Amplitudes for Monopoles

Ofri Telem (UC Berkeley)

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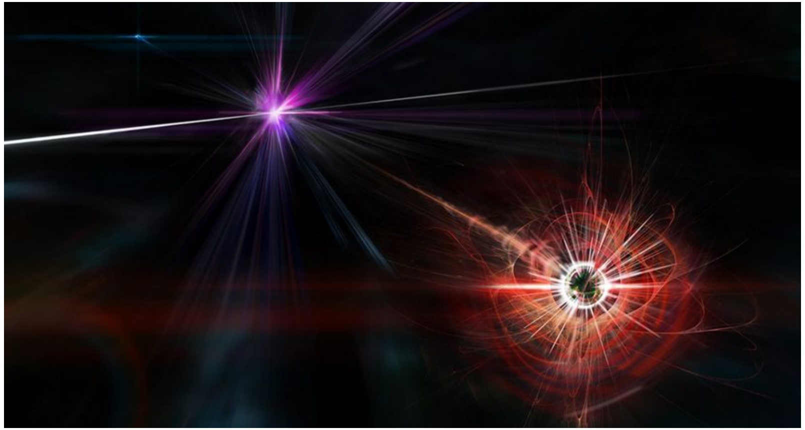
Upshot

- We solved the 40+ year old problem of finding a consistent S-matrix for monopoles & charges
- To do this we had to rethink some of the basic tenets of the S-matrix
- We fixed all 3-pt amplitudes and wrote the most generic 2->2 scattering amplitude
- Our formalism utilizes only symmetry not dynamics

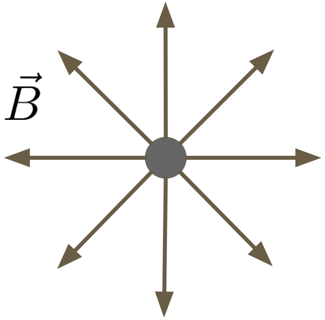
Monopoles:
On-Shell Success Where Lagrangian Field Theory Fails



Magnetic Monopoles

Sources of U(1) field with non-trivial winding number $\pi_1[\mathrm{U}(1)] = \mathbb{Z}$

$$\vec{B} = \frac{g}{r^2}\,\hat{r}$$



- At $r \gg m^{-1}$ effectively abelian Dirac '31
- At $r \sim m^{-1}$ have non-abelian cores 't Hooft / Polyakov '74

We won't care.
For us they are just scattering particles

- Lead to charge quantization Dirac '31, Wu & Yang '76

The Completeness Conjecture

Polchinski '03

- In any theoretical framework that requires charge to be quantized, there will exist magnetic monopoles
- In any fully unified theory, for every gauge field there will exist electric and magnetic sources with the minimum relative Dirac quantum

> "... the existence of magnetic monopoles seems like one of the safest bets that one can make about physics not yet seen."
>
> Polchinski '03

No global symmetries in gravity conjecture $\longrightarrow$ Completeness conjecture

Palti '19 (see also Banks Seiberg '11)

Monopoles: Where “No” Lagrangian Exists

- Since the days of Dirac, no clear way to write a local, Lorentz invariant Lagrangian for abelian monopoles & electric charges

    - Schwinger approach: non-local Lagrangian — Schwinger ‘66

    - Zwanziger approach: local Lagrangian, — Zwanziger ‘71
      loss of manifest Lorentz by introducing Dirac string

- The S-matrix for charge-monopole scattering is local and Lorentz invariant *up to a phase*, but the local Lagrangian is Lorentz-violating

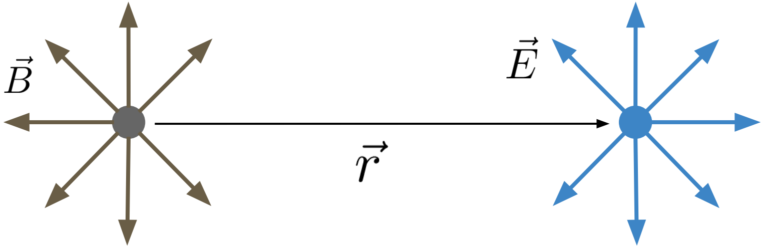
An On-Shell Opportunity

- The S-matrix has to be “special” in some way, otherwise why no local, Lorentz invariant Lagrangian?

- Dirac quantization should play a leading role
    - $q \equiv e\,g$ is half integer. Other half integers for the S-matrix? - Spins and helicities!
    - Helcities & spins are associated with 1 particle states
    - $q \equiv e\,g$ associated with charge-monopole pairs

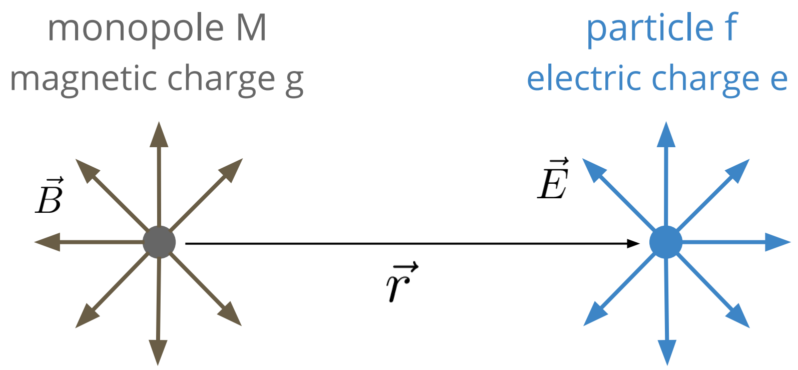
“*pairwise*” helicity?

Charge - Monopole Scattering:
A Non-Relativistic Prelude



Monopole and Charge: Extra Classical Angular Momentum

Thomson 1904



$$\vec{J}_{\text{field}} = \frac{1}{4\pi}\int \vec{r}\,' \times \left(\vec{E}\times\vec{B}\right)\, d^3r\,' = -\frac{g}{4\pi}\int \left(\vec{\nabla}\,'\cdot\vec{E}\right)\,\hat{r}\,'\, d^3r' = -eg\hat{r}$$

Distance independent!

In the quantum theory $\vec{J}_{\text{field}}$ quantized $\longrightarrow$ $eg = \dfrac{n}{2}$ Dirac quantization

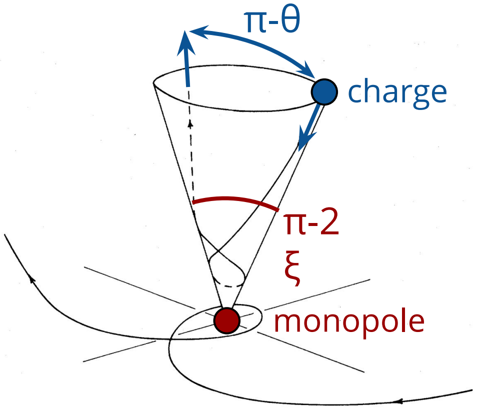
Saha 1936

Classical NR Charge-Monopole Scattering

Schwinger 1976   Boulware 1976

Conserved angular momentum: $\vec{L} = m\,\vec{r} \times \dot{\vec{r}} - eg\,\hat{r}$

$\vec{L} \cdot \hat{r} = -eg$ → motion on a cone

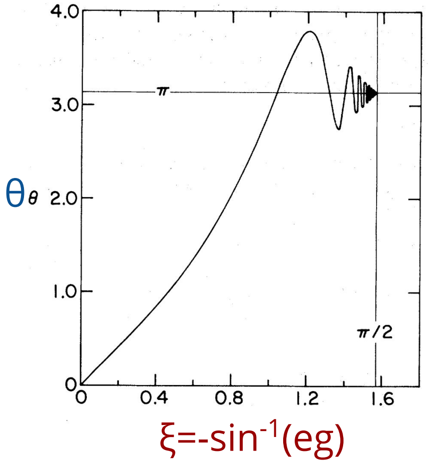


Scattering angle vs. “cone angle”:

$$\cos^2\left(\frac{\theta}{2}\right) = \cos^2(\xi)\,\sin^2\left(\frac{\pi}{2\cos\xi}\right)$$

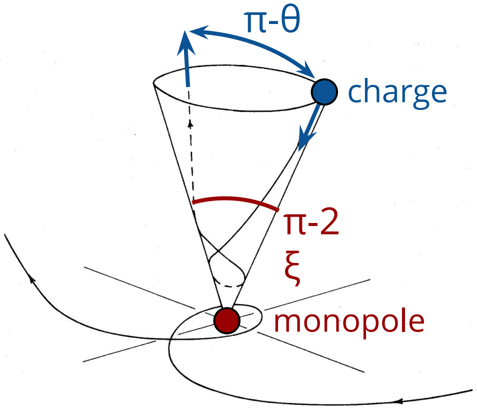
peaks at 1,2, 3... windings around the cone

winding = non-perturbative effect



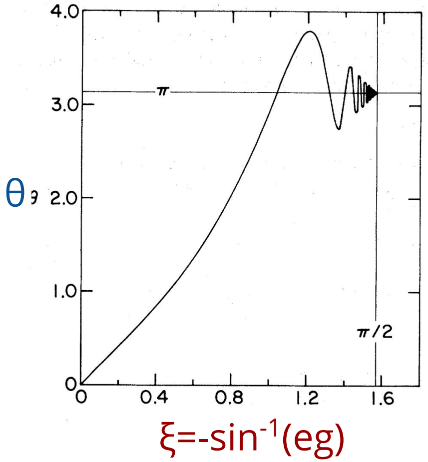
Classical NR Charge-Monopole Scattering

Schwinger 1976 Boulware 1976

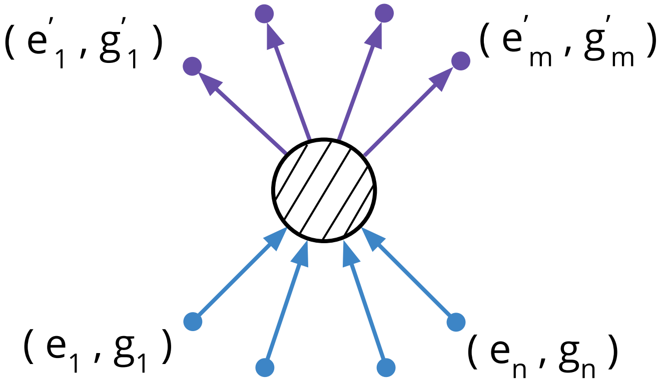


Take home: constants of motion deformed by long range EM interaction

angle of “hard” scattering (hitting the tip) constrained by “soft” cone

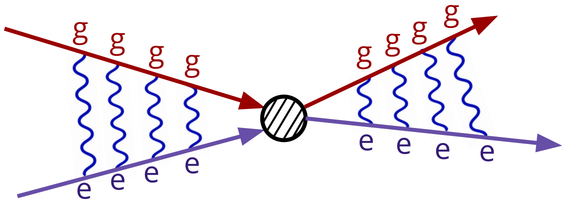


The S-Matrix for Charges, Monopoles and Dyons



The Problem

- Want to scatter quantum charges and monopoles
- The quantum charges and monopoles source a classical EM field
- The classical EM field deforms the definition of the quantum angular momentum operator
- How can we write a consistent S-matrix?



3.3 *Symmetries of the S-Matrix* 119

In virtually all known field theories, the effect of interactions is to add an interaction term V to the Hamiltonian, while leaving the momentum and angular momentum unchanged:

$$H = H_0 + V \ , \qquad \mathbf{P} = \mathbf{P}_0 \ , \qquad \mathbf{J} = \mathbf{J}_0 \ . \qquad (3.3.18)$$

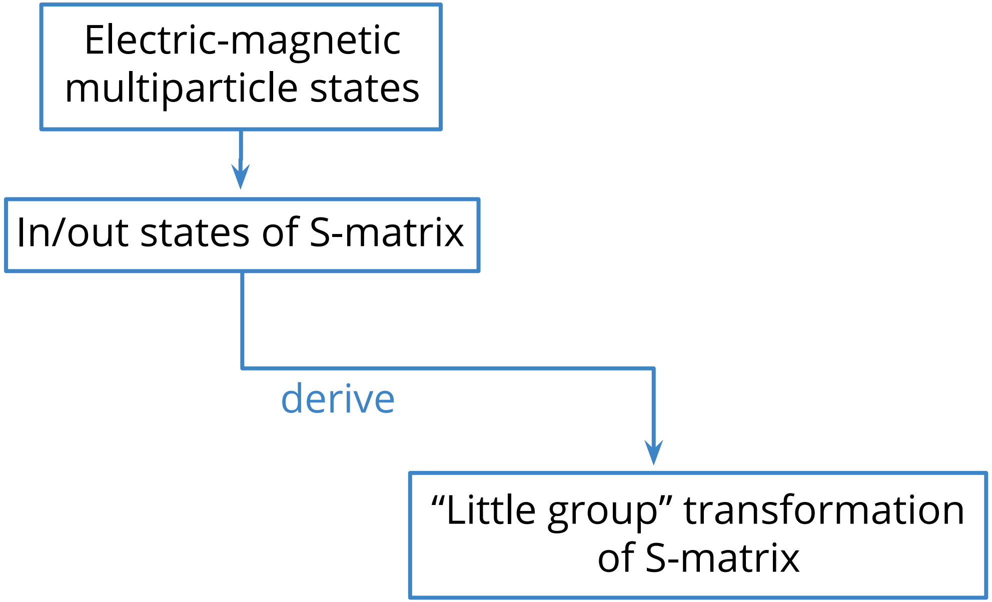
(The only known exceptions are theories with topologically twisted fields, such as those with magnetic monopoles, where the angular momentum of states depends on the interactions.)

S. Weinberg, The Quantum Theory of Fields Vol. 1

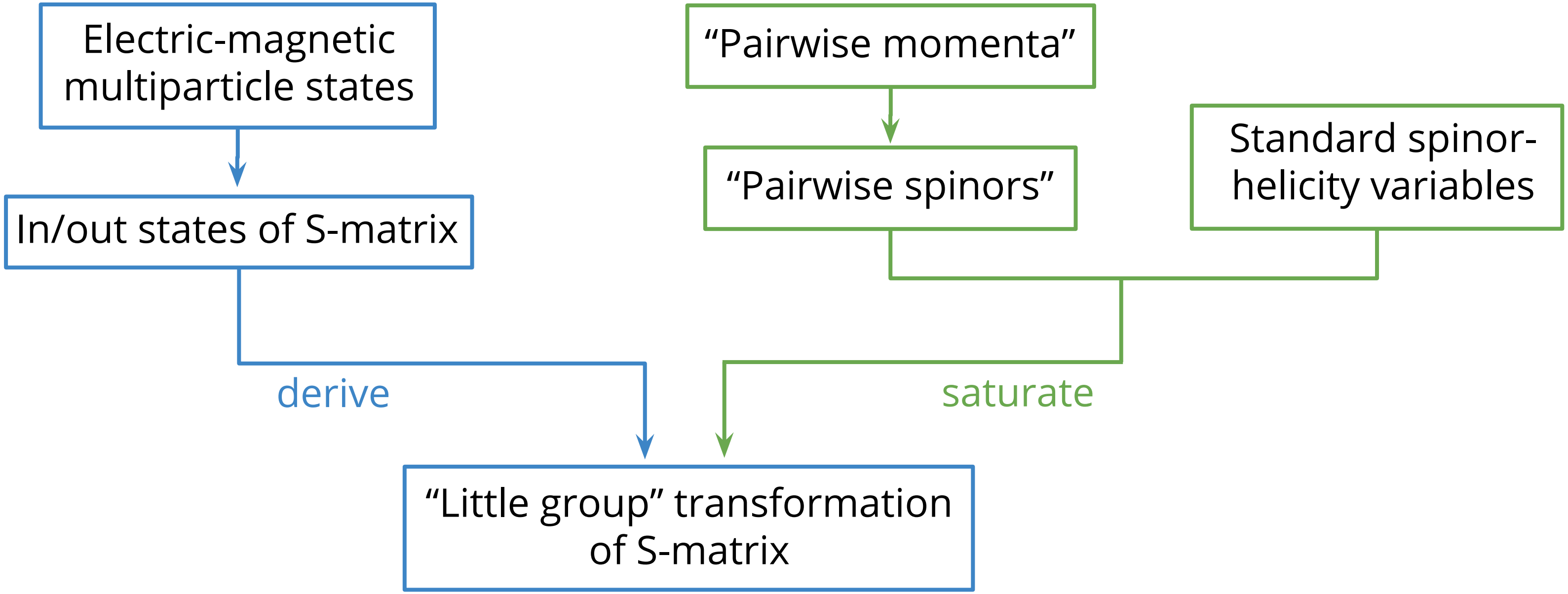
Idea: Re-Define the Hilbert Space

- We re-define the asymptotic multiparticle states of the S-matrix to include J_{EM}
- The new multiparticle are by definition not tensor products of single particles
- We derive the modified transformation rule for the electric-magnetic S-matrix
- We then use it to construct amplitudes for monopoles

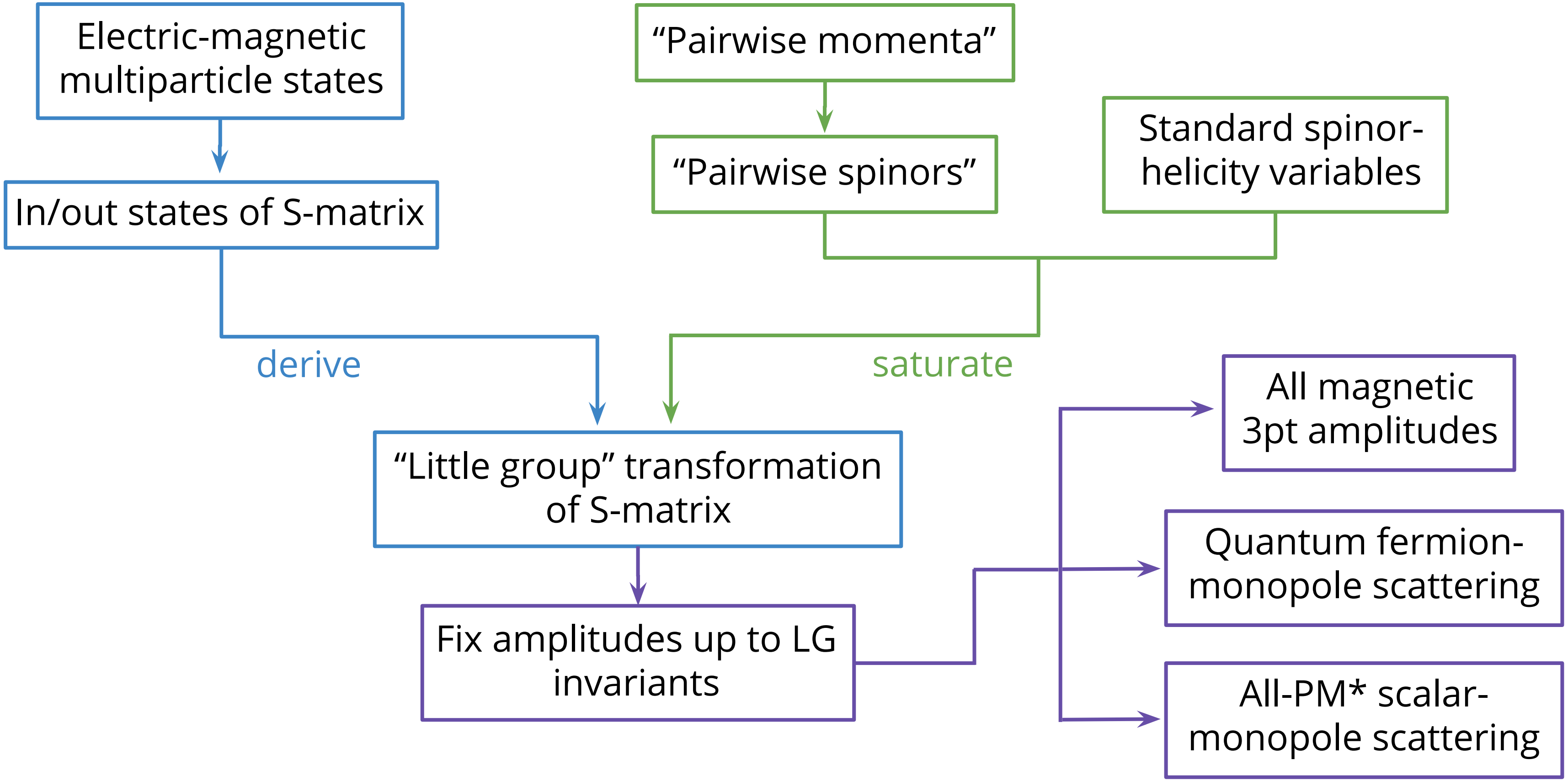
Talk Flowchart



Talk Flowchart



Talk Flowchart



* All orders in (q/J)

Talk Flowchart

Electric-magnetic multiparticle states

Defining Relativistic Quantum States

Wigner '37, Bargmann & Wigner '48

- Relativistic Quantum states are defined via their irreducible representations under Poincaré

Little group (**LG**)= compact subgroup of Lorentz which leaves a reference momentum invariant

Massive irreps. : $k = (m, 0, 0, 0) \longrightarrow$ little group SU(2), particles labeled by spin

Massless irreps. : $k = (E, 0, 0, E) \longrightarrow$ little group U(1)*, particles labeled by helicity

$$U(\Lambda)\,|p;\sigma\rangle \quad \text{induced from} \quad D(W)_{\sigma'\sigma}\,|k;\sigma'\rangle$$

Lorentz irrep. Little group irrep.

- Mutiparticle states? Usually tensor products of single particle states

The Quantum State of Scalar Monopole & Charge

Zwanziger '72

- How does Lorentz act on a 2-particle state with a scalar monopole and a scalar charge?
    - Naively, because they are scalars: $U(\Lambda)\,|p_1,\, p_2\rangle = |\Lambda p_1,\, \Lambda p_2\rangle$

      can't be true because that implies no $q_{12} \equiv e_1 g_2 - e_2 g_1$ contribution to the angular momentum

    - Instead: $U(\Lambda)\,|p_1, p_2; q_{12}\rangle = e^{i\, q_{12}\, \phi(p_1, p_2, \Lambda)}\,|\Lambda p_1, \Lambda p_2; q_{12}\rangle$

      where φ is a *pairwise* little group phase associated with *both* momenta

Wigner's Method for Scalar Charge-Monopole States

Zwanziger '72

Reference momenta in COM frame:

$$p_c = \sqrt{\frac{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}{s}}$$

$$(k_1)_\mu = (E_1^c, 0, 0, +p_c) \qquad (k_2)_\mu = (E_2^c, 0, 0, -p_c) \qquad E_{1,2}^c = \sqrt{m_{1,2}^2 + p_c^2}$$

Pairwise Little Group (LG) - All Lorentz transformations which leave both $k_{1,2}$ invariant

- Always just a U(1) - rotations around the z-axis
- Charge-monopole pairs labeled by their pairwise LG charge q_{12}

$$U\left[R_z(\phi)\right] |k_1, k_2\ ;\ q_{12}\rangle \ \equiv\ e^{iq_{12}\phi}\ |k_1, k_2\ ;\ q_{12}\rangle$$

Wigner's Method for Scalar Charge-Monopole States

Zwanziger '72

- Define canonical Lorentz transformation L_p as the COM → Lab transformation

$$p_1 = L_p k_1 \qquad p_2 = L_p k_2$$

- Wigner's trick: $$U(\Lambda)\,|p_1, p_2\,;\,q_{12}\rangle = U\left(L_{\Lambda p}\right)\, \underbrace{U\left(L^{-1}_{\Lambda p}\Lambda L_p\right)}_{\text{Pairwise LG rotation } = e^{iq_{12}\phi(p_1,p_2,\Lambda)}} |k_1, k_2\,;\,q_{12}\rangle$$

$$\boxed{U(\Lambda)\,|p_1, p_2; q_{12}\rangle = e^{i\,q_{12}\,\phi(p_1,p_2,\Lambda)}\;|\Lambda p_1, \Lambda p_2\,;\,q_{12}\rangle}$$

where $R_z(\phi) \equiv L^{-1}_{\Lambda p}\,\Lambda\,L_p.$ This is the *electric-magnetic two scalar state*

Electric-Magnetic Multiparticle States

hep-th/2010.13794

- We can easily generalize the two scalar state to any *electric-magnetic multiparticle states*

$$U(\Lambda)\,|p_1,\ldots,p_n\,;\,\sigma_1,\ldots,\sigma_n\,;\,q_{12},q_{13},\ldots q_{n-1,n}\rangle =$$

$$\underbrace{e^{i\sum_{i<j} q_{ij}\phi(p_i,p_j,\Lambda)}}_{\text{Pairwise LG}}\prod_{i=1}^{n}\underbrace{\mathcal{D}^{i}_{\sigma'_i\sigma_i}}_{\text{Single particle LG}}\,|\Lambda p_1,\ldots,\Lambda p_n\,;\,\underbrace{\sigma'_1,\ldots,\sigma'_n}_{\text{Spins / helicities}}\,;\,\underbrace{q_{12},q_{13},\ldots,q_{n-1,n}}_{\text{Pairwise helicities}}\rangle$$

$\mathcal{D}^{i}_{\sigma'_i\sigma_i}$ are the matrices (phases) for each single particle massive (massless) LG

- Electric-magnetic multiparticle states are *not* direct products of single particle states!

Talk Flowchart

Electric-magnetic multiparticle states

↓

In/out states of S-matrix

The Electric-Magnetic S-Matrix

- To define the S-matrix, we define electric-magnetic in- and out- states as

$$U(\Lambda)\ |p_1,\ldots,p_n\,;\,\pm\rangle = \prod_i \mathcal{D}\,(W_i)\ |\Lambda p_1,\ldots,\Lambda p_n\,;\,\pm\rangle\ e^{\pm i\Sigma}$$

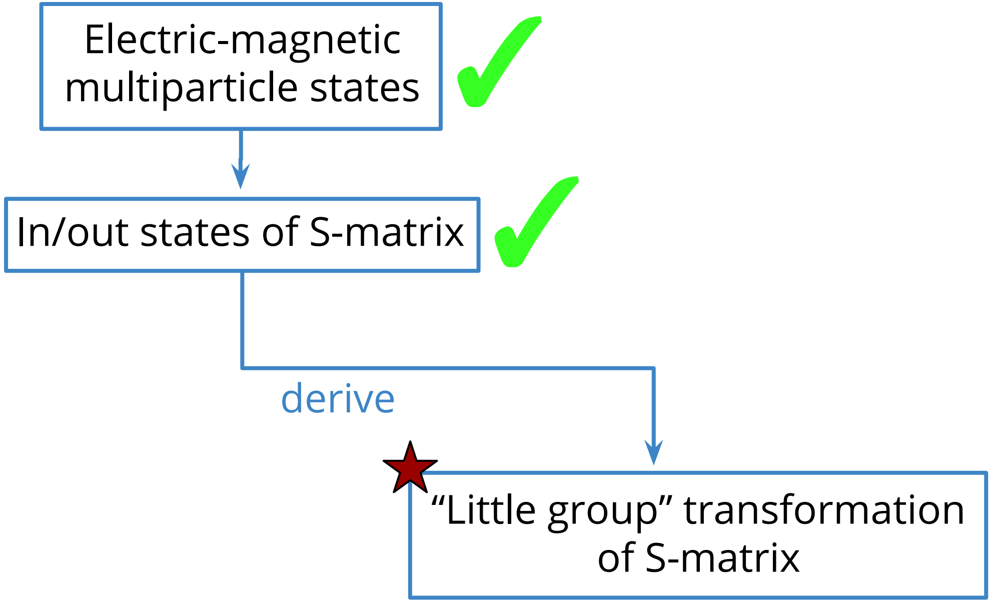
+ for 'in'

- for 'out'

where $\Sigma \equiv \sum_{i>j}^{n} q_{ij}\,\phi\,(p_i,p_j,\Lambda)$.

- The ± for the pairwise LG phase of the in / out state is very important!
- Has to be there to reproduce the angular momentum in the E&M field in the classical limit

Talk Flowchart



The Electric-Magnetic S-Matrix

- The S-matrix then transforms as:

$$S\left(p_1',\ldots,p_m' \mid p_1,\ldots,p_n\right) \equiv \langle p_1',\ldots,p_m';- \mid p_1,\ldots,p_n;+\rangle$$

$$= \langle p_1',\ldots,p_m';- \left| U(\Lambda)^\dagger\, U(\Lambda) \right| p_1,\ldots,p_n;+\rangle$$

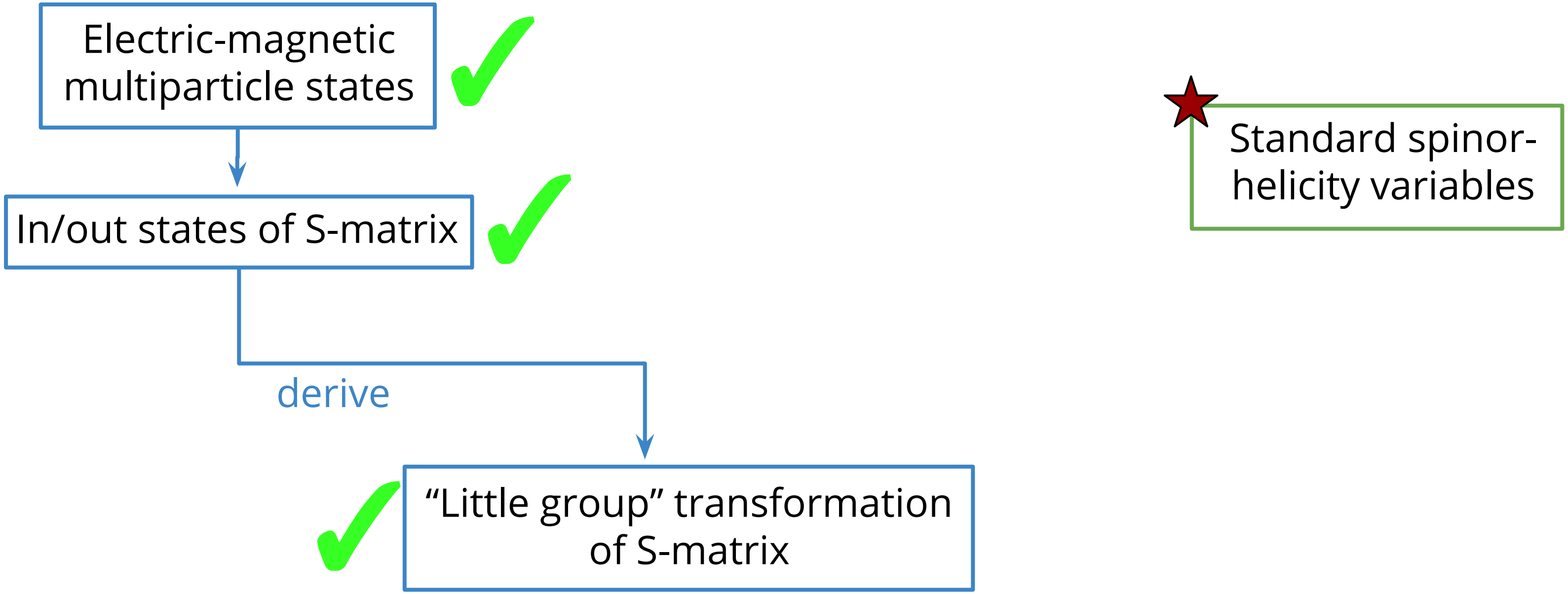
$$= e^{i(\Sigma_+ + \Sigma_-)} \prod_{i=1}^{m} \mathcal{D}\left(W_i\right)^\dagger \prod_{j=1}^{n} \mathcal{D}\left(W_j\right)\; S\left(\Lambda p_1',\ldots,\Lambda p_m' \mid \Lambda p_1,\ldots,\Lambda p_n\right)$$

with $\quad \Sigma_+ \equiv \sum_{i>j}^{n} q_{ij}\phi\left(p_i,p_j,\Lambda\right) \qquad \Sigma_- \equiv \sum_{i>j}^{m} q_{ij}\phi\left(p_i',p_j',\Lambda\right)$

- The extra *pairwise LG phase* is the key element in our formalism

- Every electric-magnetic S-matrix must transform with this phase by construction!

Talk Flowchart



The Standard Spinor-Helicity Formalism

De Causmaecker et al. '82
Parke, Taylor '86
....
Arkani-Hamed at al. '17

Standard definition: spinor helicity variables transform covariantly under the single particle LGs

Massless:

$$\underbrace{\Lambda_\alpha^{\ \beta}}_{\text{Lorentz trans.}} |p_i\rangle_\beta = \underbrace{e^{+\frac{i}{2}\phi(p_i,\Lambda)}}_{\text{LG phase}} |\Lambda p_i\rangle_\alpha \,, \qquad [p_i|_{\dot\beta} \underbrace{\tilde\Lambda^{\dot\beta}_{\ \dot\alpha}}_{\text{Lorentz trans.}} = \underbrace{e^{-\frac{i}{2}\phi(p_i,\Lambda)}}_{\text{LG phase}} [\Lambda p_i|_{\dot\alpha}$$

Massive:

$$\underbrace{\Lambda_\alpha^{\ \beta}}_{\text{Lorentz trans.}} |\mathbf{p_i}\rangle_\beta^I = \underbrace{\mathcal{D}^I_J(W_i)}_{\text{LG SU(2)}} |\mathbf{\Lambda p_i}\rangle_\alpha^J \,, \qquad [\mathbf{p_i}|_{I\dot\beta} \underbrace{\tilde\Lambda^{\dot\beta}_{\ \dot\alpha}}_{\text{Lorentz trans.}} = \underbrace{\mathcal{D}^{\dagger J}_I(W_i)}_{\text{LG SU(2)}} [\mathbf{\Lambda p_i}|_{J\dot\alpha}$$

Building Amplitudes - Like Playing ``Little Group Sudoku”

No monopole example: **1**-massive vector, **2**-massive scalar, 3-massless vector with helicity -1

$$S(p_1, p_2, p_3) \;=\; \underbrace{\mathcal{D}(W_1)}_{\substack{\text{particle 1} \\ \text{SU(2) rot.}}}\, \underbrace{e^{-i\phi_3}}_{\substack{\text{particle 3} \\ \text{U(1) rot.}}}\, S(\Lambda p_1, \Lambda p_2, \Lambda p_3)$$

- Need two $|\mathbf{p}_1^I\rangle_\alpha \equiv |\mathbf{1}\rangle_\alpha$, each one spin-½, to reproduce the spin-1 $\mathcal{D}(W_1)$
- Need two $|p_3\rangle_\beta \equiv |3\rangle_\beta$, each one helicity -½, to reproduce the $e^{-i\phi_3}$
- All spinor indices must be contracted!

$$\boxed{S \;=\; N\, \langle \mathbf{1}3\rangle^2}$$

Magnetic Amplitudes - Need Extra Building Blocks!

Monopole example: **1**-massive vector, **2**-massive scalar, 3-massless vector with helicity -1

2 has *electric* charge e, 3 has *magnetic* charge g, q_{23}=eg=2

$$S(p_1, p_2, p_3) \;=\; \underbrace{e^{-2i\phi_{23}}}_{\substack{\text{particles 2\&3} \\ \text{pairwise rot.}}} \underbrace{\mathcal{D}(W_1)}_{\substack{\text{particle 1} \\ \text{SU(2) rot.}}} \underbrace{e^{-i\phi_3}}_{\substack{\text{particle 3} \\ \text{U(1) rot.}}} S(\Lambda p_1, \Lambda p_2, \Lambda p_3)$$

- How can we account for the pairwise phase? Need a new kind of spinors!

Need New Building Blocks for the S-Matrix: Pairwise Spinors

Need a new kind of spinors $|p^\flat_{ij}\rangle$ transforming with a *pairwise* phase, i.e.

$$\Lambda_\alpha^\beta \left|p^\flat_{ij}\right\rangle_\beta = e^{\frac{i}{2}\phi_{ij}} \left|\Lambda p^\flat_{ij}\right\rangle_\alpha$$

- The spinor $|p^\flat_{ij}\rangle$ should be associated with *both* p_i and p_j
- It should have a U(1) phase even though particles i and j can be massive
- The U(1) phase has to be the same as the one in the transformation of the S-matrix

Talk Flowchart

Electric-magnetic multiparticle states ✓

↓

In/out states of S-matrix ✓

derive

↓

✓ "Little group" transformation of S-matrix

"Pairwise momenta"

Standard spinor-helicity variables ✓

Definition: Pairwise Momenta

Null linear combinations; "pairwise momenta"

$$\begin{cases} p_{ij}^{\flat+} = \dfrac{1}{E_i^c + E_j^c}\left[\left(E_j^c + p_c\right) p_i - \left(E_i^c - p_c\right) p_j\right] \\ p_{ij}^{\flat-} = \dfrac{1}{E_i^c + E_j^c}\left[\left(E_i^c + p_c\right) p_j - \left(E_j^c - p_c\right) p_i\right] \end{cases}$$

$$E_i^c = \sqrt{m_i^2 + p_c^2} \qquad p_c = \sqrt{\left((p_i \cdot p_j)^2 - m_i^2 m_j^2\right)/s_{ij}}$$

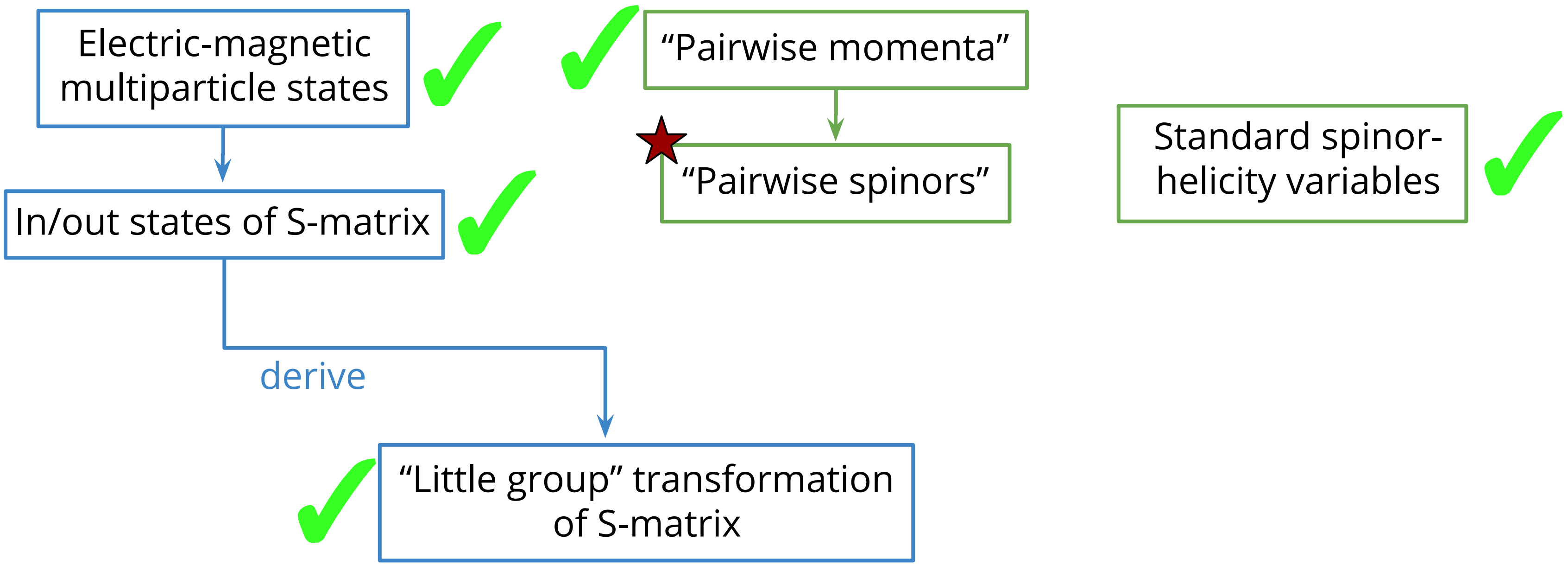
In the COM frame of particles i and j:

$$p_i \to k_i = (E_i^c, 0, 0, p_c) \qquad p_{ij}^{\flat+} \to k_{ij}^{\flat+} = (p_c, 0, 0, p_c)$$

$$p_j \to k_j = (E_j^c, 0, 0, -p_c) \qquad p_{ij}^{\flat-} \to k_{ij}^{\flat-} = (p_c, 0, 0, -p_c)$$

In this frame the pairwise momenta are null vectors with the same spatial parts as p_i , p_j

Talk Flowchart



Definition: Pairwise Spinors

In the COM frame of particles i and j, we define the *pairwise spinors* $|k_{ij}^{\flat\pm}\rangle$, $\left[k_{ij}^{\flat\pm}\right|$ so that

$$k_{ij}^{\flat\pm} \cdot \sigma_{\alpha\dot{\alpha}} = |k_{ij}^{\flat\pm}\rangle_{\alpha} \left[k_{ij}^{\flat\pm}\right|_{\dot{\alpha}}$$

Explicitly:

$$\left|k_{ij}^{\flat+}\right\rangle_{\alpha} = \sqrt{2p_c} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad , \quad \left|k_{ij}^{\flat-}\right\rangle_{\alpha} = \sqrt{2p_c} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\left[k_{ij}^{\flat+}\right|_{\dot{\alpha}} = \sqrt{2p_c}\,(1 \quad 0) \quad , \quad \left[k_{ij}^{\flat-}\right|_{\dot{\alpha}} = \sqrt{2p_c}\,(0 \quad 1)$$

This mirrors the definition of regular spinor-helicity variables, only with pairwise momenta

In other reference frames? Perform a Lorentz boost!

Pairwise Momenta & Spinors: Boosting Away from the COM Frame

Remember the canonical Lorentz boost from the quantum states?

$$p_i = L_p\, k_i \qquad p_j = L_p\, k_j$$

By linearity, we also have

$$p_{ij}^{\flat+} = L_p\, k_{ij}^{\flat+} \qquad p_{ij}^{\flat-} = L_p\, k_{ij}^{\flat-}$$

And also the spinor version

$$\left|p_{ij}^{\flat\pm}\right\rangle_{\alpha} = (\mathcal{L}_p)_{\alpha}^{\beta} \left|k_{ij}^{\flat\pm}\right\rangle_{\beta} \quad , \quad \left[p_{ij}^{\flat\pm}\right|_{\dot\alpha} = \left[k_{ij}^{\flat\pm}\right|_{\dot\beta} \left(\tilde{\mathcal{L}}_p\right)^{\dot\beta}_{\dot\alpha}$$

Pairwise Spinors: LG Transformation

- By another "Wigner trick" we get

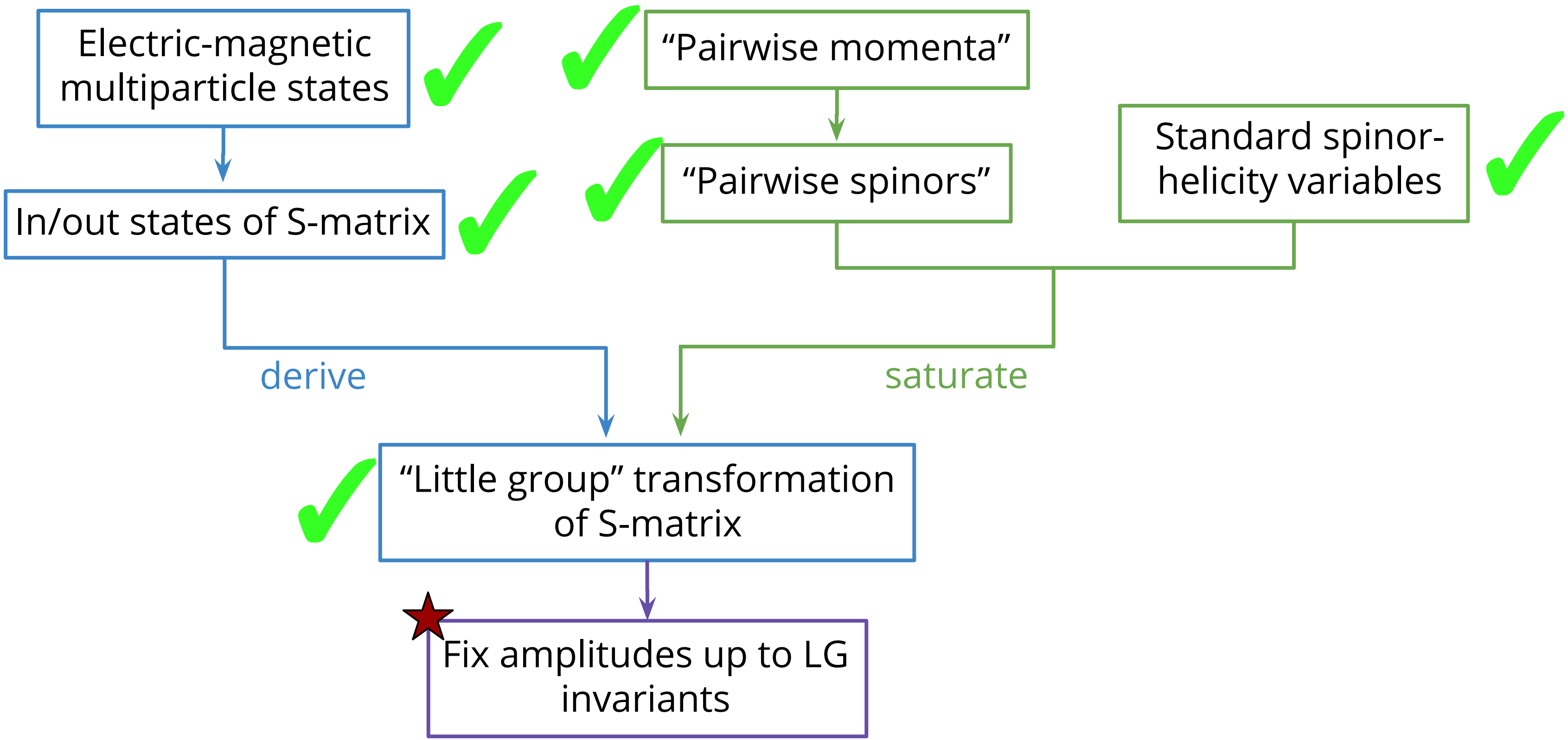
$$\Lambda_\alpha^{\ \beta} \left|p_{ij}^{\flat\pm}\right\rangle_\beta = \left(\mathcal{L}_{\Lambda p}\right)_\alpha^{\ \beta} \left(\mathcal{L}_{\Lambda p}^{-1}\Lambda\mathcal{L}_p\right)_\beta^{\ \gamma} \left|k_{ij}^{\flat\pm}\right\rangle_\gamma \longrightarrow$$

$$\Lambda_\alpha^{\ \beta} \left|p_{ij}^{\flat\pm}\right\rangle_\beta = e^{\pm\frac{i}{2}\phi_{ij}} \left|\Lambda p_{ij}^{\flat\pm}\right\rangle_\gamma$$

Same pairwise phase as the quantum states (Because the canonical boost is the same)

- Now we can use them as building blocks for electric-magnetic amplitudes!

Talk Flowchart



Constructing Electric-Magnetic Amplitudes

- We showed that the electric-magnetic S-matrix transforms as

$$S\left(\Lambda p_1', \ldots, \Lambda p_m' \mid \Lambda p_1, \ldots, \Lambda p_n\right) =$$

$$e^{-i(\Sigma_- + \Sigma_+)} \prod_{i=1}^{m} \mathcal{D}\left(W_i\right) \prod_{j=1}^{n} \mathcal{D}\left(W_j\right)^{\dagger} \; S\left(p_1', \ldots, p_m' \mid p_1, \ldots, p_n\right)$$

- To fix amplitudes up to LG invariants, we play “little group Sudoku” with an additional *pairwise* phase and *pairwise* spinors

- Our results are fully *non-perturbative*, as we never rely on a perturbative expansion

1st Surprise: Technically, No Forward Scattering!

Remember the beginning of every QFT textbook?

$$S_{\alpha\beta} = \delta(\alpha - \beta) - 2i\pi\delta^{(4)}\left(p_\alpha - p_\beta\right)\mathcal{A}_{\alpha\beta}$$

1st Surprise: Technically, No Forward Scattering!

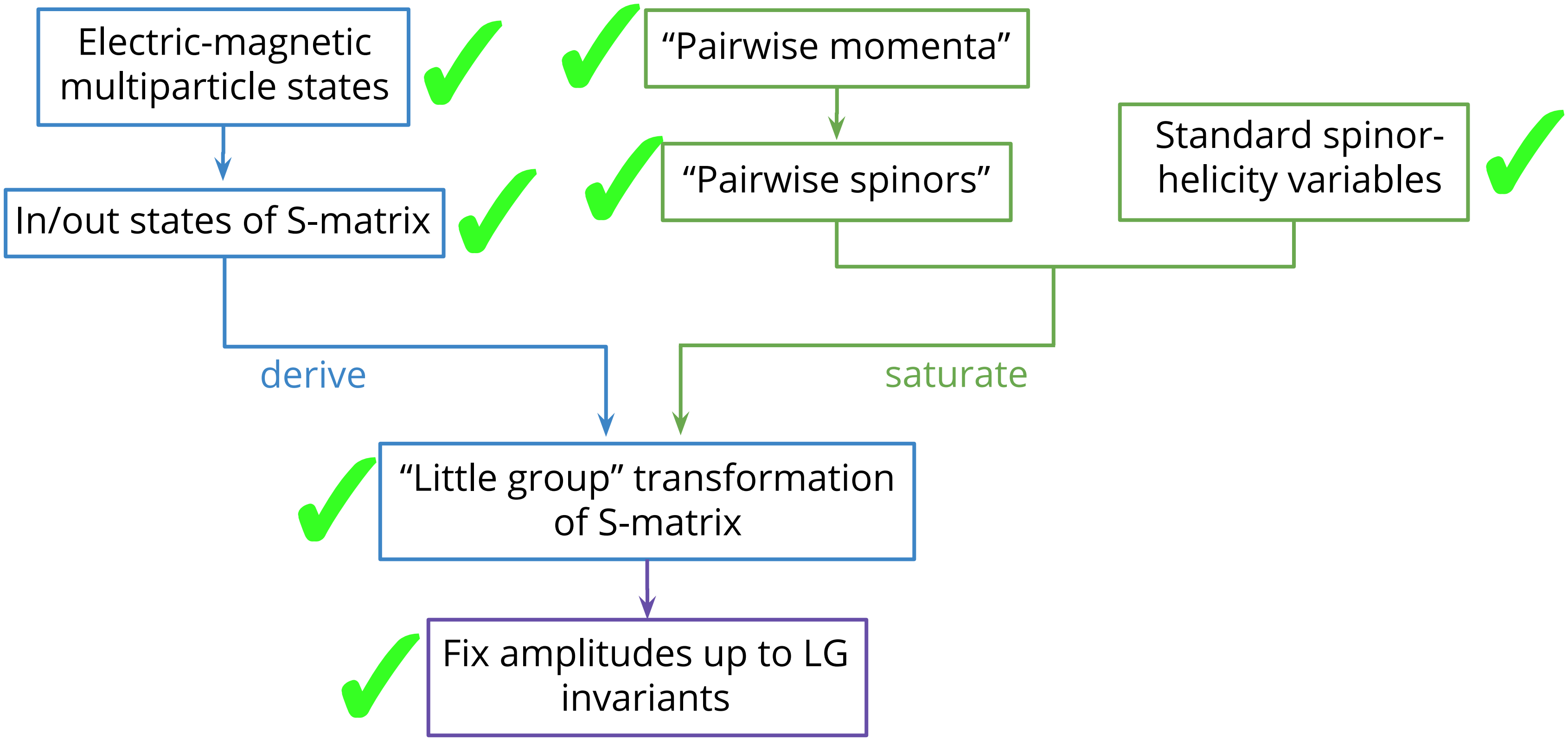
Remember the beginning of every QFT textbook?

$$S_{\alpha\beta} = \cancel{\delta(\alpha - \beta)} - 2i\pi\delta^{(4)}\left(p_\alpha - p_\beta\right)\mathcal{A}_{\alpha\beta}$$

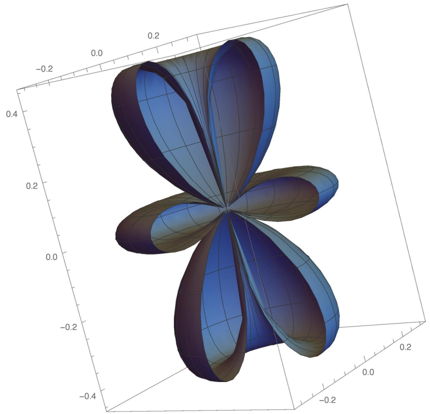
doesn’t transform with the pairwise LG phase!

- Forward scattering always involves the in-state incurring an (unphysical) phase
- This unphysical phase encodes the “Dirac string” dependence
- In certain case, forward scattering completely forbidden (fermion-monopole)

Talk Flowchart

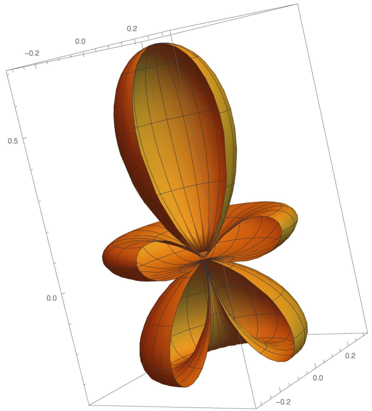


Results



$$Y_{\frac{5}{2},-\frac{1}{2}}(\theta,\phi)$$

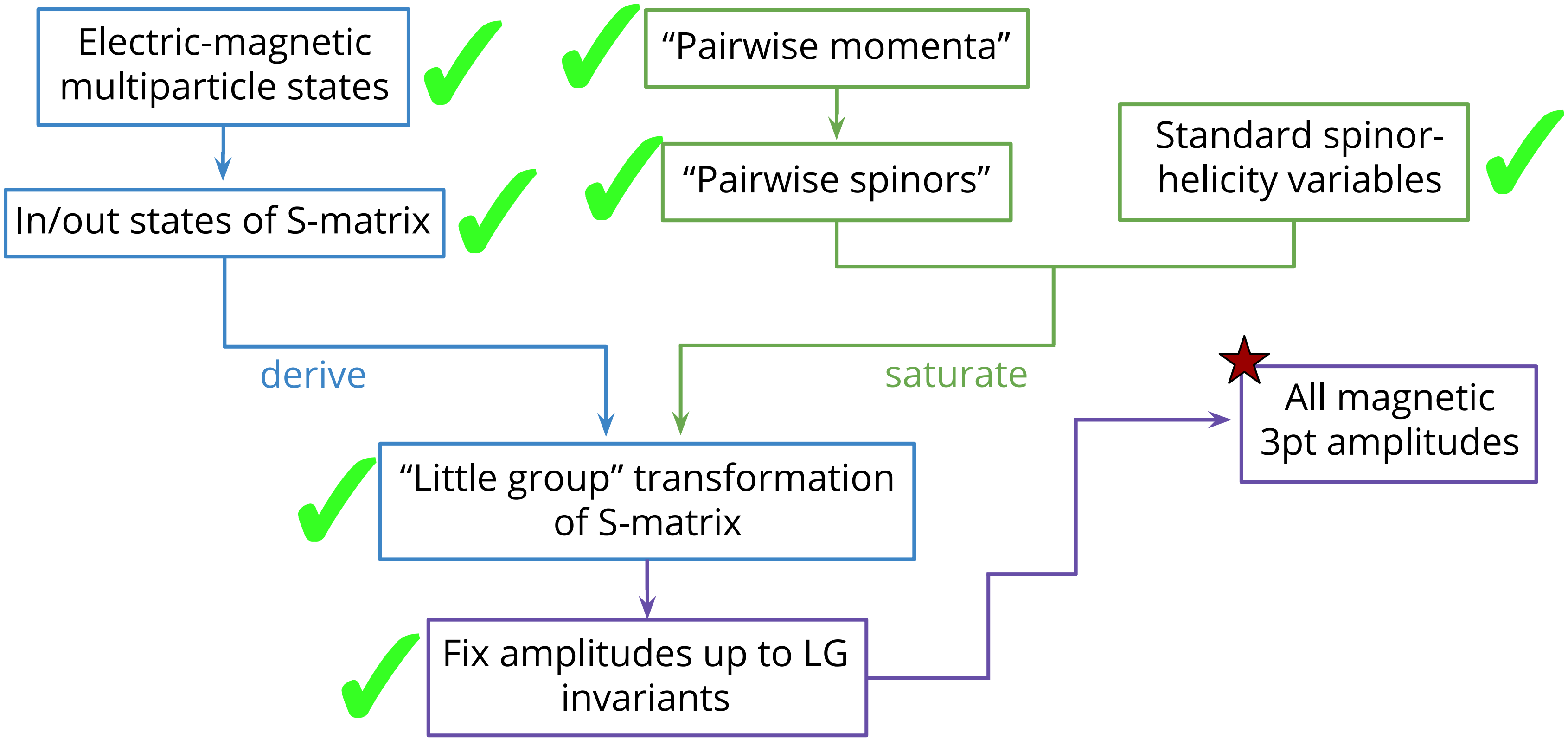
Spherical Harmonics



$${}_{\frac{1}{2}}Y_{\frac{5}{2},-\frac{1}{2}}(\theta,\phi)$$

Monopole - Spherical Harmonics

Talk Flowchart

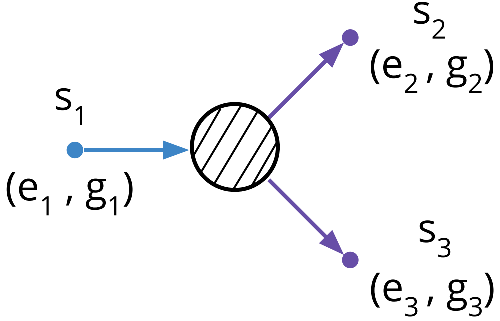


All 3-pt Electric-Magnetic Amplitudes

- Pairwise LG + individual LGs allow us to classify all 3-pt amplitudes
- This generalizes the massive amplitude formalism by Arkani-Hamed at al. ‘17
- Our amplitudes & selection rules reduce to theirs for $q = 0$

Example 3-Massive Electric-Magnetic Amplitudes

$q_{23} \equiv e_2 g_3 - e_3 g_2$



- To saturate the individual SU(2) LG for each particle, need

$$\left(\left\langle \mathbf{1}\right|^{2s_1}\right)^{\{\alpha_1\ldots\alpha_{2s_1}\}} \left(\left\langle \mathbf{2}\right|^{2s_2}\right)^{\{\beta_1\ldots\beta_{2s_2}\}} \left(\left\langle \mathbf{3}\right|^{2s_3}\right)^{\{\gamma_1\ldots\gamma_{2s_3}\}}$$

- In the q=0 case, contracted with a bunch of pairwise LG inert $\epsilon_{\delta\rho},\ p_\delta{}^{\dot\delta} p_{\dot\delta\rho}$

- In the our case, contracted with a bunch of $\left|p_{ij}^{\flat\pm}\right\rangle_\delta$ with overall pairwise LG weight $-q_{23}$

Example 3-Massive Electric-Magnetic Amplitudes

- Unique result with correct pairwise LG weight:

$$S = \left(\langle \mathbf{1}|^{2s_1}\right)^{\{\alpha_1\ldots\alpha_{2s_1}\}} \left(\langle \mathbf{2}|^{2s_2}\right)^{\{\beta_1\ldots\beta_{2s_2}\}} \left(\langle \mathbf{3}|^{2s_3}\right)^{\{\gamma_1\ldots\gamma_{2s_3}\}} \times$$

$$\sum_i a_i \left(|p_{23}^{\flat +}\rangle^{\hat{s}+q} |p_{23}^{\flat -}\rangle^{\hat{s}-q}\right)_{\{\alpha_1,\ldots,\alpha_{2s_1}\}\{\beta_1,\ldots,\beta_{2s_2}\}\{\gamma_1,\ldots,\gamma_{2s_3}\}} \qquad \hat{s} = \sum s_i$$

ŝ±q non-negative integers → $\boxed{\textit{Selection rule: } |q| \leq \hat{s}}$

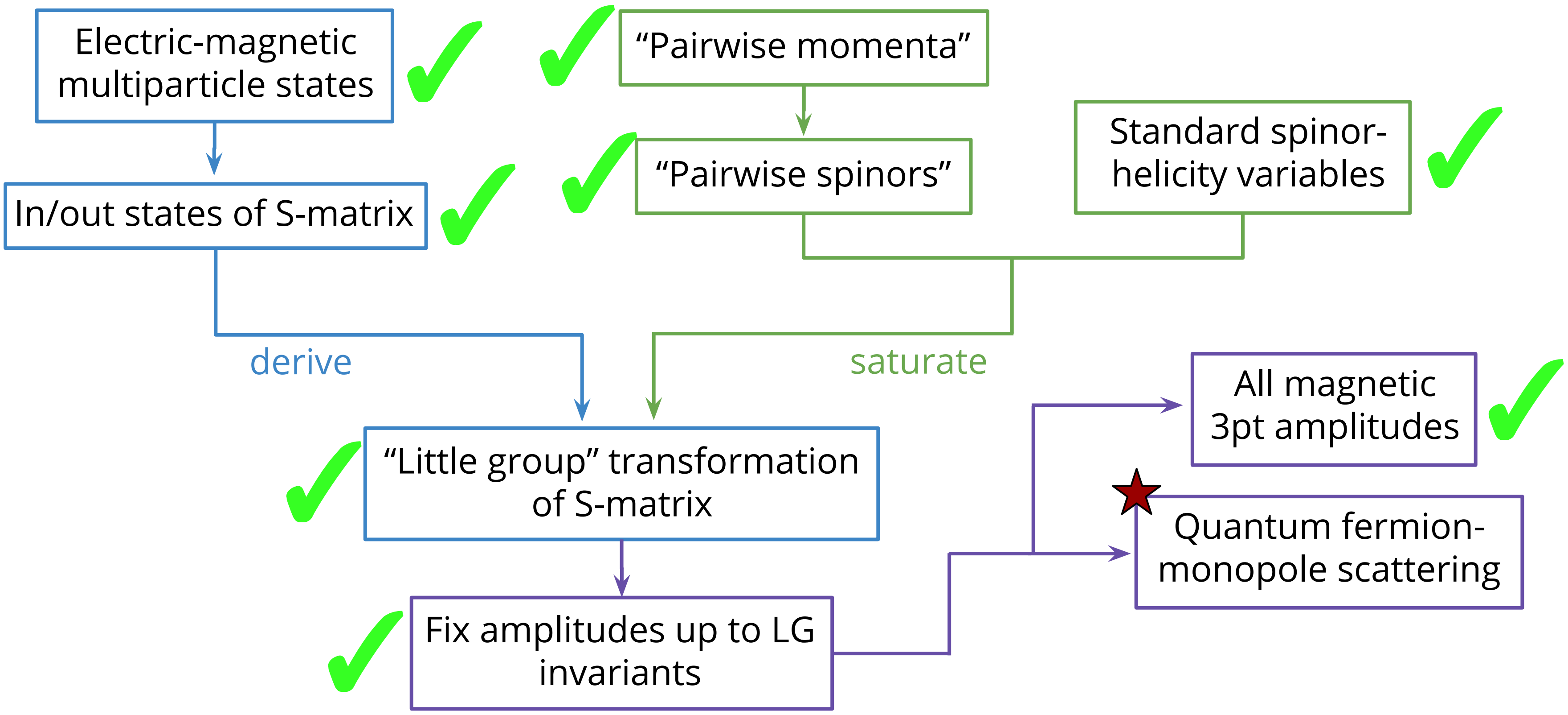
In particular a *massive scalar dyon* cannot decay to *two massive scalar dyons*

(Unless all 3 are dual to electric charges simultaneously)

All 3-pt Electric-Magnetic Amplitudes

Kinematics	Selection Rule	Amplitude
Incoming massive particle two outgoing massive particles	$\lvert q\rvert \leq \hat{s}$	$S^{q}_{\{\alpha_1,\ldots,\alpha_{2s_1}\}\{\beta_1,\ldots,\beta_{2s_2}\}\{\gamma_1,\ldots,\gamma_{2s_3}\}} = \sum_{i=1}^{C} a_i \left(\lvert w\rangle^{\hat{s}-q}\lvert r\rangle^{\hat{s}+q}\right)_{\{\alpha_1,\ldots,\alpha_{2s_1}\}\{\beta_1,\ldots,\beta_{2s_2}\}\{\gamma_1,\ldots,\gamma_{2s_3}\}}$
Incoming massive particle, outgoing massive particle + massless particle, unequal mass	$\lvert h+q\rvert \leq \hat{s}$	$S^{h,q,\ \text{unequal}}_{\{\alpha_1,\ldots,\alpha_{2s_1}\}\{\beta_1,\ldots,\beta_{2s_2}\}} = \sum_{i=1}^{C}\sum_{j,k} a_{ijk}\,\langle ur\rangle^{\max(j+k,0)}\langle vw\rangle^{\max(-j-k,0)} \left(\lvert u\rangle^{\frac{\hat{s}}{2}-h-j}\lvert v\rangle^{\frac{\hat{s}}{2}+h+k}\lvert w\rangle^{\frac{\hat{s}}{2}-q+j}\lvert r\rangle^{\frac{\hat{3}}{2}+q-k}\right)_{\{\alpha_1,\ldots,\alpha_{2s_1}\}\{\beta_1,\ldots,\beta_{2s_2}\}}$
Incoming massive particle, outgoing massive particle + massless particle, equal mass	none	$S^{h,q,\ \text{equal}}_{\{\alpha_1\ldots\alpha_{2s_1}\}\{\beta_1\ldots\beta_{2s_2}\}} = \sum_{i=1}^{C}\sum_{j}\sum_{k=-j}^{j} x^{h+q+j}\langle ur\rangle^{\max[2q+j-k,0]}\langle vw\rangle^{\max[-2q-j+k,0]} \left(\lvert u\rangle^{j+k}\lvert w\rangle^{j-k}\epsilon^{\hat{s}-j}\right)_{\{\alpha_1\ldots\alpha_{2s_1}\}\{\beta_1\ldots\beta_{2s_2}\}}$
Incoming massive particle, two outgoing massless particles	$\lvert\Delta-q\rvert \leq s$	$S^{q}_{\{\alpha_1,\ldots,\alpha_{2s}\}} = \sum_{ij} a_{ij}\left(\lvert u\rangle^{s/2-i-\Delta}\lvert v\rangle^{s/2-j+\Delta}\lvert w\rangle^{s/2+j-q}\lvert r\rangle^{s/2+i+q}\right)_{\{\alpha_1,\ldots,\alpha_{2s}\}}$ $[uv]^{\max[\Sigma+(s-i-j)/2,0]}\langle uv\rangle^{\max[-\Sigma-(s+i+j)/2,0]}(\langle uw\rangle[vr])^{\frac{1}{2}\max[i-j,0]}([uw]\langle vr\rangle)^{\frac{1}{2}\max[j-i,0]}$

Talk Flowchart



Fermion-Monopole Scattering: Solving a 45-year Mystery

In 1977 Kazama, Yang and Goldhaber considered the quantum scattering of a fermion in the field of a static monopole, by solving the Dirac equation in its background.

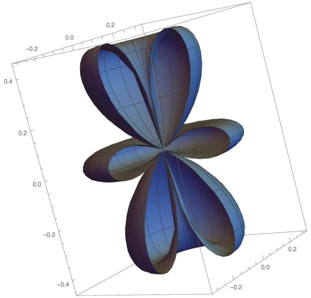
They found several counterintuitive results which we can now explain

Fermion-Monopole Scattering: NRQM Result

1. The partial wave decomposition doesn't start at $J=0$, but at $J=|q|-\frac{1}{2}$ with $q=eg$

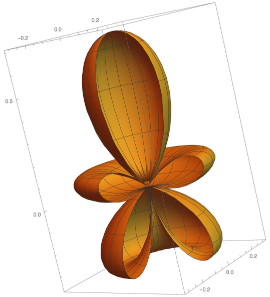
Fermion-Monopole Scattering: NRQM Result

2. The angular wavefunctions are not Y_{lm} but are actually ${}_qY_{lm}$ - *monopole* spherical harmonics



$$Y_{\frac{5}{2},-\frac{1}{2}}(\theta,\phi)$$

Spherical Harmonics

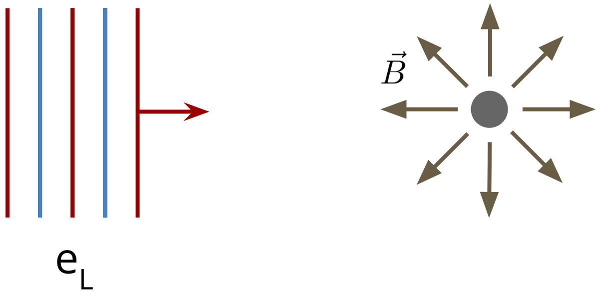


$${}_{\frac{1}{2}}Y_{\frac{5}{2},-\frac{1}{2}}(\theta,\phi)$$

Monopole - Spherical Harmonics

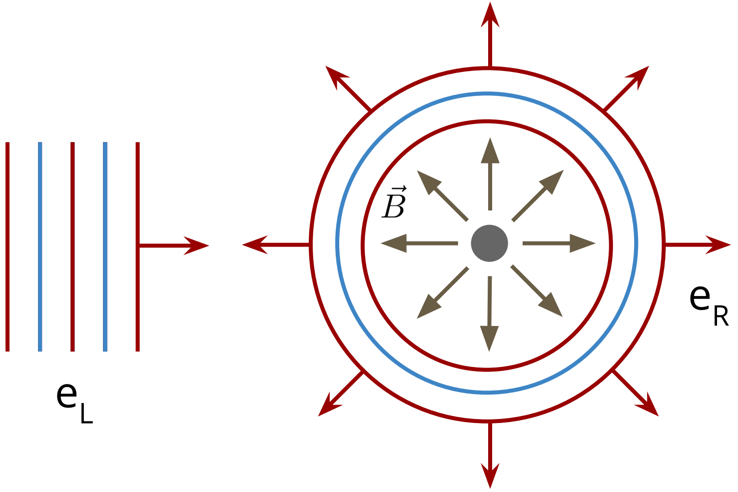
Fermion-Monopole Scattering: NRQM Result

3. At the lowest partial wave, the fermion helicity must flip

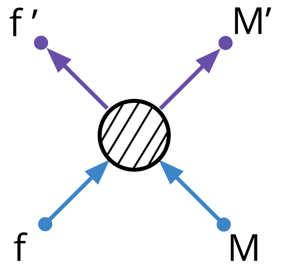


Fermion-Monopole Scattering: NRQM Result

3. At the lowest partial wave, the fermion helicity must flip



Fermion-Monopole Scattering: Our Method



- Most general partial wave with the right pairwise helicity:

Massive spinor for incoming fermion

$$S_J = \frac{2J+1}{s^{J+1}} \sum_{\sigma=\pm} a_\sigma \langle \mathbf{f} p^{\flat\sigma}_{fM}\rangle \left(\langle p^{\flat+}_{fM}|^{J+q_\sigma} \langle p^{\flat-}_{fM}|^{J-q_\sigma}\right)^{\{\alpha_1,\ldots,\alpha_{2J}\}} \times$$

Right amount of pairwise spinors for overall pairwise helicity q

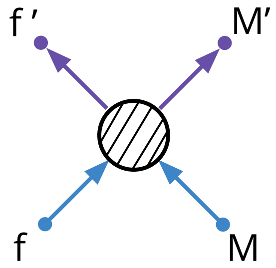
2J spinor indices contracted between in- and out- state for J partial wave

$$\sum_{\sigma'=\pm} a'_{\sigma'} \langle \mathbf{f}' p^{\flat\sigma'}_{f'M'}\rangle \left(|p^{\flat+}_{f'M'}\rangle^{J+q_{\sigma'}} |p^{\flat-}_{f'M'}\rangle^{J-q_{\sigma'}}\right)_{\{\alpha_1,\ldots,\alpha_{2J}\}}$$

$$q_\pm \equiv q \mp \frac{1}{2}$$

Massive spinor for incoming fermion

Fermion-Monopole Scattering: Our Method

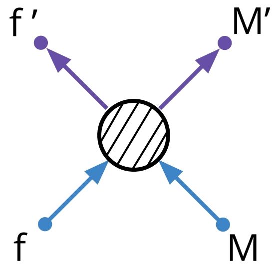


- Substituting the explicit values & contracting spinors symmetrically,

$$\boxed{S_J = \frac{2J+1}{s} \sum_{\sigma,\sigma'=\pm} a_\sigma a_{\sigma'} \, \langle \mathbf{f} p^{\flat\sigma}_{fM} \rangle \, \langle \mathbf{f}' p^{\flat\sigma'}_{f'M'} \rangle \; \mathcal{D}^{J*}_{q_\sigma, -q_{\sigma'}}(\alpha, \beta, \gamma)}$$

$$\underbrace{\mathcal{D}^{J}_{q_\sigma, -q_{\sigma'}}\overbrace{(\alpha, \beta, \gamma)}^{\text{Euler rotation between in and out directions}}}_{\text{Wigner D-matrix}} \equiv \langle J, q_\sigma | e^{-i\alpha J_z} e^{-i\beta J_y} e^{-i\gamma J_z} | J, q_{\sigma'} \rangle$$

The Fermion-Monopole "Jacob-Wick" Formula

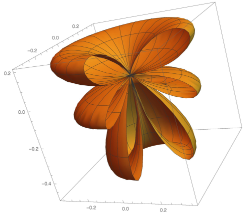


$$S_J \;=\; \frac{2J+1}{s} \sum_{\sigma,\sigma'=\pm} a_\sigma a_{\sigma'} \; \langle \mathbf{f} p^{\flat\sigma}_{fM} \rangle \, \langle \mathbf{f}' p^{\flat\sigma'}_{f'M'} \rangle \;\; \mathcal{D}^{J*}_{q_\sigma, -q_{\sigma'}}(\alpha, \beta, \gamma)$$

- This equation has nearly* all of the NRQM result hidden in it!

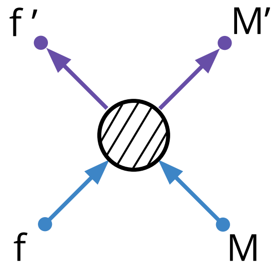
1. The Wigner D-matrix vanishes unless $J \geq |q_\sigma| \geq |q| - \frac{1}{2}$
2. Wigner D-matrices are equivalent to a *monopole* spherical harmonics

$$\mathcal{D}^{l*}_{q,m}(\Omega) = \sqrt{\frac{4\pi}{2l+1}}\; {}_qY_{l,m}(-\Omega)$$



* The numbers $a_\sigma a_{\sigma'} = e^{i\delta(J)}$ are phase shifts, extrapolated from NRQM

The Fermion-Monopole "Jacob-Wick" Formula



$$\boxed{S_J \;=\; \frac{2J+1}{s} \sum_{\sigma,\sigma'=\pm} a_\sigma a_{\sigma'} \;\langle \mathbf{f} p^{\flat\sigma}_{fM} \rangle \, \langle \mathbf{f}' p^{\flat\sigma'}_{f'M'} \rangle \;\; \mathcal{D}^{J*}_{q_\sigma, -q_{\sigma'}}(\alpha,\beta,\gamma)}$$

- This equation has nearly* all of the NRQM result hidden in it!

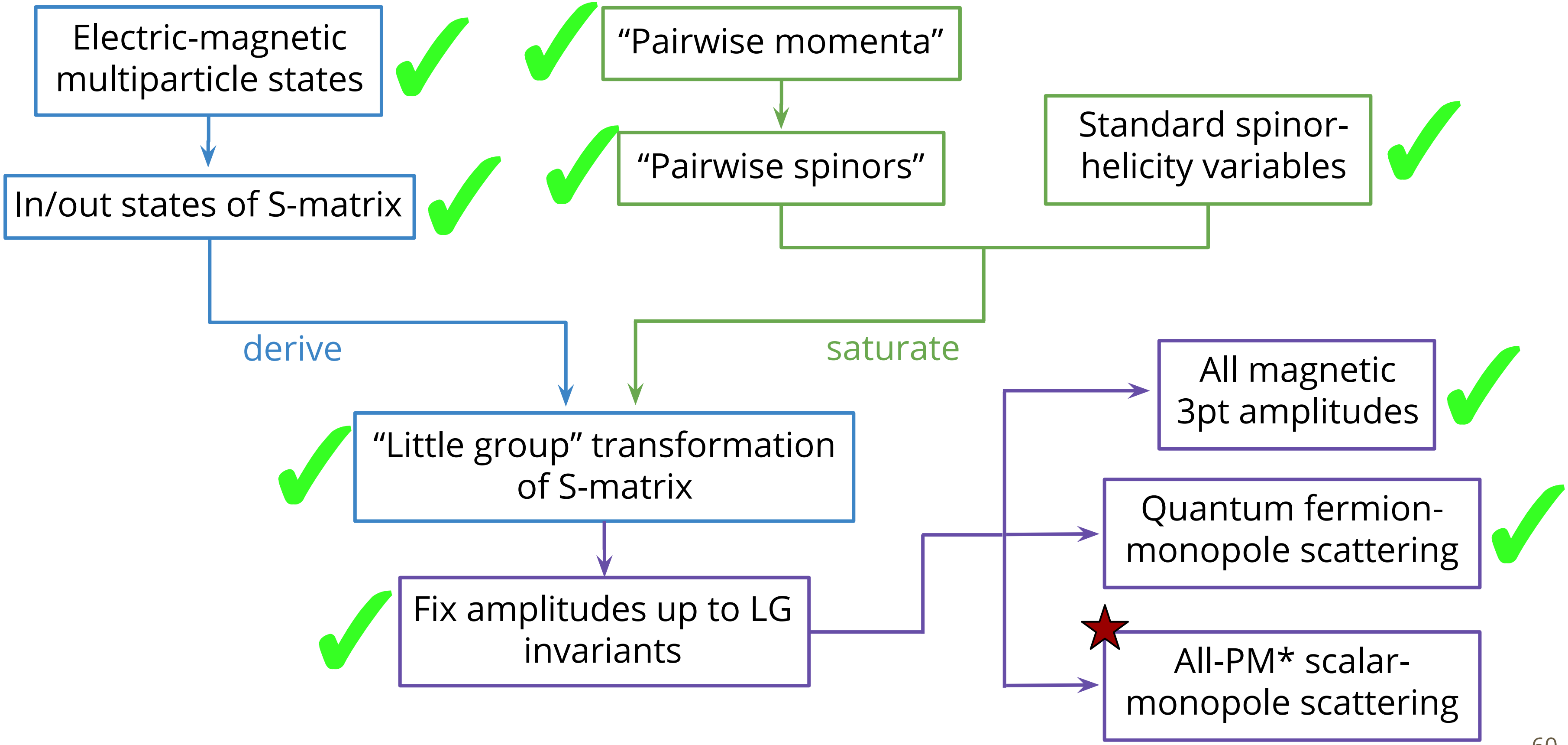
3. At the lowest J=|q|-½, the D-matrix vanishes unless

    - For q<0 : σ=σ′=+ ⟶ RH f going to LH f ′
    - For q>0 : σ=σ′=- ⟶ LH f going to RH f ′

Converting from the "All outgoing" convention

Helicity flip emerges from pairwise LG selection rule ✓

Talk Flowchart



* All orders in (q/J)

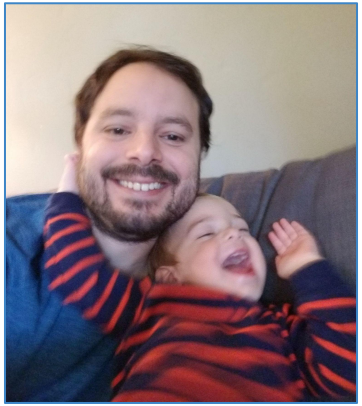
Conclusions

- Solved the problem of constructing Lorentz covariant electric-magnetic amplitudes
- Identified electric-magnetic multiparticle states that are not direct products
- Defined the pairwise LG, helicity and spinor-helicity variables
- Fixed all 3-pt amplitudes
- Fixed all angular dependence of $2 \to 2$ scattering and reproduced lowest PW helicity-flip

Future Directions

- The quantum dyon - Taub-NUT double copy: out soon!
- Generalization to celestial amplitudes Lippstreu '21
- Supersymmetrize! Decay of ¼-BPS dyons in N=4 SYM, walls of marginal stability in N=2 ?
- Massless mutually non-local particles: what's the S-matrix at an Argyres-Douglas point?
- Higher dimensions: D-brane magnetic scattering
- On-shell derivation of the Rubakov-Callan effect

Thank You!



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