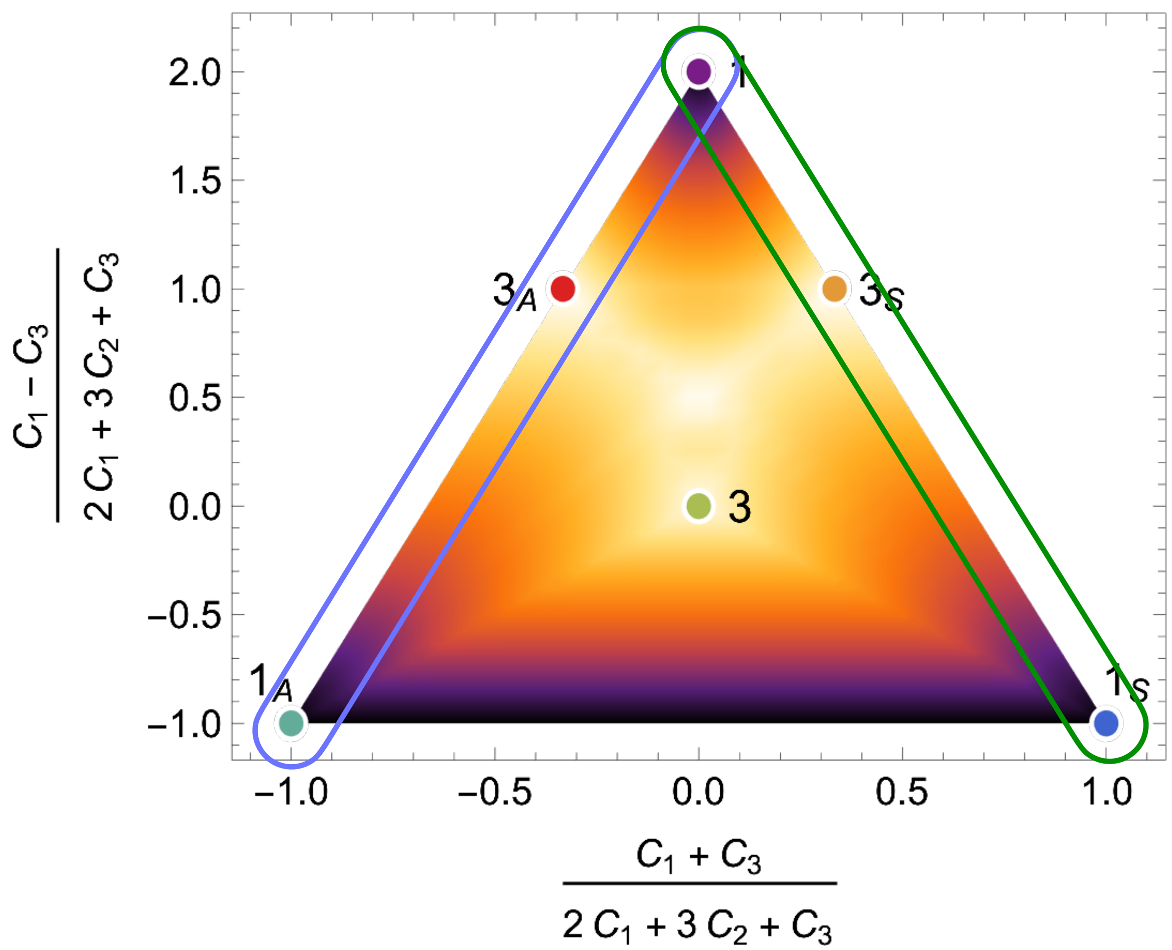
Example 4: SM Higgs

- 1, 1S, 1A are extremal. Triangular cone.

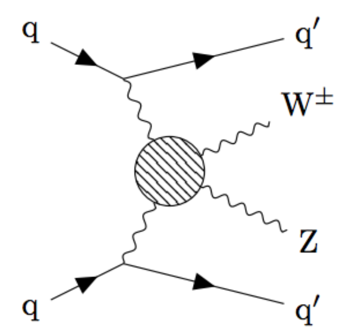
$\vec{g}_1 = (1,0,-1)$ $\vec{g}_{1S} = (0,0,2)$ $\vec{g}_{3S} = (4,0,-2)$

$\vec{g}_3 = (0,1,0)$ $\vec{g}_{1A} = (-2,2,0)$ $\vec{g}_{3A} = (2,2,-4)$

- Take the cross section of triangular cone



- Bounds on “aQGC”



$$\mathcal{O}_{S,0} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi\right] \times \left[(D^\mu\Phi)^\dagger D^\nu\Phi\right]$$
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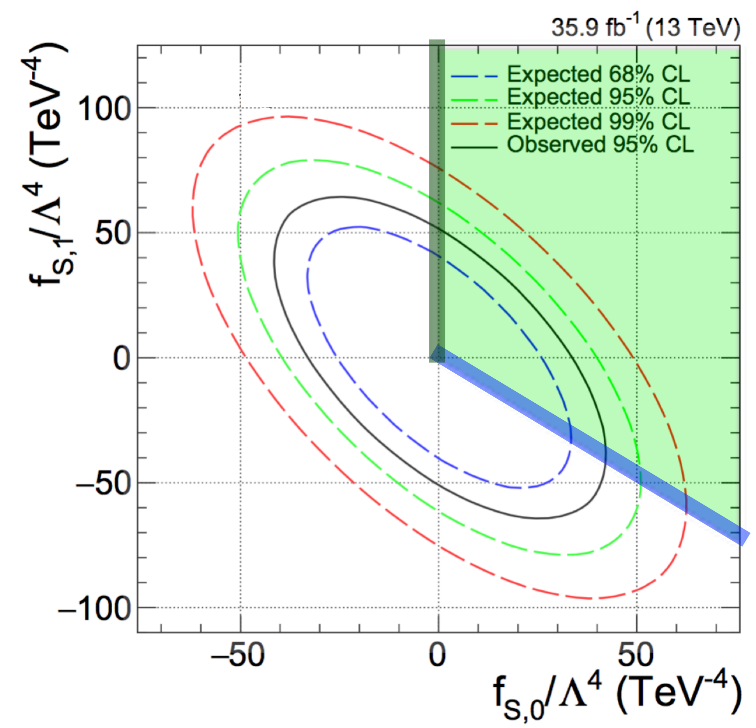

WZjj (CMS-PAS-SMP-18-001)

Example 5: photon

✦ Operators

[C. Murphy, 2005.00059]

$Q_{B^4}^{(1)}$	$C_1\ (B_{\mu\nu}B^{\mu\nu})(B_{\rho\sigma}B^{\rho\sigma})$
$Q_{B^4}^{(2)}$	$C_2\ (B_{\mu\nu}\widetilde{B}^{\mu\nu})(B_{\rho\sigma}\widetilde{B}^{\rho\sigma})$
$Q_{B^4}^{(3)}$	$C_3\ (B_{\mu\nu}B^{\mu\nu})(B_{\rho\sigma}\widetilde{B}^{\rho\sigma})$ Parity violating

✦ Amplitude

$\mathcal{M}^{ijkl} = 8\times$

	B_xB_x	B_yB_y	B_xB_y	B_yB_x
B_xB_x	$2C_1$	C_1-C_2	C_3	$-C_3$
B_yB_y	C_1-C_2	$2C_1$	C_3	$-C_3$
B_yB_x	C_3	C_3	$2C_2$	C_1-C_2
B_xB_y	$-C_3$	$-C_3$	C_1-C_2	$2C_2$

✦ Very similar to 2 scalars with SO(2).

✦ Difference: (i,j) exchange in $\boldsymbol{m}^{ij}$ was a symmetry (either symmetric or antisymmetric) in the scalar case, but here it corresponds to **parity**.

Example 5: photon

- ***m*** matrices: same as 2-scalar EFT

$$m_{\mathbf{1_S}} = \begin{array}{c|c|c|} & \phi_1 & \phi_2 \\ \hline \phi_1 & 1 & 0 \\ \hline \phi_2 & 0 & 1 \\ \hline \end{array}, \qquad m_{\mathbf{1_A}} = \begin{array}{c|c|c|} & \phi_1 & \phi_2 \\ \hline \phi_1 & 0 & 1 \\ \hline \phi_2 & -1 & 0 \\ \hline \end{array}$$

$$m_{\mathbf{2}}^{\alpha} = \left(\begin{array}{c|c|c|} & \phi_1 & \phi_2 \\ \hline \phi_1 & 1 & 0 \\ \hline \phi_2 & 0 & -1 \\ \hline \end{array}, \begin{array}{c|c|c|} & \phi_1 & \phi_2 \\ \hline \phi_1 & 0 & 1 \\ \hline \phi_2 & 1 & 0 \\ \hline \end{array} \right)$$

With $\phi_1 \to B_x, \ \phi_2 \to B_y$

- Generators: SO(2) projectors

$$P_{\mathbf{1}_S}^{i(j|k|l)} = \frac{1}{2}\delta^{ij}\delta^{kl}$$

$$P_{\mathbf{1}_A}^{i(j|k|l)} = \frac{1}{2}\left(\delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk}\right)$$

$$P_{\mathbf{2}}^{i(j|k|l)} = \frac{1}{2}\left(\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk} - \delta^{ij}\delta^{kl}\right)$$

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✦ If parity is violated, the first 2 mix

1	r
$-r$	1

i.e. infinitely many ERs

✦ Generator vectors

$$\vec{g}_1 = (1, r^2, 2r), \ \vec{g}_2 = (1, 1, 0)$$

Particle	Spin	Parity	Interaction	ER	$\vec{c}$
S_+	0	+	$\frac{g_1}{M_1} S_1 \left(B_{\mu\nu}B^{\mu\nu}\right)$	✓	$\frac{1}{2}(1,0,0)$
S_-	0	−	$\frac{g_2}{M_2} S_2 \left(B_{\mu\nu}\tilde{B}^{\mu\nu}\right)$	✓	$\frac{1}{2}(0,1,0)$
S_{mix}	0	?	$\frac{g_3}{M_3} S_3 \left(B_{\mu\nu}B^{\mu\nu} + r \times B_{\mu\nu}\tilde{B}^{\mu\nu}\right)$	✓	$\frac{1}{2}(1, r^2, 2r)$

Example 5: photon

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$$m_{\mathbf{1_S}} = \begin{array}{c|c|c|} & \phi_1 & \phi_2 \\ \hline \phi_1 & 1 & 0 \\ \hline \phi_2 & 0 & 1 \\ \hline \end{array}, \qquad m_{\mathbf{1_A}} = \begin{array}{c|c|c|} & \phi_1 & \phi_2 \\ \hline \phi_1 & 0 & 1 \\ \hline \phi_2 & -1 & 0 \\ \hline \end{array}$$

$$m_{\mathbf{2}}^{\alpha} = \left(\begin{array}{c|c|c|} & \phi_1 & \phi_2 \\ \hline \phi_1 & 1 & 0 \\ \hline \phi_2 & 0 & -1 \\ \hline \end{array}, \begin{array}{c|c|c|} & \phi_1 & \phi_2 \\ \hline \phi_1 & 0 & 1 \\ \hline \phi_2 & 1 & 0 \\ \hline \end{array} \right)$$

With $\phi_1 \to B_x, \;\; \phi_2 \to B_y$

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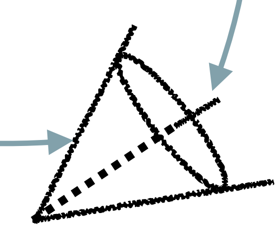


$$C_1 \geq 0, \;\; C_2 \geq 0, \;\; C_3^2 \leq 4C_1C_2$$

|P violating| < P conserving

[Remmen, Rodd,1908.09845] [T. Trott, 2011.10058]

Example 6: SM W boson, gluons

✦ *W* boson: SO(2) x SU(2)

$$O_{T,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}]$$
$$O_{T,2} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}]$$
$$O_{T,1} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}]$$
$$O_{T,10} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\tilde{W}^{\alpha\beta}]$$
$$O_W = \varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$$

✦ Bounds (elastic only covers first 4)

$$F_{T,2} \geq 0,$$
$$4F_{T,1} + F_{T,2} \geq 36\bar{a}_W^2,$$
$$F_{T,2} + 8F_{T,10} \geq 36\bar{a}_W^2,$$
$$8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0,$$
$$12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10} \geq 0,$$
$$4F_{T,0} + 4F_{T,1} + 3F_{T,2} + 12F_{T,10} \geq 72\bar{a}_W^2$$

✦ See [Yamashita, Zhou, CZ, 2009.04490] for more W+B cases and applications in aQGC.

✦ Gluon: SO(2) x SU(3)

$Q_{G^4}^{(1)}$	$(G^A_{\mu\nu}G^{A\mu\nu})(G^B_{\rho\sigma}G^{B\rho\sigma})$
$Q_{G^4}^{(2)}$	$(G^A_{\mu\nu}\widetilde{G}^{A\mu\nu})(G^B_{\rho\sigma}\widetilde{G}^{B\rho\sigma})$
$Q_{G^4}^{(3)}$	$(G^A_{\mu\nu}G^{B\mu\nu})(G^A_{\rho\sigma}G^{B\rho\sigma})$
$Q_{G^4}^{(4)}$	$(G^A_{\mu\nu}\widetilde{G}^{B\mu\nu})(G^A_{\rho\sigma}\widetilde{G}^{B\rho\sigma})$

$Q_{G^4}^{(7)}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}G^{B\mu\nu})(G^C_{\rho\sigma}G^{D\rho\sigma})$
$Q_{G^4}^{(8)}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}\widetilde{G}^{B\mu\nu})(G^C_{\rho\sigma}\widetilde{G}^{D\rho\sigma})$

[C. Murphy, 2005.00059]

+ D6 3gluon operator

✦ Bounds $\vec{x}\cdot\vec{c} \geq 0$

x's are:

[0, 0, 0, 1, 0, 0, 0]	[0, 0, 6, 3, 7, 2, 0]	[24, 0, 12, 21, 15, 14, 0]	[0, 0, 96, 24, 64, 40, −81]
[0, 0, 1, 1, 1, 0, 0]	[8, 6, 1, 6, 0, 2, 0]	[24, 32, 24, 4, 8, 0, −27]	[40, 32, 80, 4, 0, 0, −189]
[2, 0, 1, 0, 0, 0, 0]	[0, 6, 3, 12, 5, 0, 0]	[48, 36, 21, 27, 25, 0, 0]	[0, 0, 24, 120, 40, 104, −81]
[0, 2, 0, 1, 0, 0, 0]	[8, 6, 1, 12, 0, 0, 0]	[32, 40, 4, 80, 0, 0, −27]	[0, 0, 120, 24, 104, 40, −81]
[0, 0, 3, 0, 2, 0, 0]	[0, 6, 6, 9, 10, 4, 0]	[0, 48, 0, 48, 0, 40, −81]	[96, 0, 144, 24, 64, 40, −81]
[0, 0, 0, 3, 0, 2, 0]	[0, 12, 0, 14, 0, 0, −9]	[24, 0, 36, 24, 16, 40, −81]	[48, 0, 96, 24, 0, 40, −243]
[1, 1, 2, 2, 0, 0, 0]	[0, 0, 8, 8, 0, 8, −27]	[0, 0, 48, 24, 32, 40, −81]	[0, 192, 168, 96, 112, 120, −405]
[6, 0, 3, 0, 2, 0, 0]	[12, 0, 14, 0, 0, 0, −27]	[0, 0, 24, 48, 16, 56, −81]	[168, 480, 168, 156, 56, 160, −729]
[4, 2, 2, 1, 2, 0, 0]	[6, 8, 12, 1, 0, 0, −27]	[88, 32, 56, 4, 40, 0, −27]	[264, 384, 156, 168, 16, 200, −729]
[0, 0, 4, 0, 0, 0, −9]	[8, 16, 4, 8, 0, 8, −27]	[96, 42, 27, 84, 25, 0, 0]	[288, 384, 216, 168, 0, 200, −891]
[6, 0, 6, 0, 5, 0, 0]	[0, 24, 0, 12, 0, 8, −27]	[96, 66, 42, 39, 50, 4, 0]	[480, 384, 480, 168, 160, 200, −729]
[0, 0, 3, 6, 5, 4, 0]	[8, 22, 1, 14, 0, 10, −27]	[120, 42, 39, 42, 40, 14, 0]	[336, 768, 672, 216, 0, 200, −2187]

7D polyhedral cone with 48 faces

[X. Li et al., 2101.01191]

Example 7: SM fermions

✦ Lepton doublet $O^1 = \partial_\mu \left(\bar{l}\gamma_\nu l\right) \partial^\mu \left(\bar{l}\gamma^\nu l\right), \quad O^3 = \partial_\mu \left(\bar{l}\gamma_\nu \tau^I l\right) \partial^\mu \left(\bar{l}\gamma^\nu \tau^I l\right)$

✦ Generators

State	Spin	Charge	$\mathcal{L}_{\rm int}$	ER	$\vec{c}$
$\mathcal{B}_1$	1	1_1	$\mathcal{B}_1^\mu(\bar{l}^c i \overleftrightarrow{D}_\mu l)$	✓	$\frac{1}{2}(1,-1) \propto \vec{g}_1$
Ξ_1	0	3_1	$\Xi_1^I(\bar{l}^c \tau^I l)$	✗	$\frac{1}{2}(-3,-1) \propto \vec{g}_2$
$\mathcal{B}$	1	1_0	$\mathcal{B}^\mu(\bar{l}\gamma_\mu l)$	✓	$\frac{1}{2}(-1,0) \propto \vec{g}_3$
$\mathcal{W}$	1	3_0	$\mathcal{W}^{I\mu}(\bar{l}\gamma_\mu \tau^I l)$	✗	$\frac{1}{2}(0,-1) \propto \vec{g}_4$

✦ Bounds:

$$C_1 + C_2 \le 0, \quad C_2 \le 0\,.$$

Notations following [de Blas, Criado, Pérez-Victoria, Santiago, 2005.00059]

✦ up/down quarks $O^1 = \partial_\mu \left(\bar{u}\gamma_\nu u\right) \partial^\mu \left(\bar{u}\gamma^\nu u\right), \quad O^8 = \partial_\mu \left(\bar{u}\gamma_\nu T^A u\right) \partial^\mu \left(\bar{u}\gamma^\nu T^A u\right)$

✦ Generators

State	Spin	Charge	$\mathcal{L}_{\rm int}$	ER	$\vec{c}$
$\mathcal{V}_{\frac{4}{3}}$	1	$(3,1)_{-\frac{4}{3}}$	$\mathcal{V}_{\frac{4}{3}}^{a\mu}\epsilon_{abc}\bar{u}^{cb} i \overleftrightarrow{D}_\mu u^c$	✓	$\frac{2}{3}(1,-3) \propto \vec{g}_1$
Ω_4	0	$(6,1)_{\frac{4}{3}}$	$\Omega_4^{\dagger ab}\bar{u}^{c(a}u^{b)}$	✗	$\frac{1}{3}(-2,-3) \propto \vec{g}_2$
$\mathcal{B}$	1	$(1,1)_0$	$\mathcal{B}^\mu \bar{u}\gamma_\mu u$	✓	$\frac{1}{2}(-1,0) \propto \vec{g}_3$
$\mathcal{G}$	1	$(8,1)_0$	$\mathcal{G}^{A\mu}\bar{u}\gamma_\mu T^A u$	✗	$\frac{1}{2}(0,-1) \propto \vec{g}_4$

✦ Bounds:

$$3C_1 + C_2 \le 0, \quad C_2 \le 0\,.$$

Example 7: SM fermions

✦ Quark doublet

$$O^{1,1} = \partial_\mu \left(\bar{q}\gamma_\nu q\right) \partial^\mu \left(\bar{q}\gamma^\nu q\right),$$
$$O^{3,1} = \partial_\mu \left(\bar{q}\gamma_\nu \tau^I q\right) \partial^\mu \left(\bar{q}\gamma^\nu \tau^I q\right),$$
$$O^{1,8} = \partial_\mu \left(\bar{q}\gamma_\nu T^A q\right) \partial^\mu \left(\bar{q}\gamma^\nu T^A q\right)$$
$$O^{3,8} = \partial_\mu \left(\bar{q}\gamma_\nu \tau^I T^A q\right) \partial^\mu \left(\bar{q}\gamma^\nu \tau^I T^A q\right)$$

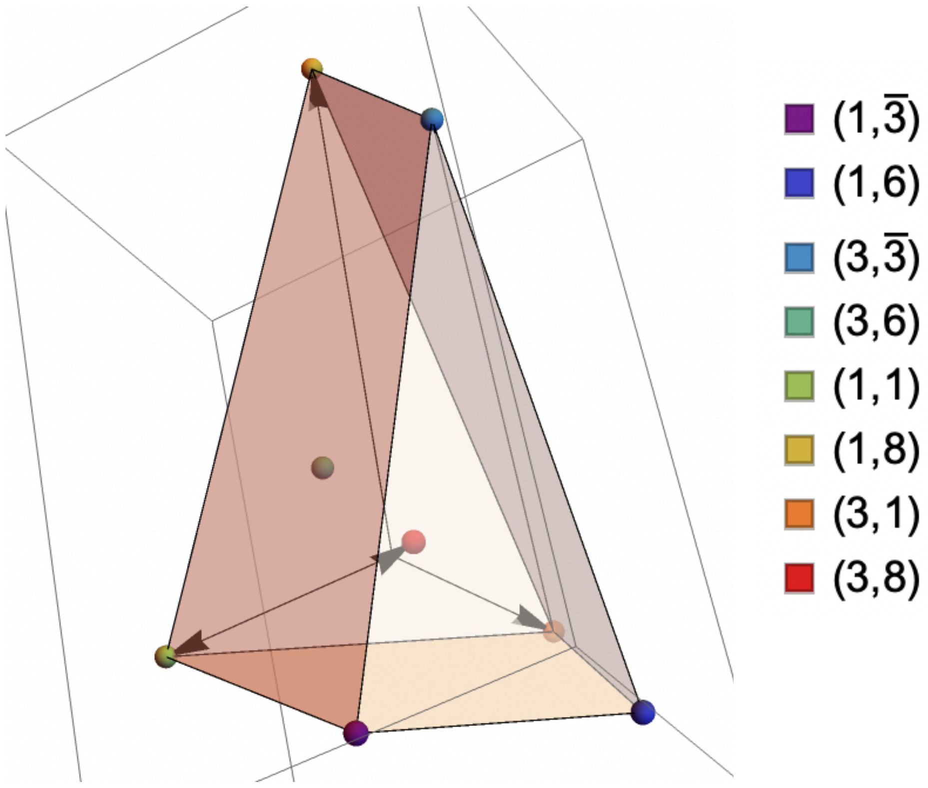
✦ Generators

State	Spin	Charge	$\mathcal{L}_{\rm int}$	ER	$\vec{c}$
ω_1	0	$(3,1)_{-\frac{1}{3}}$	$\omega_1^a \epsilon_{abc} \bar{Q}^{c^b} \epsilon Q^c$	✓	$\frac{1}{3}(-1,1,3,-3)$
$\mathcal{V}_{-\frac{1}{3}}$	1	$(3,3)_{-\frac{1}{3}}$	$\mathcal{V}^{aI}_{-\frac{1}{3}} \epsilon_{abc} \bar{Q}^{c^b} \epsilon \tau^I i \overleftrightarrow{D}_\mu Q^c$	✓	$\frac{1}{3}(3,1,-9,-3)$
$\mathcal{V}_{\frac{1}{3}}$	1	$(6,1)_{\frac{1}{3}}$	$\mathcal{V}^{\dagger ab\mu}_{\frac{1}{3}} \bar{Q}^{c(a} \epsilon i \overleftrightarrow{D}_\mu Q^{b)}$	✓	$\frac{1}{6}(2,-2,3,-3)$
Υ	0	$(6,3)_{\frac{1}{3}}$	$\Upsilon^{\dagger Iab} \bar{Q}^{c(a} \epsilon \tau^I Q^{b)}$	✗	$\frac{1}{6}(-6,-2,-9,-3)$
$\mathcal{B}$	1	$(1,1)_0$	$\mathcal{B}^\mu \bar{Q}\gamma_\mu Q$	✓	$\frac{1}{2}(-1,0,0,0)$
$\mathcal{W}$	1	$(1,3)_0$	$\mathcal{W}^{I\mu} \bar{Q}\gamma_\mu \tau^I Q$	✓	$\frac{1}{2}(0,-1,0,0)$
$\mathcal{G}$	1	$(8,1)_0$	$\mathcal{G}^{A\mu} \bar{Q}\gamma_\mu T^A Q$	✓	$\frac{1}{2}(0,0,-1,0)$
$\mathcal{H}$	1	$(8,3)_0$	$\mathcal{H}^{AI\mu} \bar{Q}\gamma_\mu T^A \tau^I Q$	✗	$\frac{1}{2}(0,0,0,-1)$

✦ Bounds:

$$\begin{pmatrix} 3 & 3 & 1 & 1 \\ 0 & 3 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 12 & 0 & 1 & 9 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} \leq 0$$

More constraining than elastic approach; See also [T. Trott, 2011.10058]



SM positivity summary

- Dim-8 bounds for self-quartics (F^4) are all solved from the generator point of view.
- Approach:
  - Compute the amplitude, map the coefficients.
  - Enumerate the ***m*** matrices and construct generators. (VE to get bounds.)
  - Map the generators to the Wilson coef. space. VE to get bounds.
- Some results for cross-quartics ($F_1^2F_2^2$) (W+B, L+R fermions). Needs a “continuous vertex enumeration”.
- For operators with more fields, may resort to a different approach. E.g. semidefinite programming.

The inverse problem

Inverse problem at the tree level

- Consider tree level UV completion: SM + $\{\mathcal{X}_{\alpha,i}\}$

$$\mathcal{L}_{\rm int} = \mathcal{X}_{\alpha i} g_{\alpha i}(\kappa_\phi J_\phi + \kappa_q J_q + \kappa_u J_u + \kappa_d J_d + \kappa_l J_l + \kappa_e J_e + \dots)$$

- Two kinds of information
  - Particle spectrum: masses, widths, overall coupling g.
    Not possible from dim-8 measurements.
  - Interaction: the currents (fixed by charge/irreps), and relative couplings $(\kappa_\phi, \kappa_q, \dots)$ We are interested in determining this.

Inverse problem at the tree level

- Q: Knowing $\vec{C} = \sum_\alpha w_\alpha \vec{g}_\alpha$, or $\mathcal{M}^{ijkl} = \sum_\alpha w_\alpha \mathcal{G}_\alpha^{ijkl}$

  or $\mathcal{M}^{ijkl} = \frac{1}{2\pi} \int_{(\epsilon\Lambda)^2}^{\infty} \frac{\mathrm{d}s}{s^3} \sum_X \left(\mathcal{M}_{ij\to X} \mathcal{M}^*_{kl\to X} + \mathcal{M}_{i\bar{l}\to X} \mathcal{M}^*_{k\bar{j}\to X} \right)$

  How do we determine $w_\alpha,\ \mathcal{G}_\alpha$? (Matching: RHS -> LHS; Inverse: LHS -> RHS)

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- In general impossible: more G (generators) than # coefficients.

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  How do we determine $w_\alpha, \ \mathcal{G}_\alpha$? (Matching: RHS -> LHS; Inverse: LHS -> RHS)

- In general impossible: more G (generators) than # coefficients.

- Exception: *M* is extremal.

Inverse problem at the tree level

- Q: Knowing $\vec{C} = \sum_\alpha w_\alpha \vec{g}_\alpha$, or $\mathcal{M}^{ijkl} = \sum_\alpha w_\alpha \mathcal{G}_\alpha^{ijkl}$

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  How do we determine $w_\alpha,\ \mathcal{G}_\alpha$? (Matching: RHS -> LHS; Inverse: LHS -> RHS)

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  - Similarly, M^{ijkl} cannot be split -> all integrands equal to *M* up to normalization.

SM Higgs

$4: H^4D^4$	
$Q_{H^4}^{(1)}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
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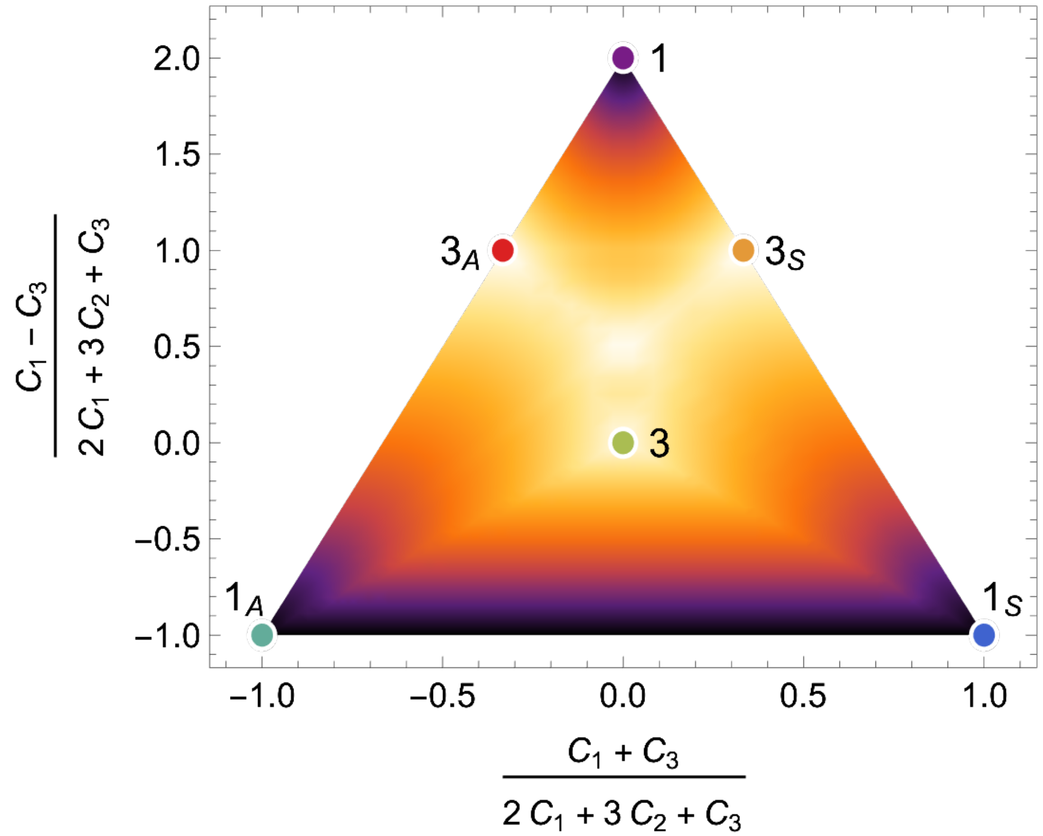
(max distance between two valid w's)



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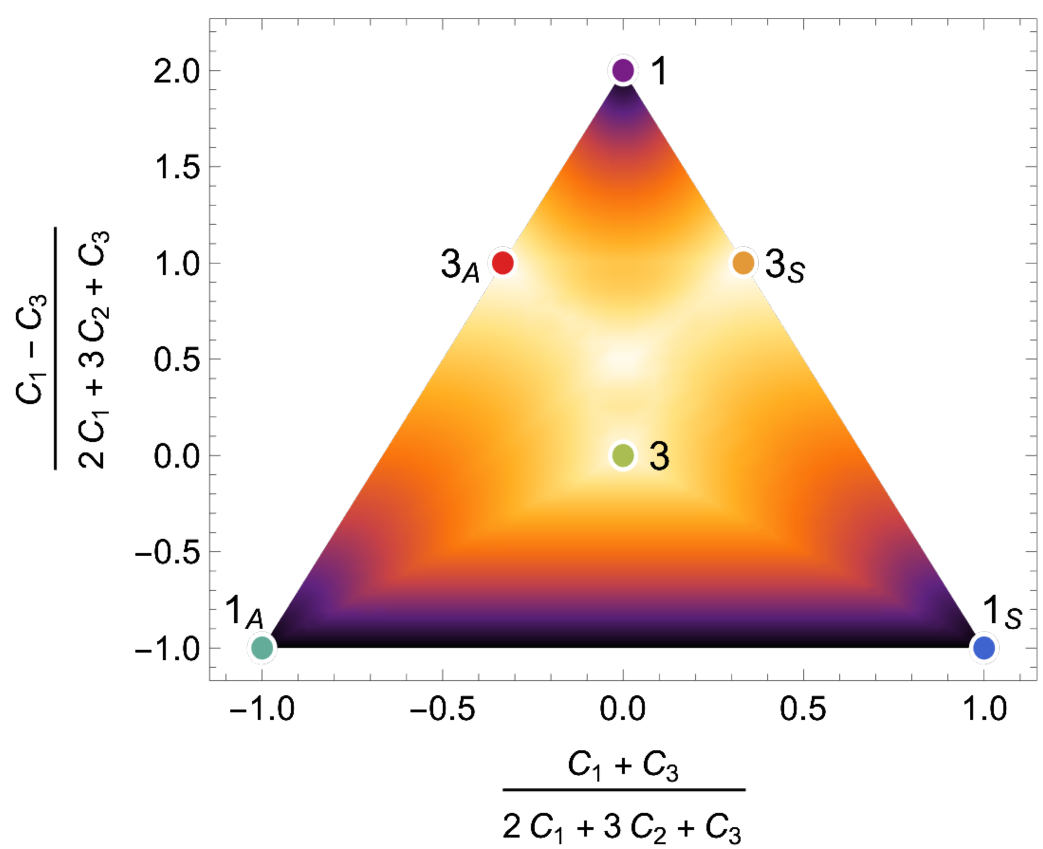

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3. Finite uncertainties: dim-8 measurement can set exclusion limit on all UV particles, in terms of g^2/M^4. (Model-independent; cannot be evaded by tuning.)

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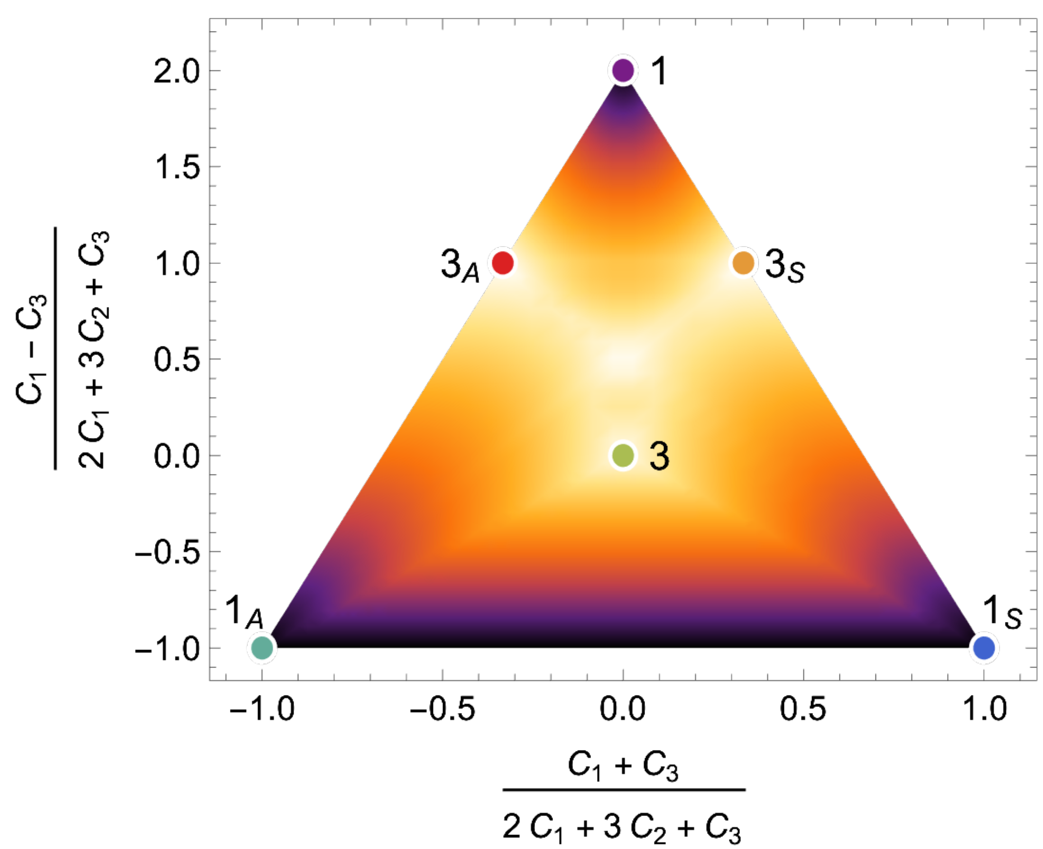

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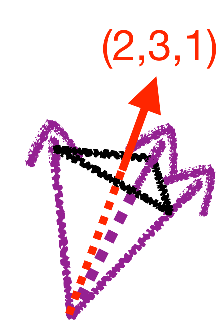
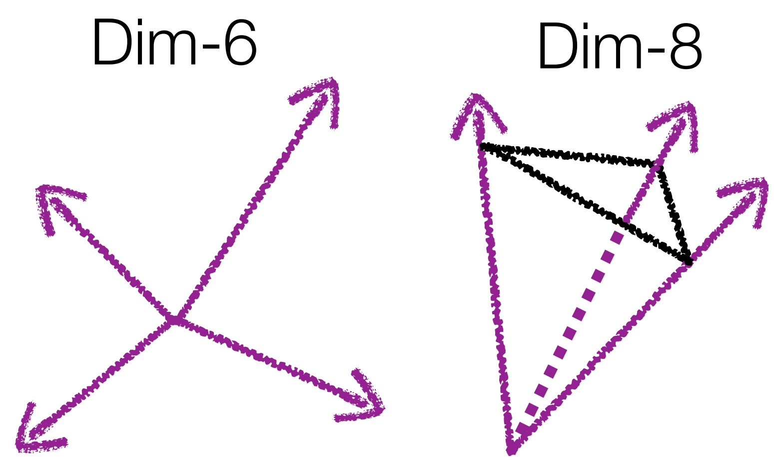


Dim-6 vs dim-8

- Generators at dim-8 form a salient cone; at dim-6 this is **not true**.
  - Can be traced back to a minus sign between $\boldsymbol{m}^{ij}\boldsymbol{m}^{kl}$ and $\boldsymbol{m}^{il}\boldsymbol{m}^{kj}$ at dim-6.
  - $\vec{C} = \sum_{\alpha} w_\alpha \vec{g}_\alpha$ has very different implications



	Dim-6	Dim-8
Unique solutions	No: $0 = \sum_\alpha \bar{w}^{(6)}_\alpha \vec{g}^{(6)}_\alpha$ has nonzero solution $w^{(6)}_\alpha \to w^{(6)}_\alpha + \lambda \bar{w}^{(6)}_\alpha, \lambda > 0$	Yes: Salient cone -> ERs always exist ER not splittable -> unique w.
Zero coefs. rule out all BSM	No: $\lambda \bar{w}^{(6)}_\alpha, \ \lambda \in \mathbf{R}^+$	Yes: 0 is an extreme point of a salient cone.
Finite uncertainty; upper bound on w.	No: $w^{(6)}_\alpha \to w^{(6)}_\alpha + \lambda \bar{w}^{(6)}_\alpha, \lambda > 0$	Yes: $\vec{C}(\lambda) \equiv \vec{C} - \lambda \vec{g}_k = \sum_{i \neq k} w_i \vec{g}_i + (w_k - \lambda)\vec{g}_k$ $\lambda_M = \max_{\vec{C}(\lambda) \in \mathbf{C}} \lambda \ \geq \ w_k$

Photon with parity violation

$$C_1\,(B_{\mu\nu}B^{\mu\nu})(B_{\rho\sigma}B^{\rho\sigma})$$
$$C_2\,(B_{\mu\nu}\widetilde{B}^{\mu\nu})(B_{\rho\sigma}\widetilde{B}^{\rho\sigma})$$
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Particle	Spin	Parity	Interaction	ER	$\vec{c}$
S_+	0	+	$\frac{g_1}{M_1}S_1\,(B_{\mu\nu}B^{\mu\nu})$	✓	$\frac{1}{2}(1,0,0)$
S_-	0	−	$\frac{g_2}{M_2}S_2\left(B_{\mu\nu}\tilde{B}^{\mu\nu}\right)$	✓	$\frac{1}{2}(0,1,0)$
$S_{\rm mix}$	0	?	$\frac{g_3}{M_3}S_3\left(\cos\theta B_{\mu\nu}B^{\mu\nu}+\sin\theta B_{\mu\nu}\tilde{B}^{\mu\nu}\right)$	✓	$\frac{1}{2}(\cos^2\theta,\sin^2\theta,2\sin\theta\cos\theta)$

- ✦ To set upper bound on one generator, when there are infinitely many of them:

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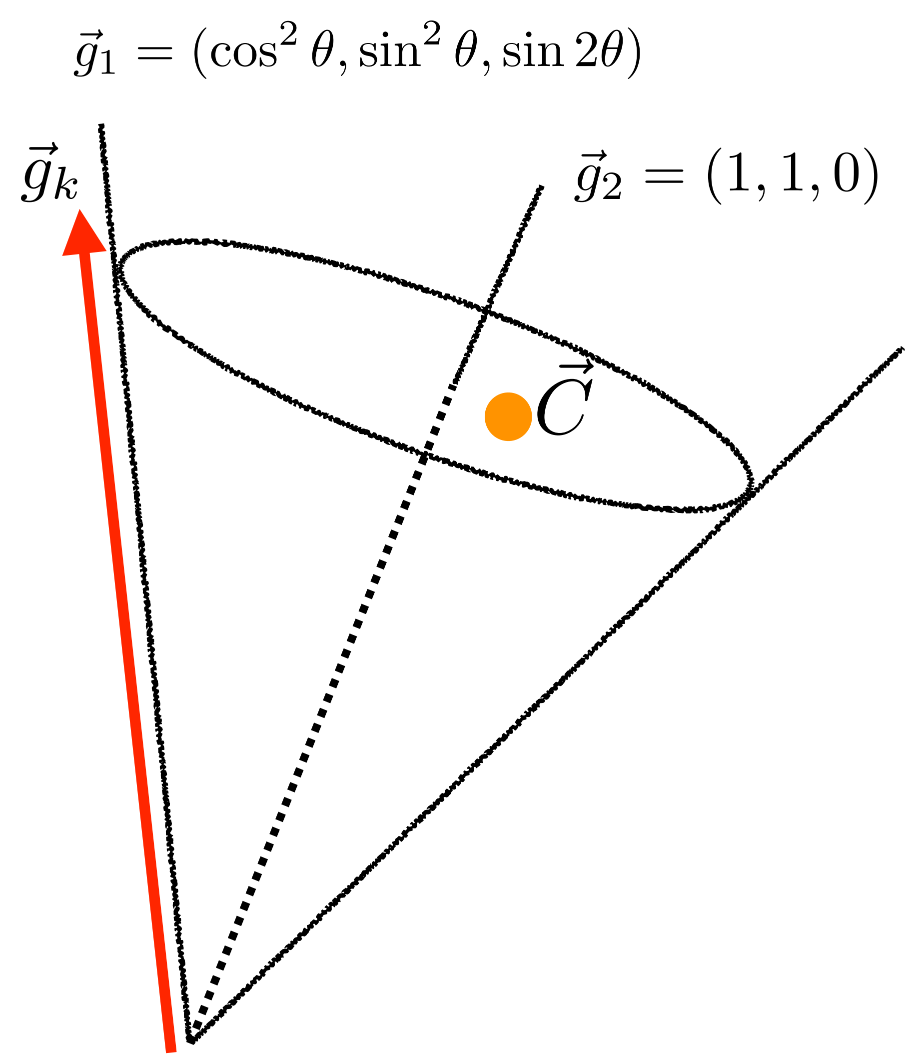

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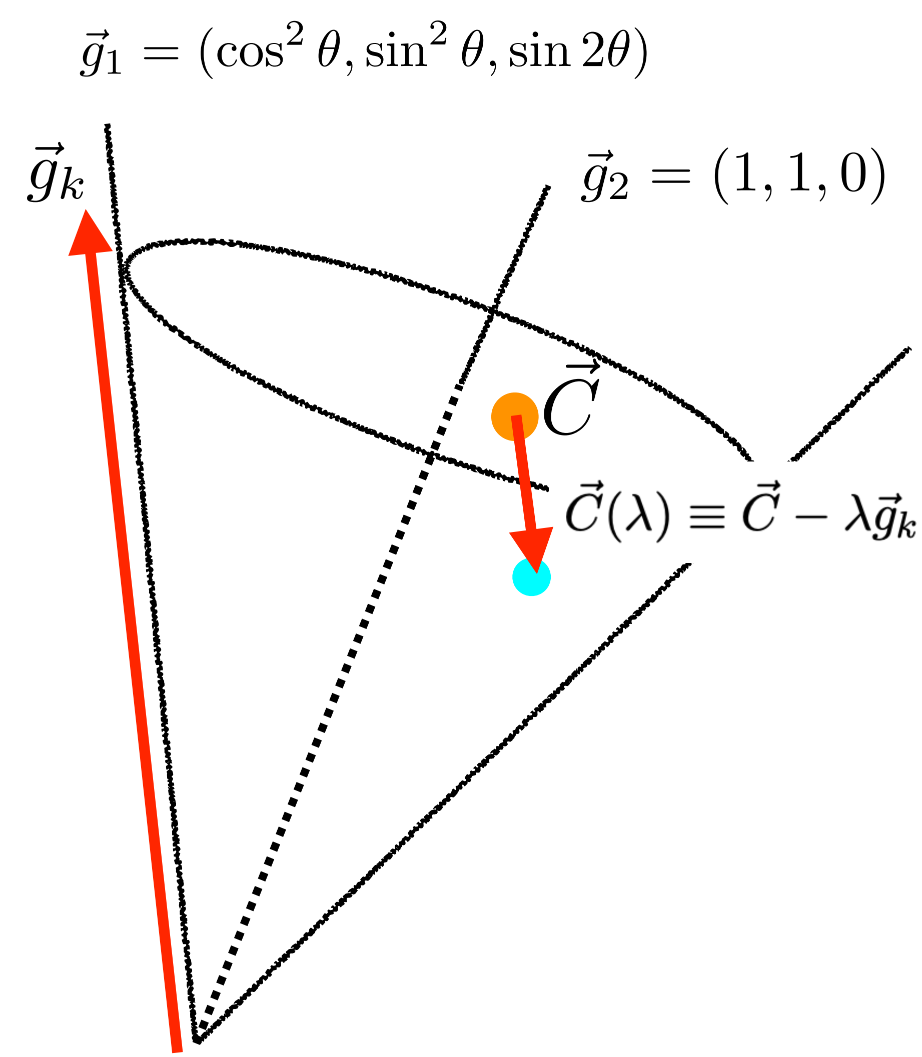

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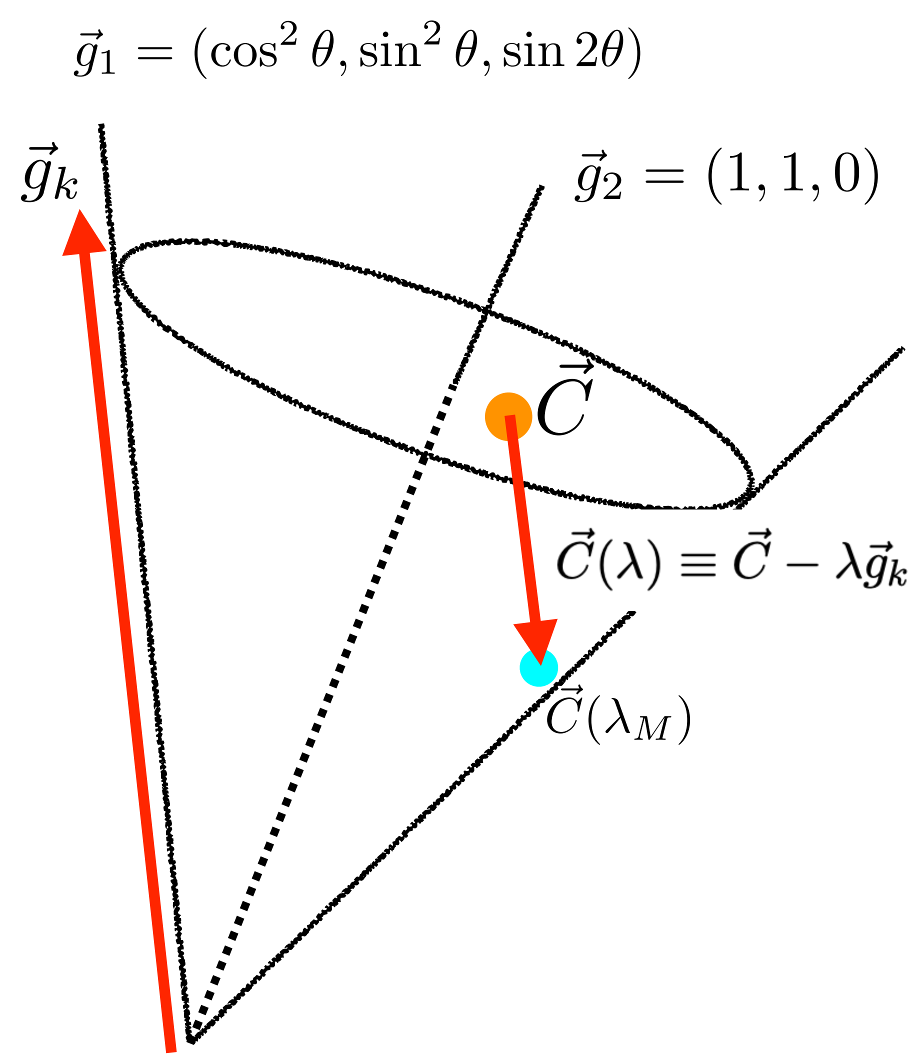
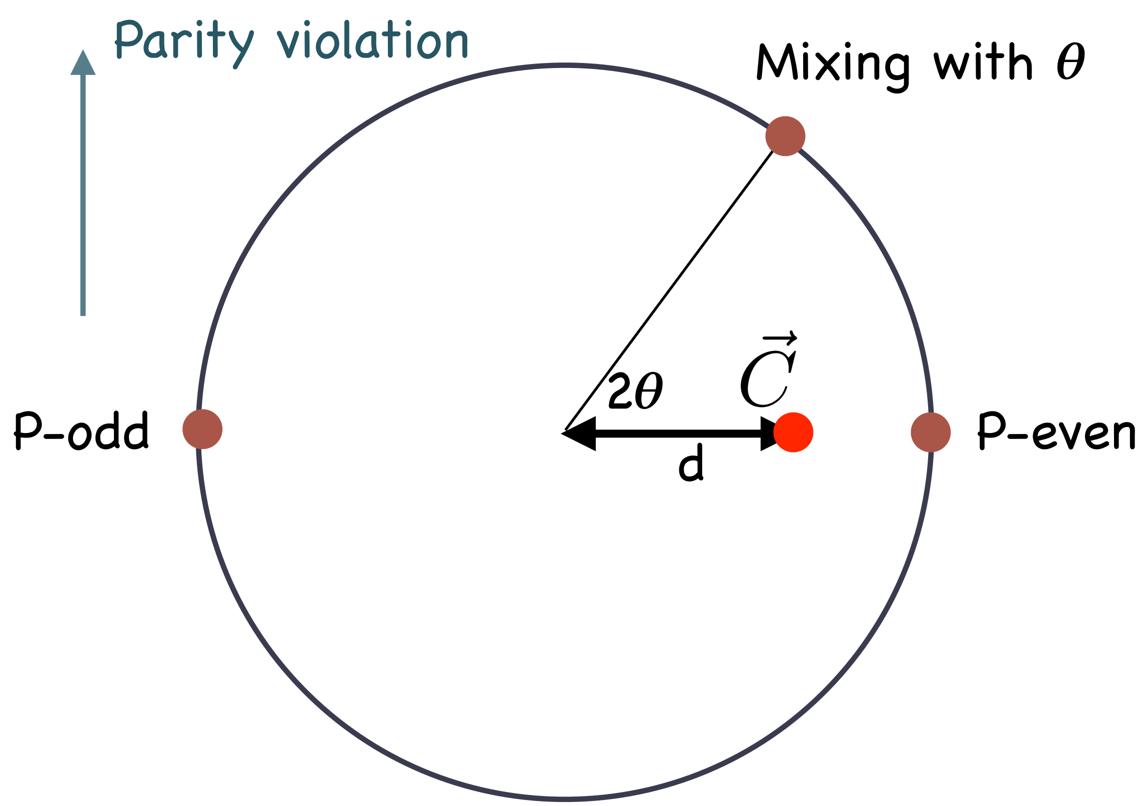
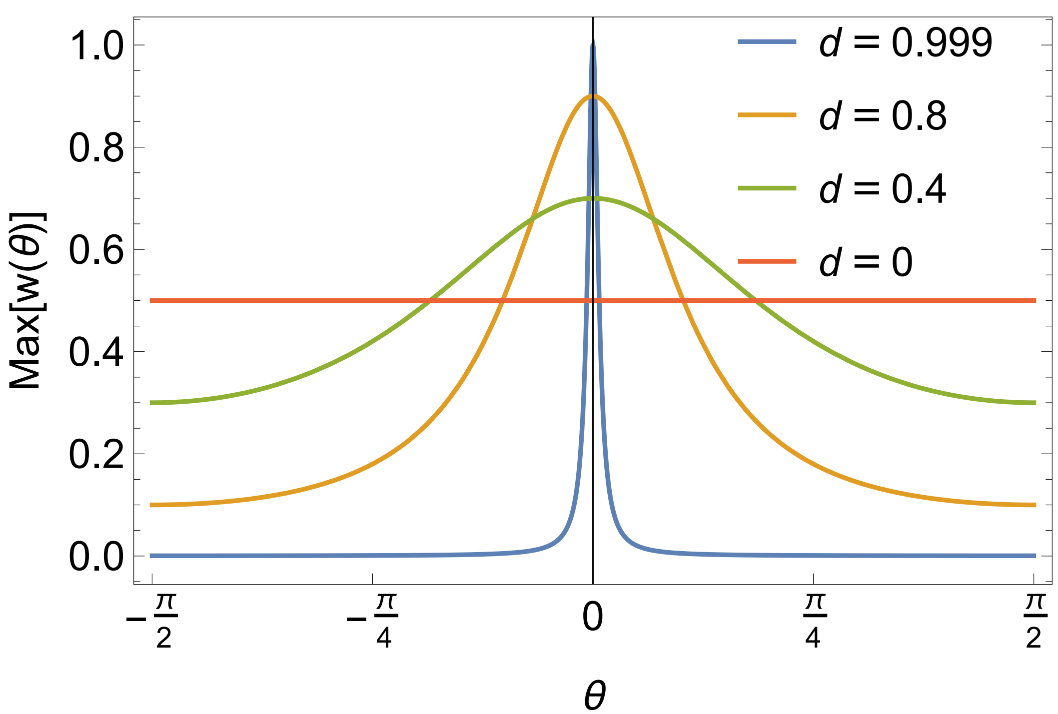


Photon with parity violation

- What is the maximum contribution from a state that couples with $\cos\theta B_{\mu\nu}B^{\mu\nu} + \sin\theta B_{\mu\nu}\tilde{B}^{\mu\nu}$?



$$\left(\frac{C_1 - C_2}{C_1 + C_2}, \frac{2C_3}{C_1 + C_2}\right)$$



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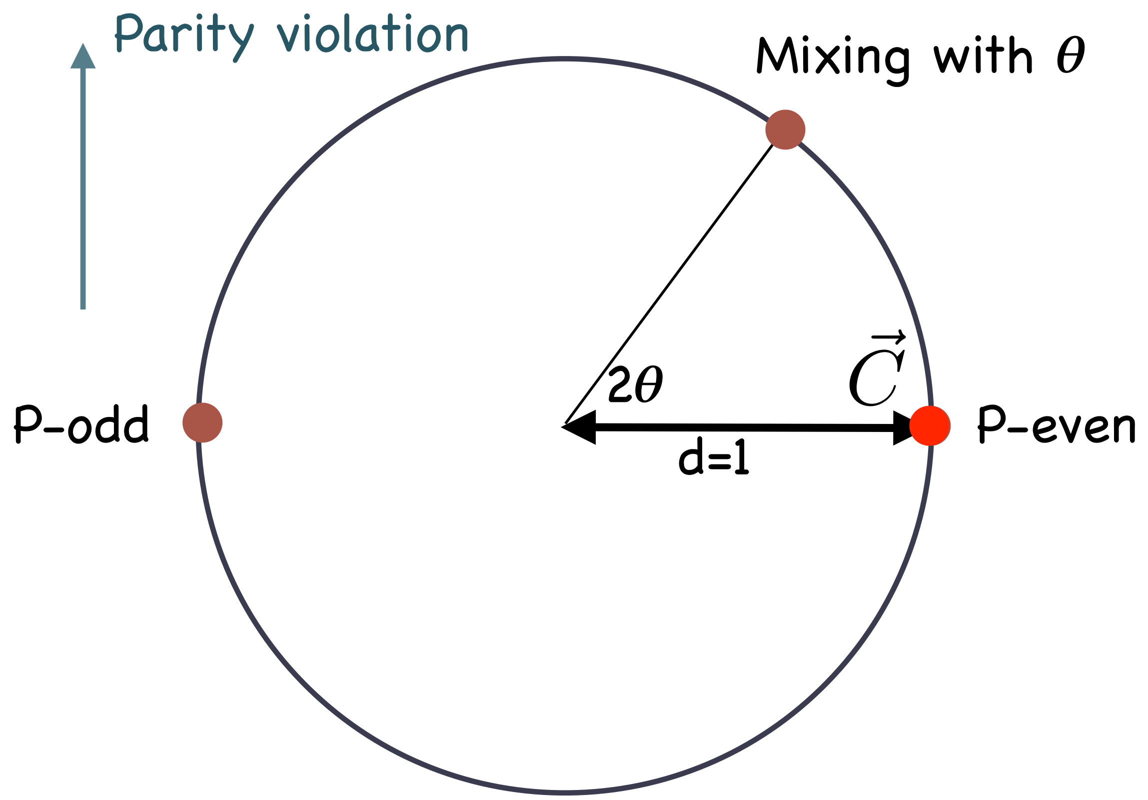

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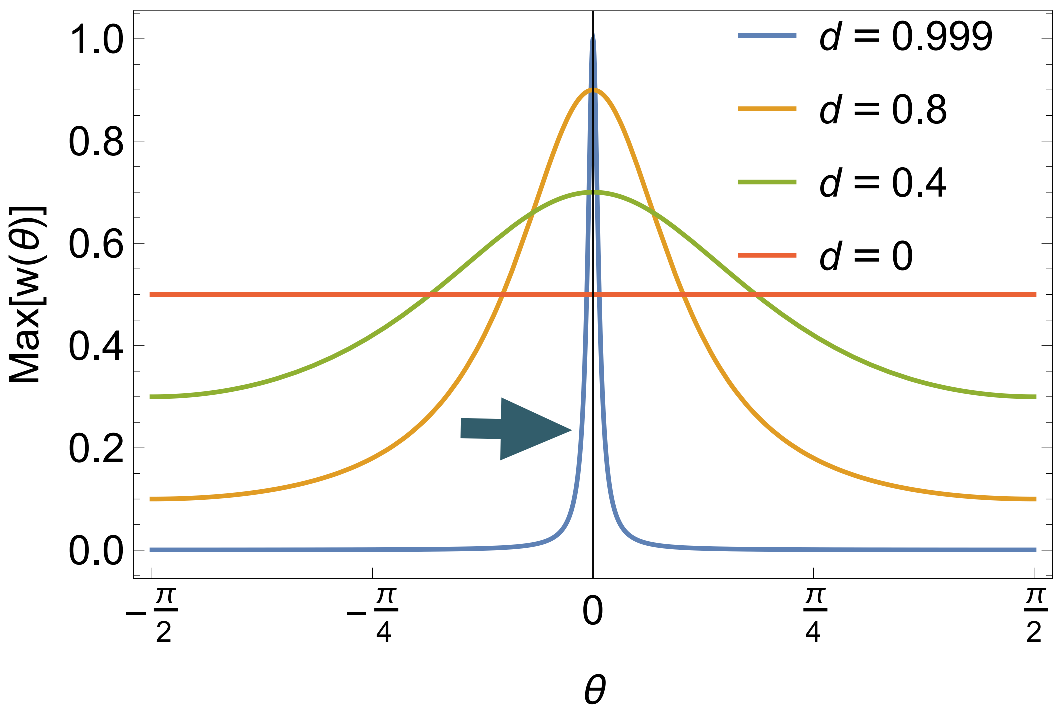

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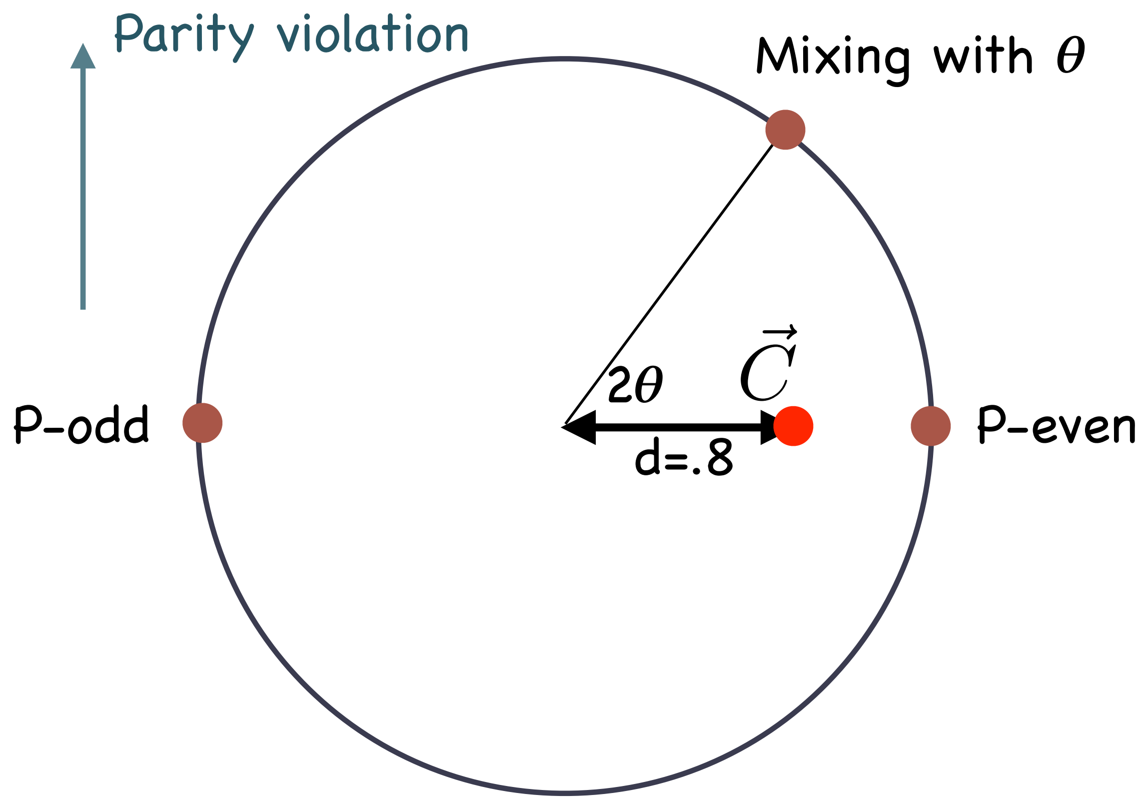

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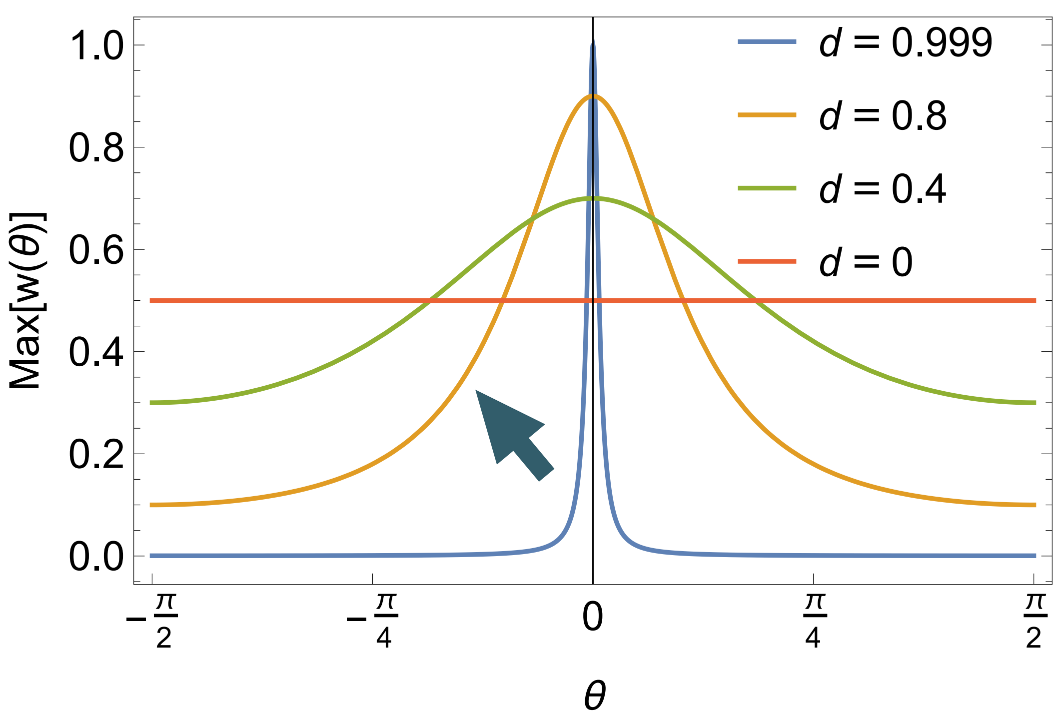

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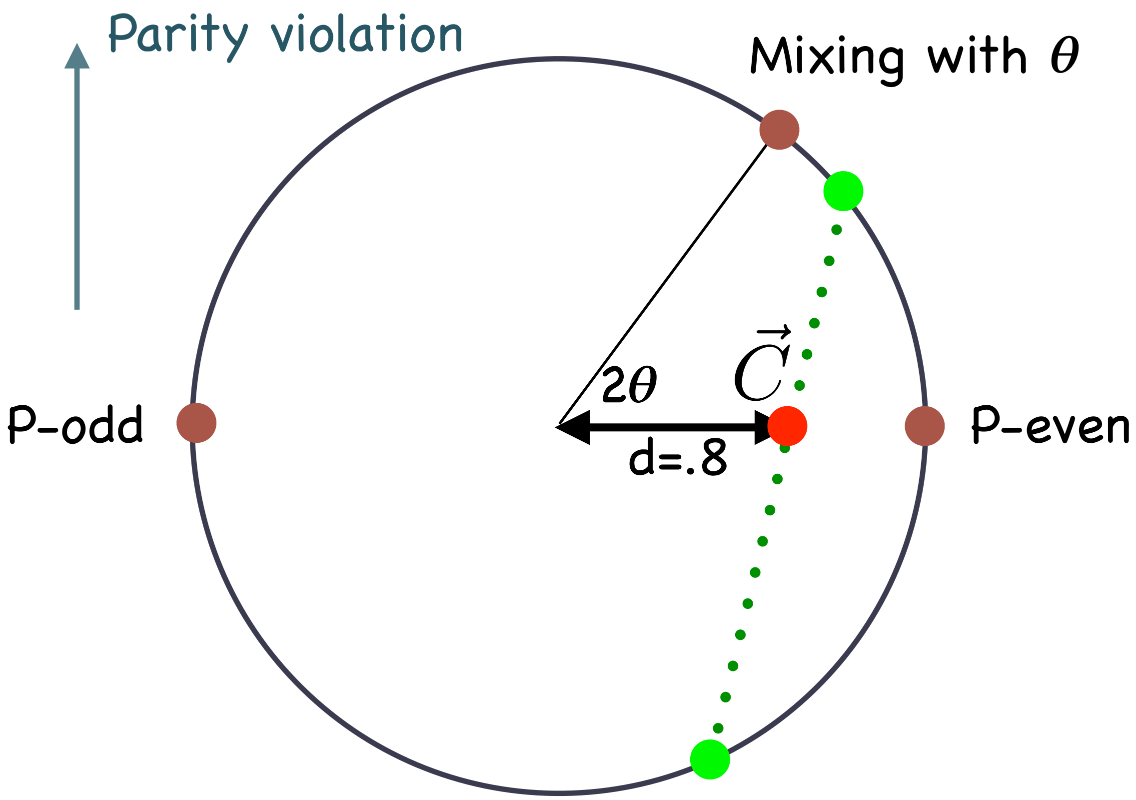

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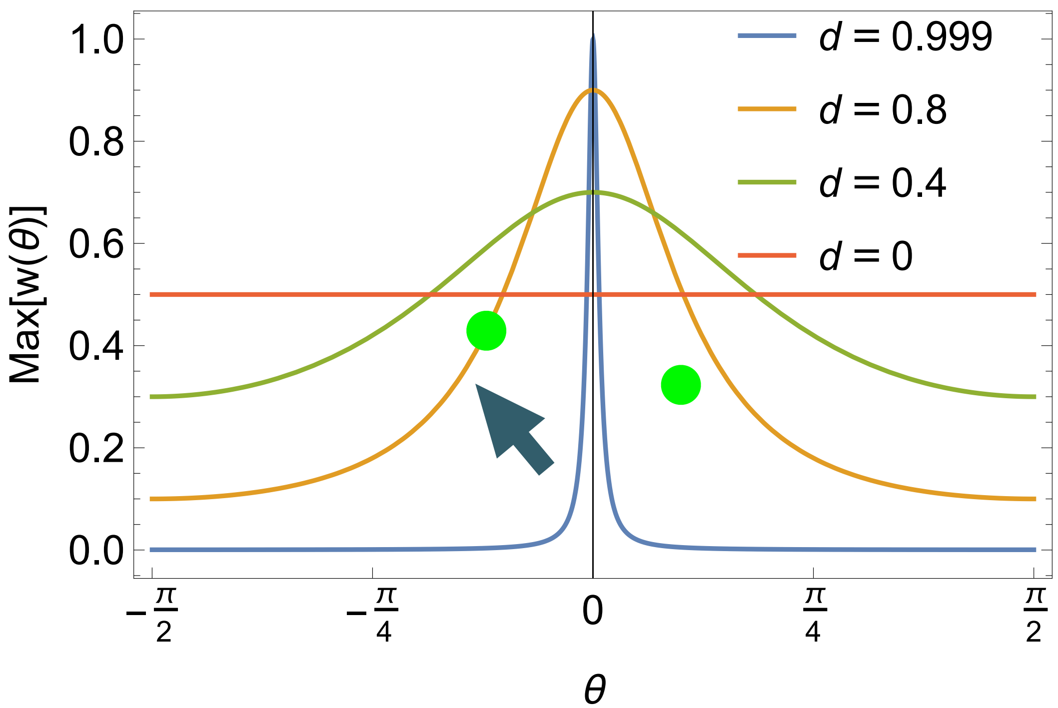
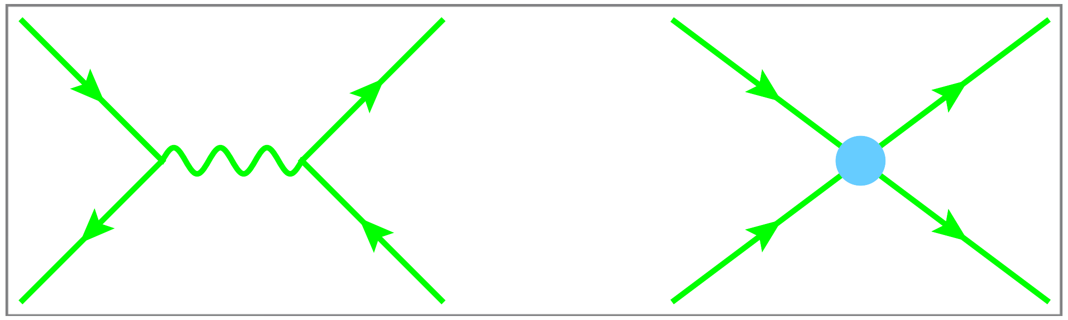


e+e- scattering at ILC

2009.02212 with B. Fuks, Y. Liu and S.-Y. Zhou

ee scattering at future lepton collider



$$O_1 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{e}\gamma_\mu e)\ ,$$
$$O_2 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{l}\gamma_\mu l)\ ,$$
$$O_3 = D^\alpha(\bar{l}e)\ D_\alpha(\bar{e}l),$$
$$O_4 = \partial^\alpha(\bar{l}\gamma^\mu l)\ \partial_\alpha(\bar{l}\gamma_\mu l)\ ,$$
$$O_5 = D^\alpha(\bar{l}\gamma^\mu\tau^I l)\ D_\alpha(\bar{l}\gamma_\mu\tau^I l)$$

$$C_1 \leq 0,$$
$$C_4 + C_5 \leq 0,$$
$$C_5 \leq 0,$$
$$C_3 \geq 0,$$
$$2\sqrt{C_1(C_4+C_5)} \geq C_2,$$
$$2\sqrt{C_1(C_4+C_5)} \geq -(C_2+C_3).$$

In ee → ee, C_5 does not give an independent contribution:

$$\vec{C}^{(8)} = (C_1, C_2, C_3, C_4)$$

UV states and interactions

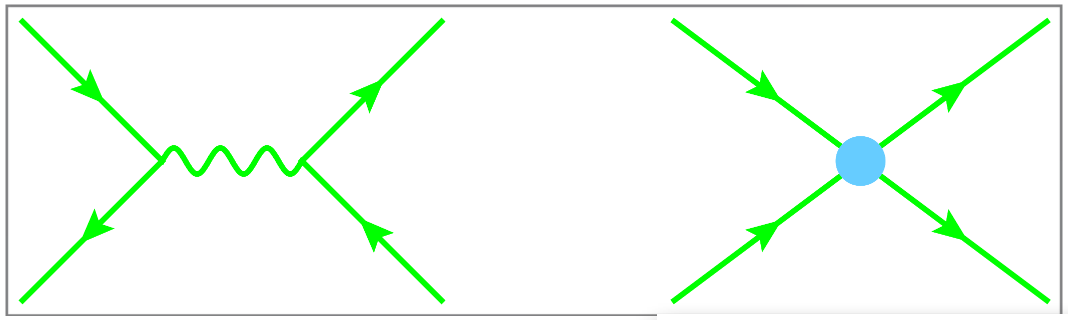
Scalar			Vector	
$D \equiv \mathbf{2}_{1/2}$	$M_L \equiv \mathbf{1}_1$	$M_R \equiv \mathbf{1}_2$	$V \equiv \mathbf{1}_0$	$V' \equiv \mathbf{2}_{-3/2}$

$$\mathcal{L}_{\rm int} = g_{Di}\bar{L}eD_i + g_{M_L i}\bar{L}^c\epsilon L M_{Li} + g_{M_R i}\bar{e}^c e M_{Ri} + g_{Vi}\Big(\bar{L}\gamma^\mu L + \kappa_i\bar{e}\gamma^\mu e\Big)V_{i\mu} + g_{V'i}(\bar{e}^c\gamma^\mu L)V_i'^\dagger + \text{h.c.},$$

Generators:

$$\vec{c}_D^{\,(8)} = (0,0,1,0),$$
$$\vec{c}_{M_L}^{\,(8)} = (0,0,0,-1),$$
$$\vec{c}_{M_R}^{\,(8)} = (-1,0,0,0),$$
$$\vec{c}_{V'}^{\,(8)} = (0,0,-1,2),$$
$$\vec{c}_{V(\kappa)}^{\,(8)} = (-\kappa^2/2,-\kappa,0,-1/2).$$

ee scattering at future lepton collider



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$$O_5 = D^\alpha(\bar{l}\gamma^\mu\tau^I l)\ D_\alpha(\bar{l}\gamma_\mu\tau^I l)$$

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$$C_4 + C_5 \le 0,$$
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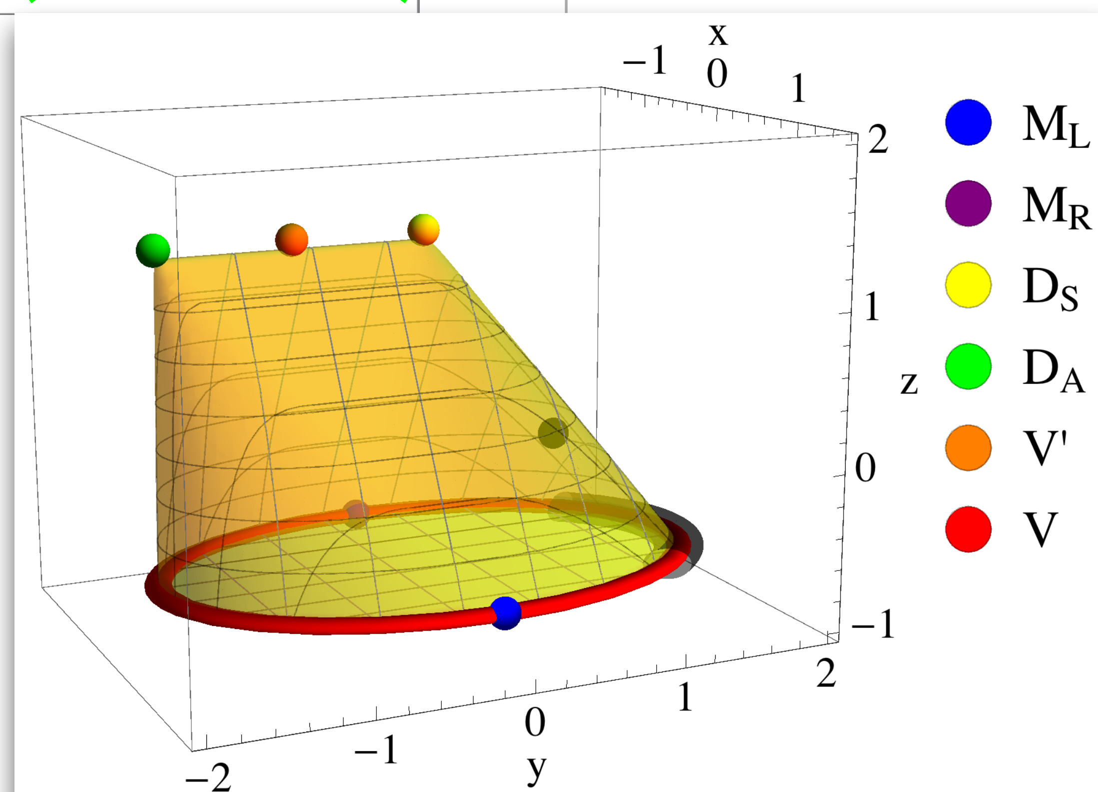

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...ractions

		Vector	
...	$M_R \equiv \mathbf{1}_2$	$V \equiv \mathbf{1}_0$	$V' \equiv \mathbf{2}_{-3/2}$

$$\ldots_{M_L i}\bar{L}^c\epsilon L M_{Li} + g_{M_R i}\bar{e}^c e M_{Ri}$$
$$\ldots \bar{e}\gamma^\mu e\Big) V_{i\mu} + g_{V'i}(\bar{e}^c\gamma^\mu L)V_i'^\dagger$$

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ee scattering at future lepton collider

Scenario	Beam polarization $P(e^-, e^+)$	Runs (luminosity @ energy), [ab^{-1}] @ [GeV] 1	2	3	4
CEPC	None	2.6@161	5.6@240		
FCC-ee	None	10@161	5@240	0.2@350	1.5@365
ILC-500	(−80%, 30%)	0.9@250	0.135@350	1.6@500	
	(80%, −30%)	0.9@250	0.045@350	1.6@500	
ILC-1000	(−80%, 30%)	0.9@250	0.135@350	1.6@500	1.25@1000
	(80%, −30%)	0.9@250	0.045@350	1.6@500	1.25@1000
CLIC	(−80%, 0%)	0.5@380	2@1500	4@3000	
	(80%, 0%)	0.5@380	0.5@1500	1@3000	

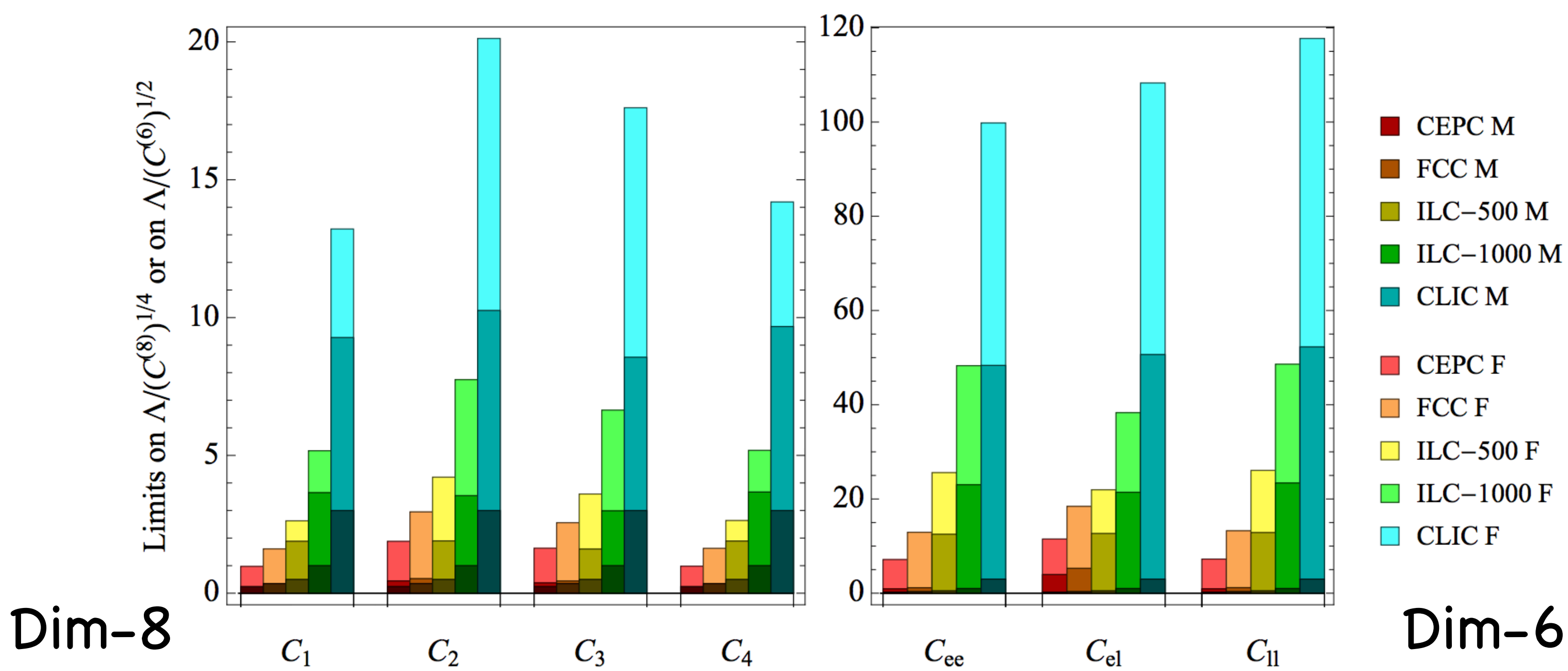


FIG. 3. Limits on the new physics characterization scale Λ_c (in TeV) for the various considered future lepton colliders. 'M' denotes marginalized limits (all other coefficients being floating) while 'F' denotes individual limits (all other coefficients being vanishing). In addition, we represent by the darkest color the largest center-of-mass energy of each collider project.

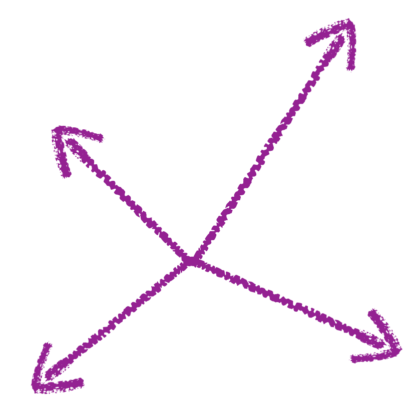
UV states

- Assume D-type scalar extension, $g_D = 0.8$, $M_D = 2$ TeV
- At ILC (with 1 TeV run), global fit ->

$$C_{ee} = 0 \pm 0.0024, \qquad C_{el} = -0.08 \pm 0.0035,$$
$$C_{ll} = 0 \pm 0.0023,$$
$$C_1 = 0 \pm 0.0074, \qquad C_2 = 0 \pm 0.0077,$$
$$C_3 = 0.04 \pm 0.020, \qquad C_4 = 0 \pm 0.0071.$$

- What to conclude at dim-6?
  - If assume the SM is only supplemented by D-type scalar,

$$M_D/g_D \in [2.45, 2.56] \text{ TeV}.$$



  - If assume the SM is extend by D and V',

$$\frac{g_D^2}{2M_D^2} - \frac{g_{V'}^2}{M_{V'}^2} = 0.08 \pm 0.0035 \text{ TeV}^{-2}.$$

  - If assuming more complicated models, not much to be concluded about the existence of UV states.

UV states

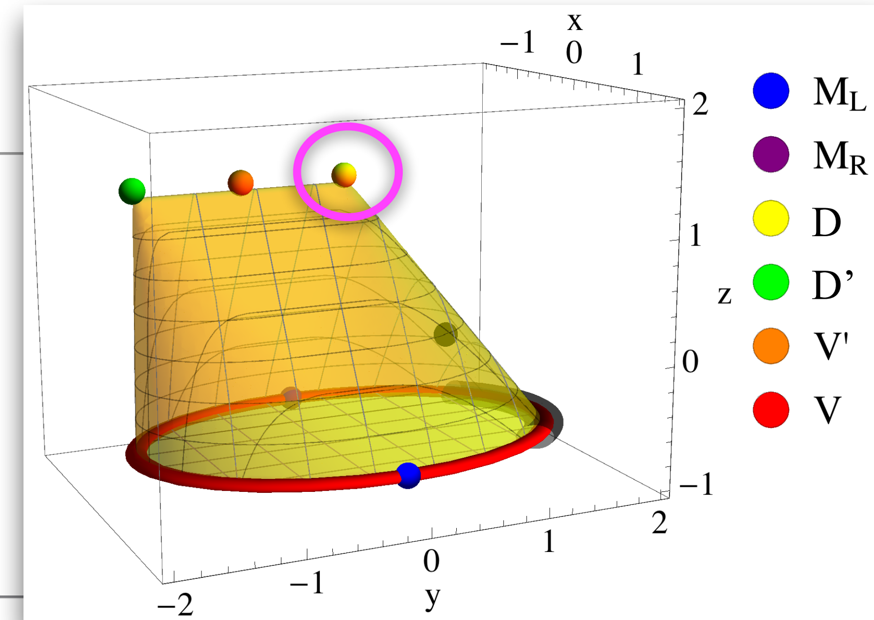
- What to conclude at dim-8?

- Upper bound on all states $$\lambda_M = \max_{\vec{C}(\lambda)\in\mathbf{C}} \lambda \geq w_k$$

  - Take into account EXP errors i.e. convex optimization

$$\begin{aligned} \text{maximum} \quad & \lambda \\ \text{subject to} \quad & \vec{C} - \lambda \vec{C}_k \in \mathbf{C} \\ \text{and} \quad & \chi^2\left(\vec{C}, \vec{C}_{\text{EXP}}\right) \leq \chi^2_c \end{aligned}$$

UV states



- What to conclude at dim-8?
- Upper bound on all states $\lambda_M = \max_{\vec{C}(\lambda)\in\mathbf{C}} \lambda \geq w_k$
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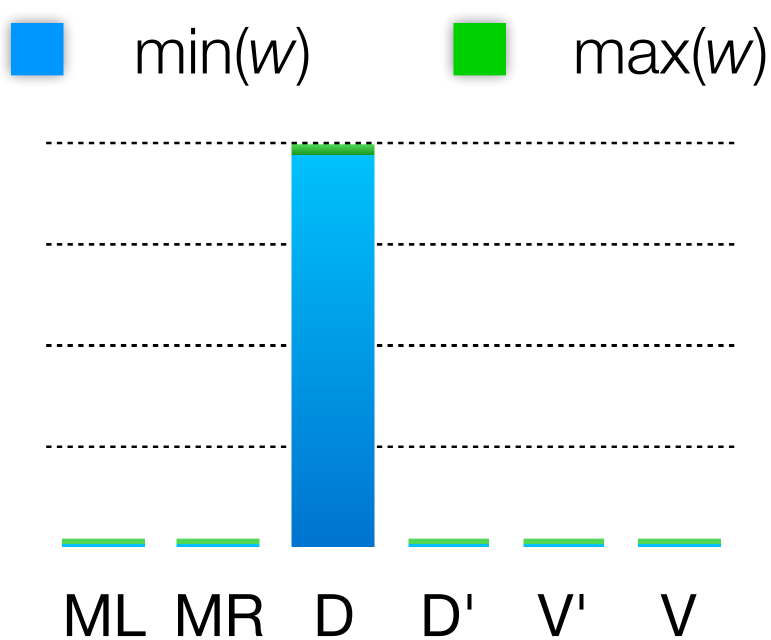
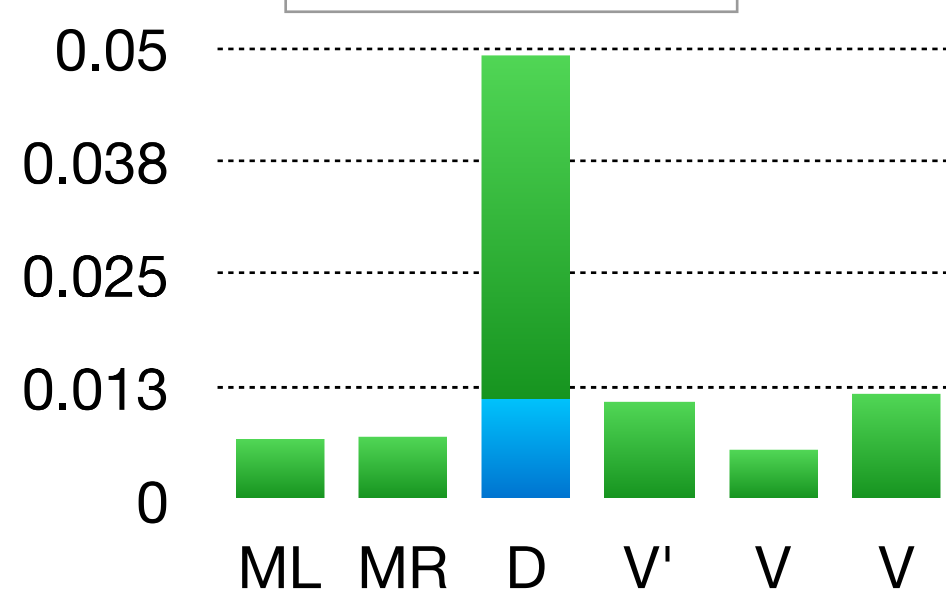
$$\begin{aligned} &\text{maximum} \quad \lambda \\ &\text{subject to} \quad \vec{C} - \lambda \vec{C}_k \in \mathbf{C} \\ &\qquad\quad \text{and} \quad \chi^2\left(\vec{C}, \vec{C}_{\rm EXP}\right) \leq \chi^2_c \end{aligned}$$

- D is an ER:



With exp. error:



UV states

Scalar			Vector	
$D \equiv \mathbf{2}_{1/2}$	$M_L \equiv \mathbf{1}_1$	$M_R \equiv \mathbf{1}_2$	$V \equiv \mathbf{1}_0$	$V' \equiv \mathbf{2}_{-3/2}$

$$\mathcal{L}_{\rm int} = g_{Di}\bar{L}eD_i + g_{M_L i}\bar{L}^c\epsilon L M_{Li} + g_{M_R i}\bar{e}^c e M_{Ri} + g_{Vi}\Big(\bar{L}\gamma^\mu L + \kappa_i \bar{e}\gamma^\mu e\Big)V_{i\mu} + g_{V'i}(\bar{e}^c\gamma^\mu L)V_i'^\dagger + \text{h.c.},$$

- Dim-8 measurement would universally exclude all alternative hypothesis:

X	$\vec{c}_X^{(8)}$	$\lambda_{\rm max}$	$M_X/\sqrt{g_X}$
M_L	$(0,0,0,-1)$	0.0067	≥ 3.5 TeV
M_R	$(-1,0,0,0)$	0.0069	≥ 3.5 TeV
V (with $\kappa = 1$)	$(-1/2,-1,0,-1/2)$	0.0055	≥ 3.7 TeV
V (with $\kappa = -1$)	$(-1/2,1,0,-1/2)$	0.0116	≥ 3.0 TeV
V'	$(0,-1,2,0)$	0.0109	≥ 3.1 TeV

$$M_D/\sqrt{g_D} \in [2.1, 3.1]\text{TeV}$$

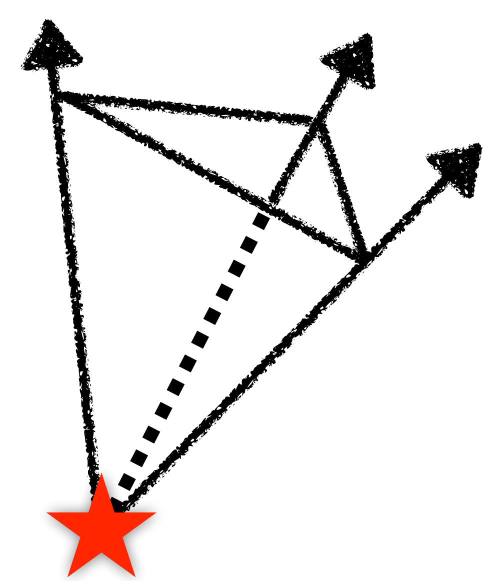
Testing the SM

Scalar			Vector	
$D \equiv \mathbf{2}_{1/2}$	$M_L \equiv \mathbf{1}_1$	M [illegible] $_2$	$V \equiv \mathbf{1}_0$	$V' \equiv \mathbf{2}_{-3/2}$

$$\mathcal{L}_{\text{int}} = g_{Di}\bar{L}eD_i + g_{M} \text{[illegible]} M_{Li} + g_{M_R i}\bar{e}^c e M_{Ri}$$
$$+ g_{Vi}\Big(\bar{L}\gamma^\mu L + \kappa_i \bar{e} \text{[illegible]} \Big) \text{[illegible]} g_{V'i}(\bar{e}^c\gamma^\mu L)V_i'^\dagger$$
$$+ \text{h.c.},$$

- If all dim-8 coefficients are consistent with 0, all states can be excluded to above certain scales

X	$\lambda_{\max}$	$M_X/\sqrt{g_X}$
D	0.0076	$\geq$ 3.4 TeV
M_L	0.0053	$\geq$ 3.7 TeV
M_R	0.0054	$\geq$ 3.7 TeV
V'	0.0056	$\geq$ 3.7 TeV
V (with $\kappa = 1$)	0.0041	$\geq$ 4.0 TeV
V (with $\kappa = -1$)	0.0041	$\geq$ 4.0 TeV



Summary

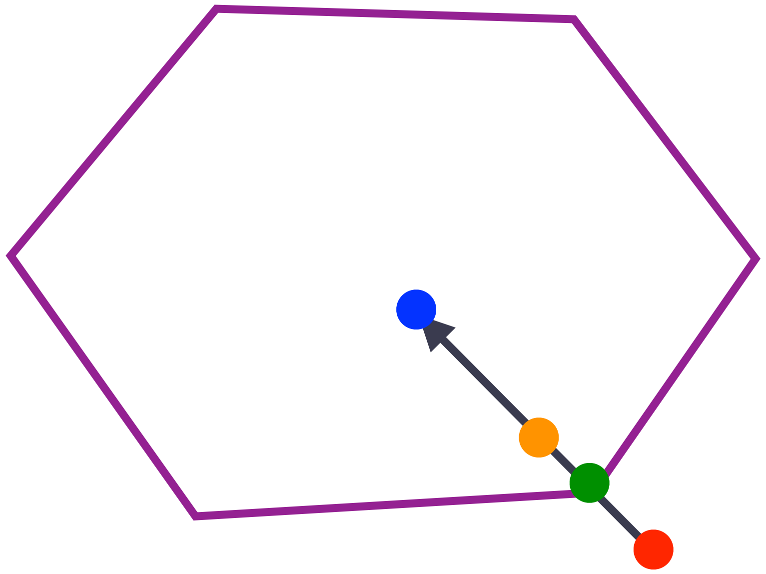
- Dispersive integral defines the positive generator of the parameter space of dim-8 Wilson coefficients.
  - Allowed region is a convex cone and can be systematically determined.

★ Outside: no UV completion

★ At the ER (or boundary): "uniquely" determine (the charge/irrep/interactions of) UV particles

★ Near an ER/boundary: limited arbitrariness in finding the UV completion

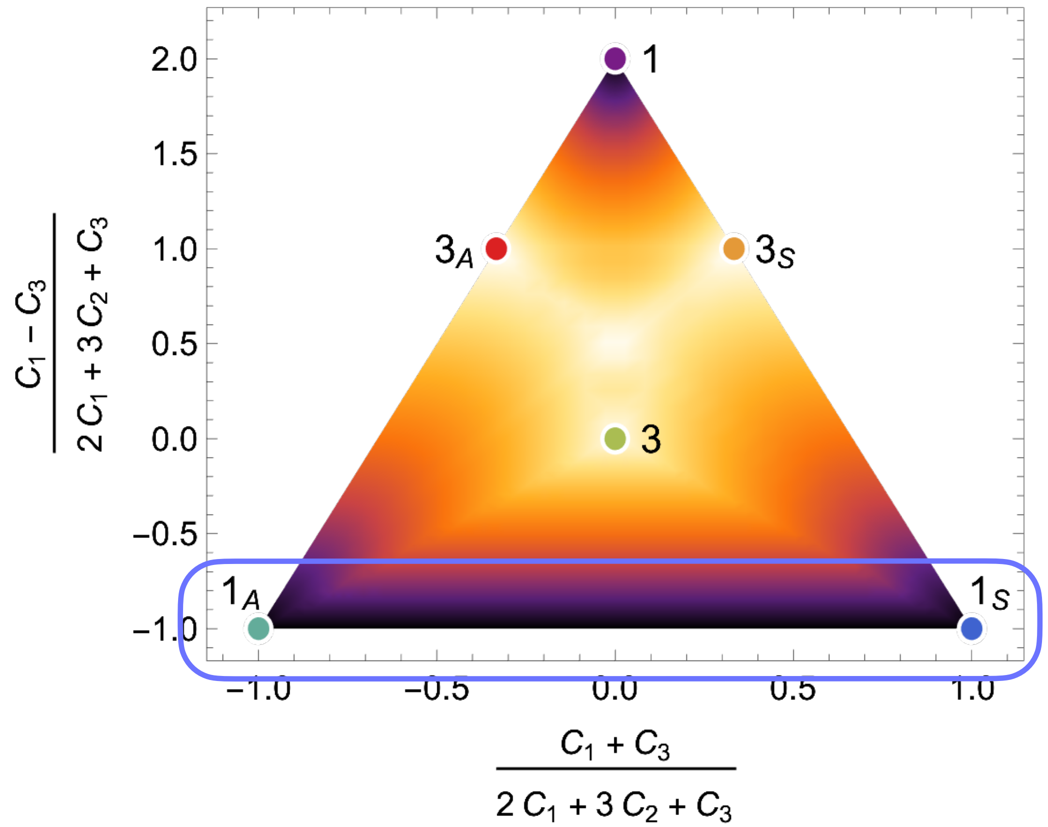
★ More inside: more arbitrariness

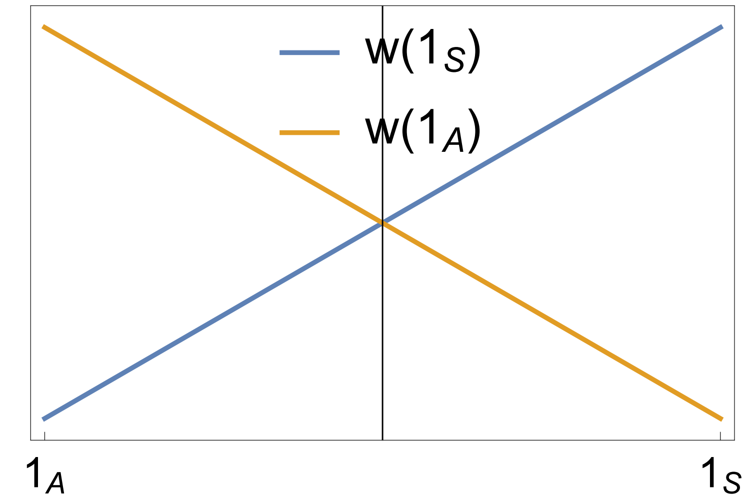


- If we care about UV model and not just coefficient measurements, dim-8 operators with s^2 dependence contains vital information.

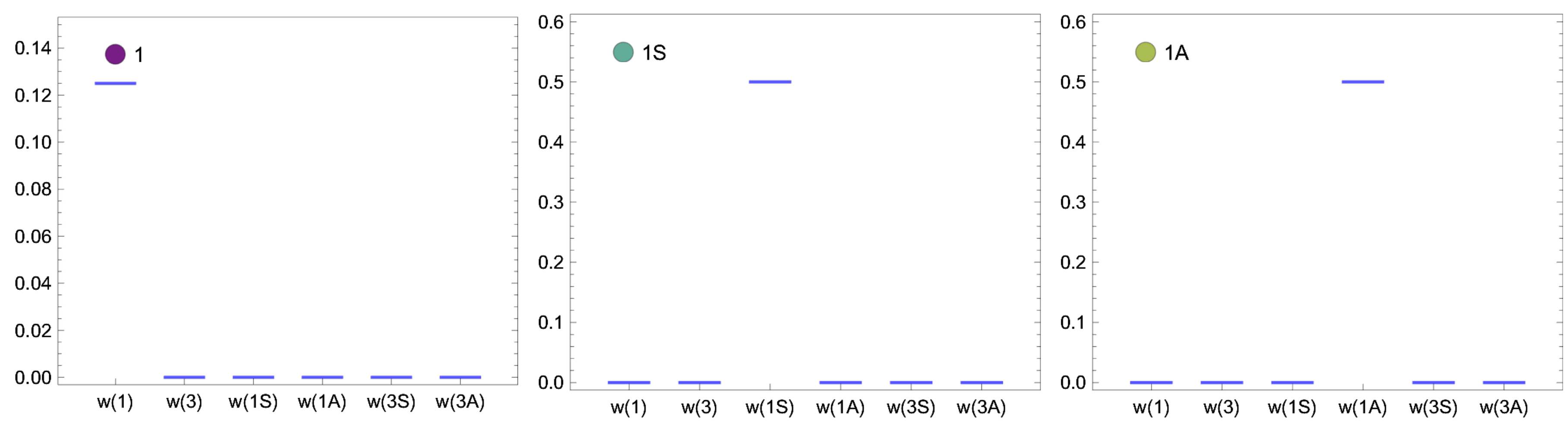
Thank you

Backups

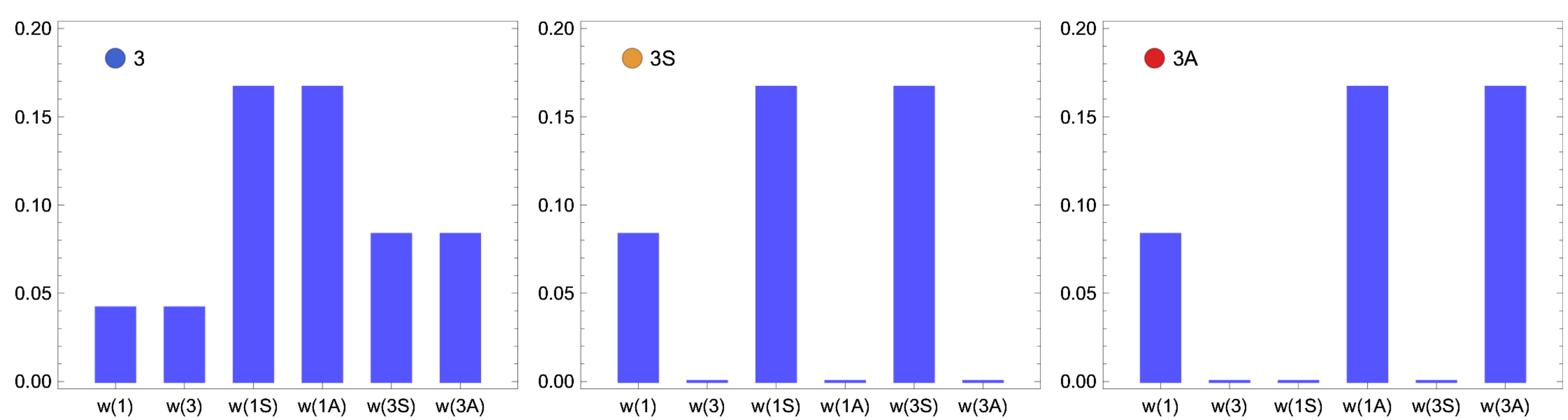




If SM is extended by "extremal" particles (1,1S,1A):



If SM is extended by "non-extremal" particles (3,3S,3A):



SM lepton doublets

$$O^1 = \partial_\mu \left(\bar{l}\gamma_\nu l\right) \partial^\mu \left(\bar{l}\gamma^\nu l\right) , \quad O^3 = \partial_\mu \left(\bar{l}\gamma_\nu \tau^I l\right) \partial^\mu \left(\bar{l}\gamma^\nu \tau^I l\right)$$

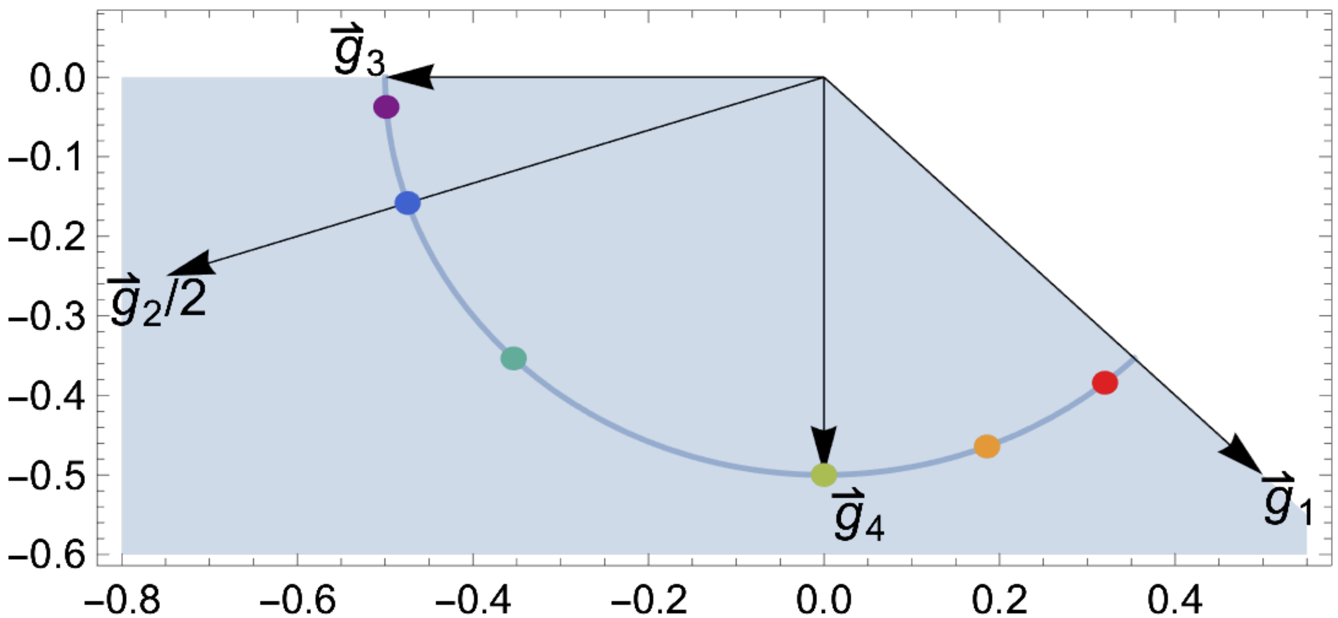
Coefficients: $\vec{C} = \sum_\alpha w_\alpha \vec{g}_\alpha$

$$w_\alpha = \sum_i \frac{g_{\alpha i}^2}{M_{\alpha i}^4} \geq 0$$

With $\vec{g}_1 = (1,-1), \quad \vec{g}_2 = (-3,-1),$

$\vec{g}_3 = (-1,0), \quad \vec{g}_4 = (0,-1)\,.$

State	Spin	Charge	$\mathcal{L}_{\rm int}$	ER	$\vec{C}$
$\mathcal{B}_1$	1	1_1	$\mathcal{B}_1^\mu(\bar{l}^c i \overleftrightarrow{D}_\mu l)$	✓	$\vec{g}_1$
Ξ_1	0	3_1	$\Xi_1^I(\bar{l}^c\tau^I l)$	✗	$\vec{g}_2$
$\mathcal{B}$	1	1_0	$\mathcal{B}^\mu(\bar{l}\gamma_\mu l)$	✓	$\vec{g}_3$
$\mathcal{W}$	1	3_0	$\mathcal{W}^{I\mu}(\bar{l}\gamma_\mu\tau^I l)$	✗	$\vec{g}_4$



Solution space of (w_1, w_2, w_3, w_4)

$$w_1 \geq 0,\ w_3 \geq 0,$$

$$w_2 = -\frac{2}{3}C_1 + \frac{1}{3}w_1 - \frac{1}{3}w_3 \geq 0,$$

$$w_4 = \frac{2}{3}C_1 - 2C_2 - \frac{4}{3}w_1 + \frac{1}{3}w_3 \geq 0$$

SM lepton doublets

- There are C values where the solution is unique (the ERs).

  $\vec{C} \propto (-1,0): \; w_2 = w_3 = w_4 = 0; \quad \vec{C} \propto (1,-1): \; w_1 = w_2 = w_4 = 0.$

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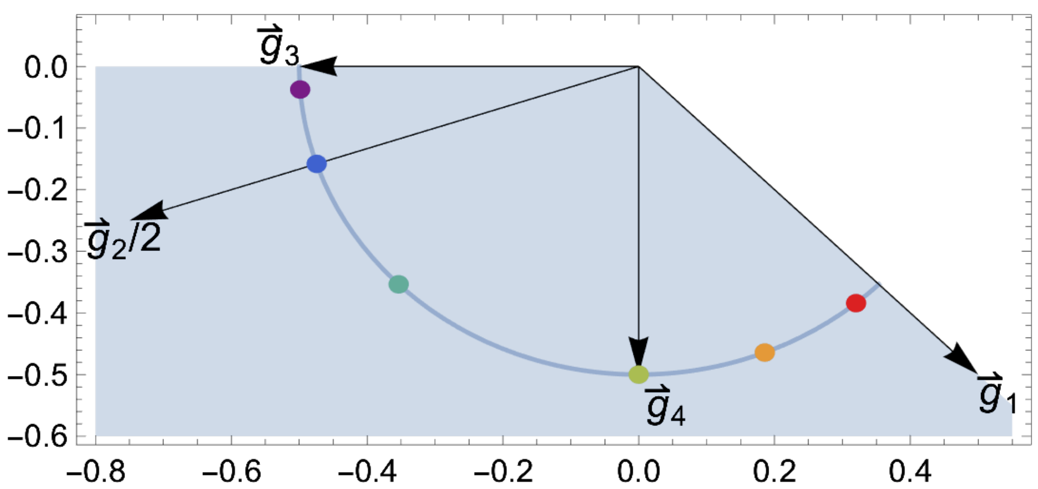
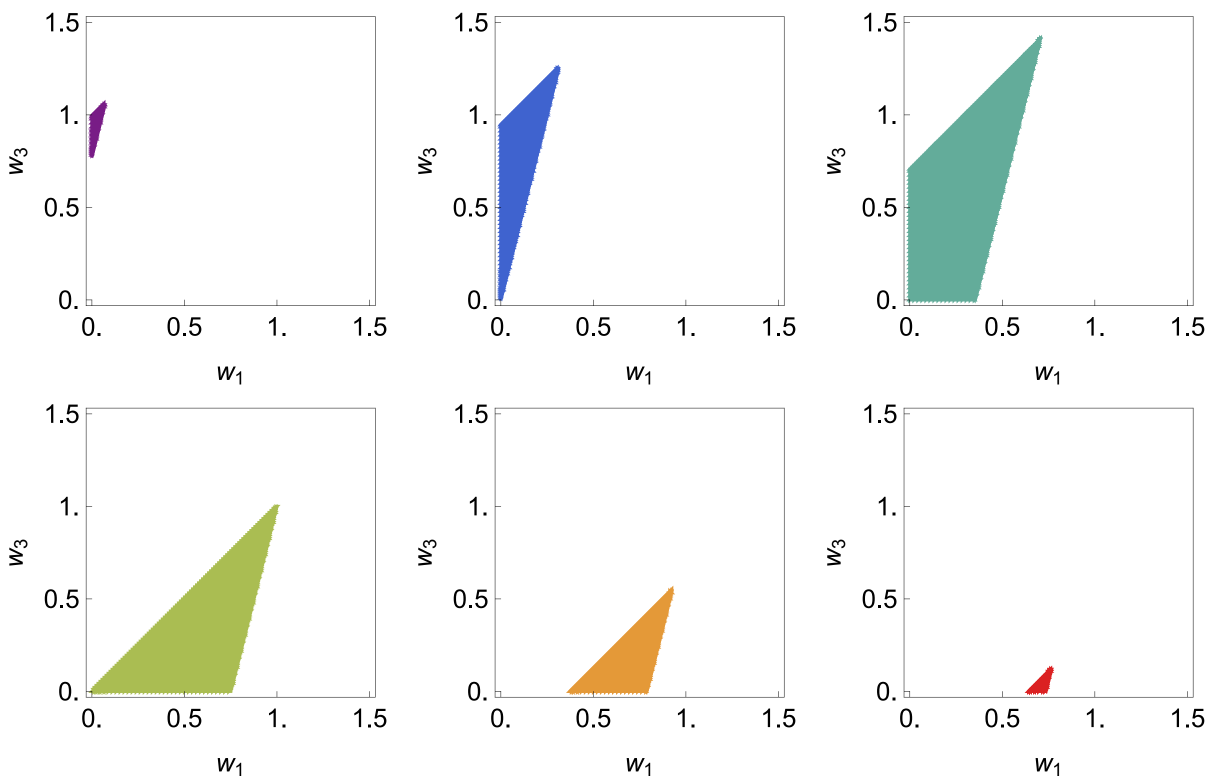
- The arbitrariness in solutions for $\vec{w}$ is always finite.





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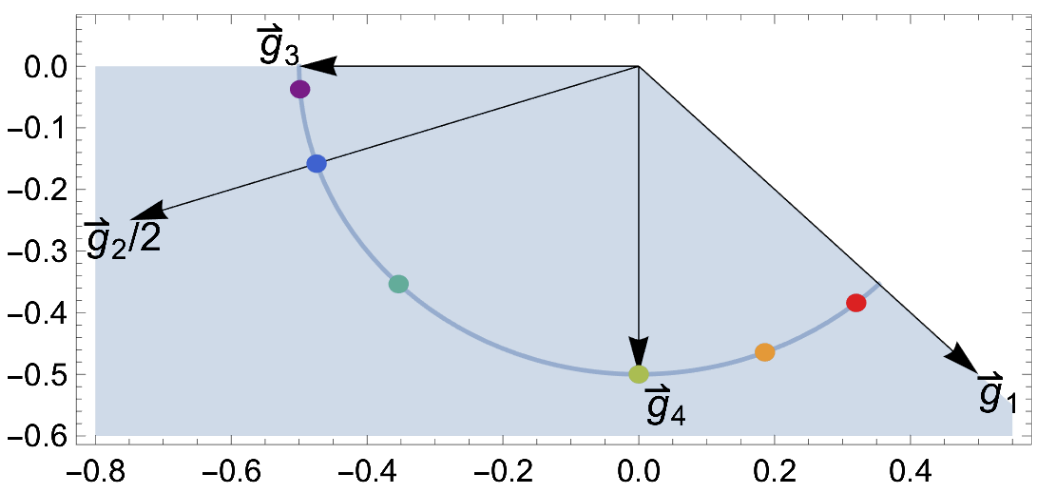
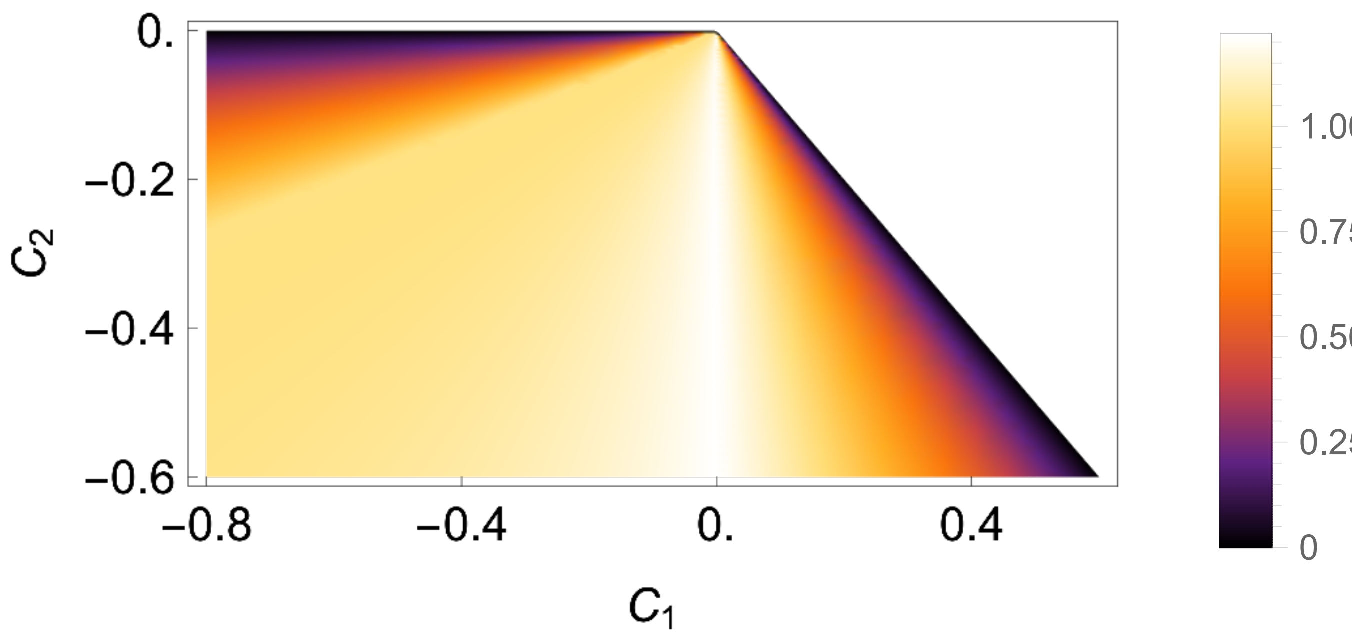
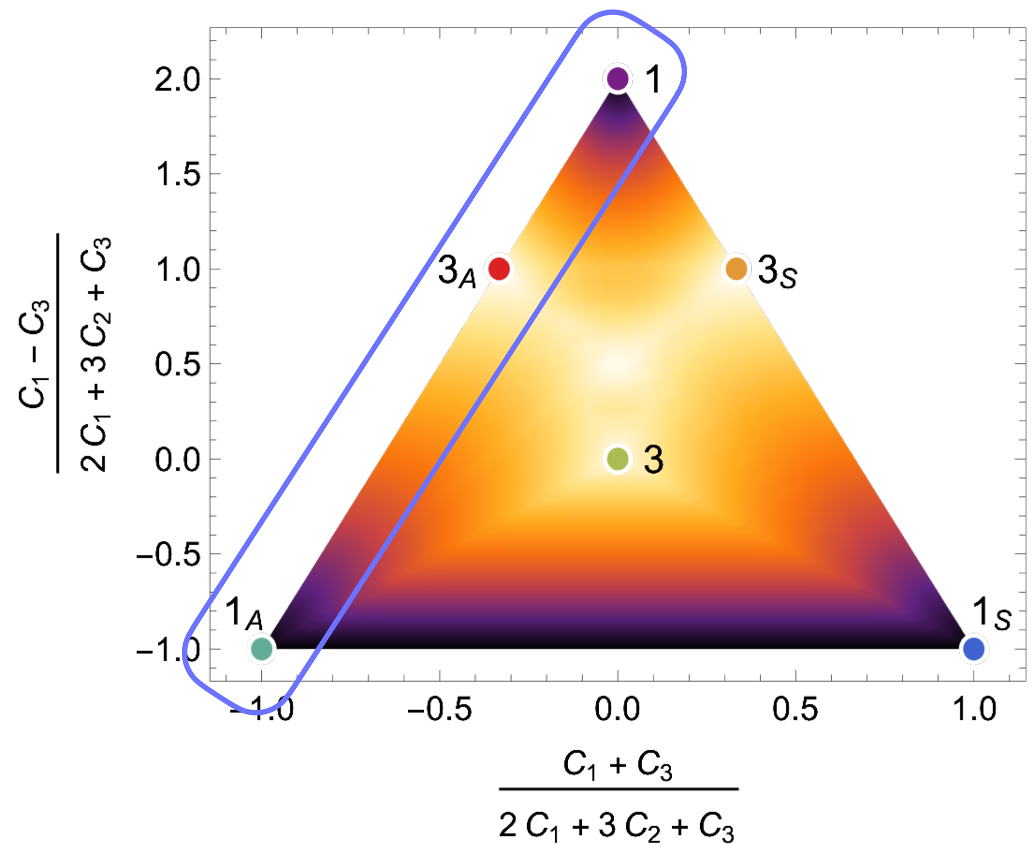
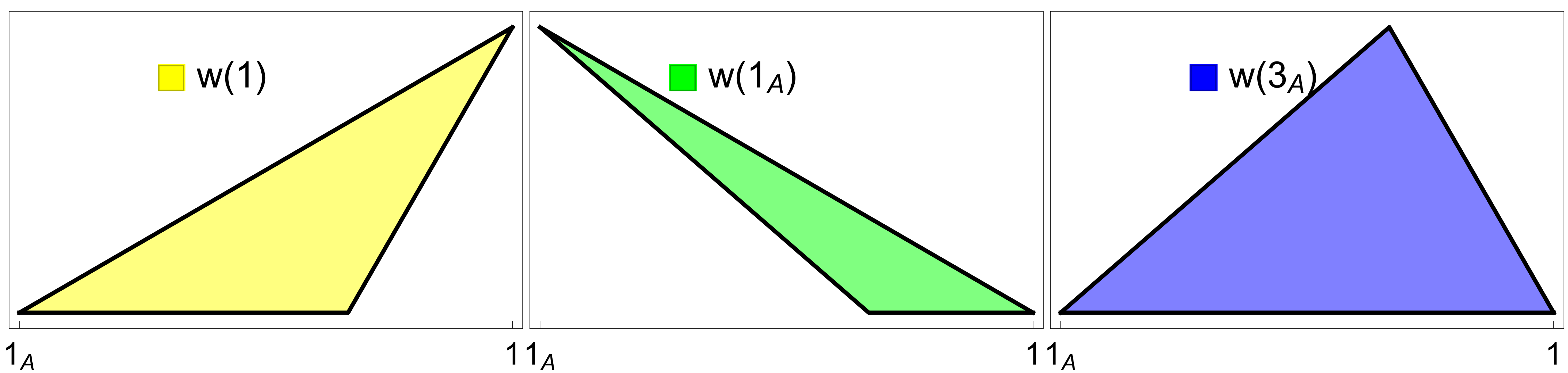


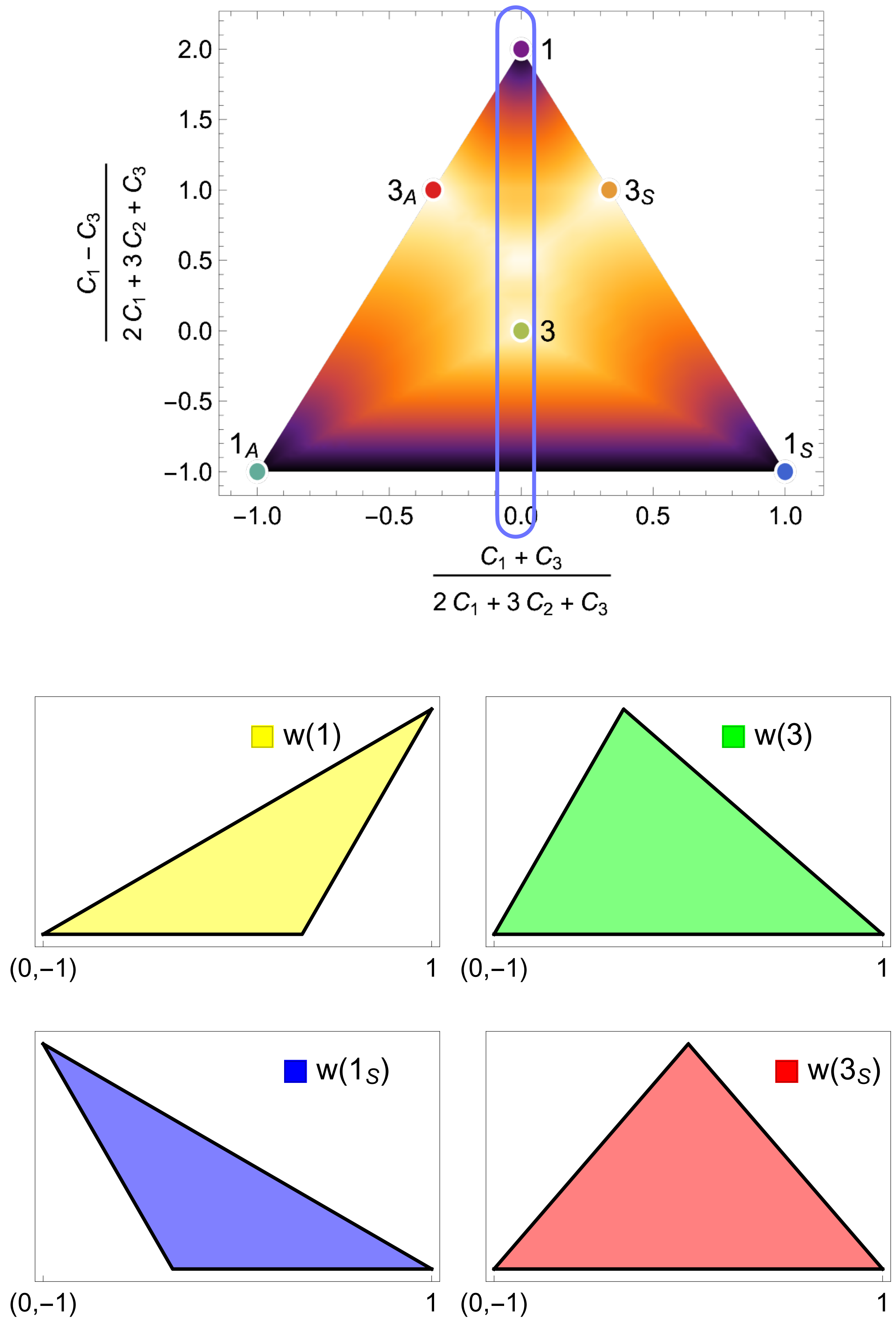




1
3_A
3_S
3
1_A
1_S
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
$\frac{C_1 - C_3}{2C_1 + 3C_2 + C_3}$
$\frac{C_1 + C_3}{2C_1 + 3C_2 + C_3}$



w(1)
$w(1_A)$
$w(3_A)$
1_A
1
1_A
1
1_A
1


1
3A
3S
3
1A
1S
C1 − C3 / 2 C1 + 3 C2 + C3
C1 + C3 / 2 C1 + 3 C2 + C3
w(1)
w(3)
w(1S)
w(3S)
(0,−1)
1